Understanding Compressible Turbulence with Time Varying Random Forcing by Low Dissipative High Order Schemes

White Paper for DOE/ASCR Workshop on Turbulent-Flow Simulations at the Exascale

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Problem and Challenges Over the past few decades direct numerical simulations (DNS) of turbulent flows played an ever increasing role in pushing the edge in numerical versus experimental comparisons.[1] The Reynolds numbers achieved in the largest DNS are of the order 10^{6-7}, i.e., comparable to the largest Reynolds numbers in laboratory experiments. DNS became a vital tool for addressing basic questions, such as universality in the sense of Kolmogorov's theory, and the dependence of the statistics on the Reynolds number and scale. Extreme-resolution DNS will provide high-quality data that are impossible to obtain experimentally. Such DNS are essential to understand intermittency and develop predictive large eddy simulations (LES).

It is well known that introduction of even modest compressibility is sufficient to generate small scales through wave steepening, enabling disparate-scale interaction.[2] Besides purely engineering applications, compressible turbulence plays an very important role in a broad range of applications in astrophysics and geophysics (galaxy formation, star formation, supernova explosions, cosmic ray acceleration, black hole accretion, planet formation, solar wind heating, etc.), and in atmospheric sciences (weather prediction and climate modeling). Each of these subjects can potentially receive a strong transformative impact from compressible DNS/LES at the exascale. Recent advances in theory of compressible turbulence have provided important reference points for validation of numerical solutions, ensuring that the flops would not be wasted.[3]

Numerical simulation is challenging due to the conflicting requirements for numerical methods to be accurate enough to resolve the small scales of turbulence but robust enough to handle shock waves without generating spurious numerical noise.

Approach and Objective Recently, very-high-order (8th and higher) very low-dissipation shock-capturing finite difference schemes were developed. Naturally, these are more expensive in terms of flops, but they also deliver a much better spectral bandwidth, which fully compensates for the higher flop load. Overall, these methods are substantially more efficient for modeling high-Re compressible fluid flows in 3D compared to the traditional Godunov-based schemes. Further work is needed, however, to extend the range of turbulent Mach numbers at which they are numerically stable. Another important advantage of this family of methods for compressible fluid dynamics is that their parallel performance does not suffer from communication bottleneck due to multidimensional FFTs at very high number of CPUs. The development of the method includes an adaptive flow sensor with automatic selection on the amount of numerical dissipation needed at each flow location for more accurate DNS and LES simulations with less tuning of parameters for flows with time varying random forcing and a wide range of flow speed regime during time-accurate evolution. An automatic selection of the different flow sensors catered to the different flow types is constructed. A Mach

[1] T. Ishihara, T. Gotoh, and Y. Kaneda (2009), Study of HighReynolds Number Isotropic Turbulence by Direct Numerical Simulation, *Annu. Rev. Fluid Mech.* 41:16580. R. Benzi, L. Biferale (2015), Homogeneous and Isotropic Turbulence: A Short Survey on Recent Developments, *J. Stat. Phys.* in press.

[2] P. H. Diamond, S.-I. Itoh and K. Itoh (2010), Modern Plasma Physics Vol. 1: Physical Kinetics of Turbulent Plasmas, p. 362.

[3] H. Aluie (2013), Scale decomposition in compressible turbulence, *Physica D* **247**, 54-65. S. Galtier and S. Banerjee (2011), Exact Relation for Correlation Functions in Compressible Isothermal Turbulence, *Phys. Rev. Lett.* **107**, 134501. S. Banerjee and S. Galtier (2013), Exact Relation with Two-point Correlation Functions and Phenomenological Approach for Compressible Magnetohydrodynamic Turbulence, *Phys. Rev. E* **87**, 013019. A.G. Kritsuk, R. Wagner and M.L. Norman (2013), Energy Cascade and Scaling in Supersonic Isothermal Turbulence, *J. Fluid Mech.* **729**, R1.

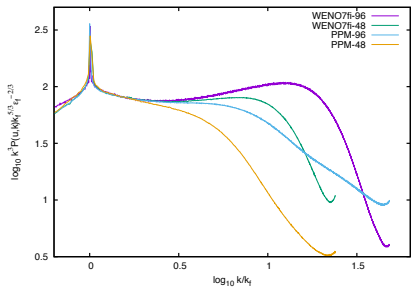

Figure 1: Compensated velocity power spectra from four simulations of 2D compressible turbulence with the PPM and WENO7fi methods. Only scales below the forcing scale λ_f resolved with 48 or 96 grid points are shown.

curve and high-frequency oscillation indicators are used to reduce the tuning of parameters in controlling the amount of shock-capturing numerical dissipation to be employed for shock-free turbulence, low speed turbulence and turbulence with strong shocks. These low dissipative high order methods will be used to address basic questions, such as universality in the sense of Kolmogorov's theory, and the dependence of the statistics on the Reynolds number and scale. Such DNS are essential to understand intermittency and develop predictive LES.

Preliminary Results and Significance Figures shown consist of a doubly periodic square with the flow fully determined by three parameters (grid resolution N, energy injection rate ε_f, and energy injection scale λ_f). The solenoidal forcing is δ-correlated in time. Comparison of a 3rd-order scheme PPM with our 7th-order scheme WENO7fi based on grid resolutions from N=2.048 to 16,384 indicates that the spectral bandwidth of WENO7fi is slightly more than two times larger than that of PPM. This means that the cost of a 2D simulation with WENO7fi, resolving the turbulence to a given degree, is approximately 4 times cheaper computationally than with PPM. For a 3D simulation the advantage of WENO7fi over PPM would be even more impressive. The key significance of the adaptive flow sensor in high order methods is that no a

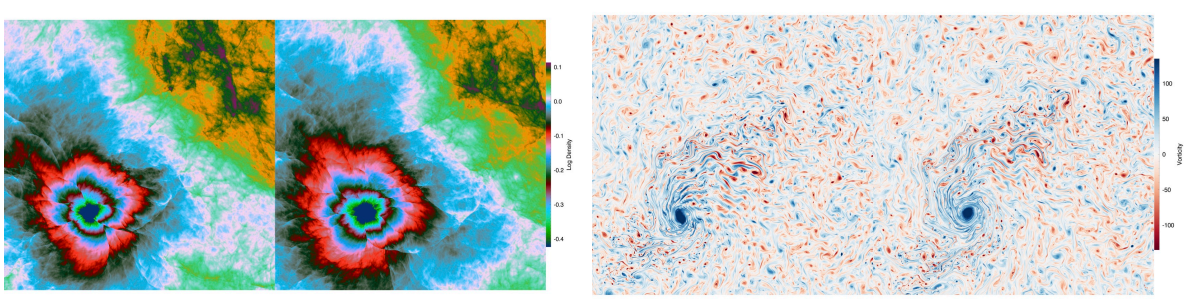

Figure 2: Density (left 2 subfigures) and vorticity (right 2 subfigures) fields for sub-volumes of PPM (left figure) and WENO7fi (right figure) simulations respectively resolving the forcing scale with 96 grid points.

priori knowledge of the flow structure of the entire evolution is needed, even for a mixture of compressible shock-free turbulence, low speed turbulence with shocklets and turbulence with strong shocks during the time evolution process. The proposed developments provide an improved predictability and reliability of CFD turbulent computations containing both low speed and high speed regimes that can be compromised by standard high order shock-capturing schemes without a proper numerical dissipation control. With the advent of exascale computing and the proper CFD tools, improved understanding of compressible turbulence with a wide range of shock speeds containing time varying random forcing will soon be within our reach.