Control of Laser-Plasma Instabilities in High Energy Density Plasmas Using Sub-Picosecond, High-Contrast, Temporal Modulations and Spatial Speckle-Pattern Scrambling: STUD Pulses to the Rescue

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Outline and Summary

• What are STUD pulses? Spike Trains of Uneven Duration and Delay
• What is the STUD pulse program? Ferrari, Mercedes, Prius versions
• The experimental evidence for gross LPI anomalies up and down the line
• How did things get so bad on NIF? > 30% energy into LPI ~ 600 kJ.
• Compare USB (Plane Wave) to RPP/SSD to STUD pulses.
• Key ideas: Plasmas tend to self-organize if you imbue them with coherent energy over long periods of time: Simple growth becomes catastrophic growth
• Fool the plasma into not forming memories of continuous pumping. Kill memory accumulation, heal previous growth spurts, spread a little growth everywhere – so not in static or very slowly varying hot spots. Scramble the beam spatial profile and turn the laser beam on and off: The Ferrari STUD pulse program
• Intersperse crossing beams by synchrony and choose which interact and scramble each other’s temporal coherence due to strong coupling SBS: The Mercedes program
• In the foot of the pulse, intersperse spikes in time between crossing beams and avoid multi-beam driven instabilities: The Prius program
• Show feasibility of all three on Jupiter & Z Beamlet with flexible STUD Pulses @ 2ω.
LPI History: “Decisions and Revisions which a Minute Will Reverse”

• Laser plasma instabilities (LPI) is a domain of study in the hands of plasma physicists. Not optics, laser physics, or nonlinear optics experts. Cultural divide.
• The two fields grew together in the early 60s as lasers developed.
• Plasma physicists started wondering what radio wave heating of the ionosphere and laser heating of pellets (plasma absorption) would look like. They made a lot of mistakes and got really excited about the prospect of externally controllable and tight focusable rapid heating. In each of the 3 patents of the laser, fusion by laser heating is mentioned as a key application. They had NO IDEA at the time just how it could happen.
• The field of nonlinear optics of plasmas got into full gear by 1972 and Rosenbluth’s seminal PRLs saying nonlinear wave-wave interactions will give rise to amplifiers and oscillators that may kill you. The modeling of the laser was trivialized and plasma folks used this new ICF/IFE prospect to work on plasma models with trivial laser models searching for magic tricks like staying below threshold (ill defined and impossible in practice) in paper after paper for decades!
• When laser technology changed, no reflection of same penetrated into the plasma community in earnest. They were busy with their toy models and solving no pressing problem like 30% reflectivity of 2 MJ on NIF. NIF kept shooting…
Hohlraums Contain Plasmas at Different Conditions \((n_e, T_e, u, f_e(v))\) in Different locations at Different Times, Made of He, Be, CH, C, Ne, SiO\(_2\), U, Au, …

NIF has \(P_{\text{drive}} > 100\) MB, and has achieved \(P_{\text{Stag}} > 150 – 200\) GB but needs \(P_{\text{Stag}} > 300\) GB to ignite at < 2 MJ.

A NIF Hohlraum is a Laser-Plasma Instability (LPI) Candy Store

Non of the plasma conditions or interaction modalities that are manifest on the NIF were ever accessed on Nova or Omega.

It’s all different and yet It remains weakly characterized, weakly diagnosed, and studied only in passing.
Adequate Stringent Control of Laser-Plasma Instabilities Is Required to Achieve Indirect or Direct Drive Ignition

• Energy coupling should be > 90% to achieve high enough $T_{\text{rad}}$

• Implosion symmetry requires controlled power balance between “inner” and “outer” beams. Soft X-ray flux on equator vs. poles in space and time must be maintained.

• Must have low capsule preheat ($T_{\text{hot}}, f_{\text{hot}}$)

• Must control SBS, SRS, $2\omega_p$ & filamentation, cross-beam energy transfer (CBET), hot electron and hard X-ray (M Band) proliferation.
What Does a Laser’s Electric Field Look Like? What Are its Degrees of Freedom?

- Polarization
- Amplitude
- Wavelength
- Frequency
- Phase

$$E_0(x,t) = \frac{1}{2} \sum_i \hat{e}_i a_{0,\text{slow},i}(x,t) \times \exp(k_i \cdot x - \omega_i(t) t + \phi_i) + c.c.$$ 

If you do not like too much coherence, what are you better off modulating?

What is most likely to disrupt resonant 3 wave instability growth?

Slight changes in frequency? Phase modulations? Polarization changes?

No. The only truly effective way is via turning the amplitude of the pump wave on and off on the instability growth time scale.

What do you mean off? Contrast of 100 or better should do the trick.
More is Different in x & t: Plasmas Self-Organize in the Presence of Coherent Energy Injection

**SHS**
- Long Time
- Montgomery 2002

**SHS**
- Short Time
- Kline 2007

**RPP Beam**
- Long Time
- Montgomery 1998

**RPP**
- Shorter Time
- Baldis 1993

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Classical steady-state SRS gain

\[ R_{SRS} = \frac{I_{SRS}}{I} = \left( \frac{l_n}{l} \right) e^G \]

\[ G \sim I \cdot L / \nu_{EPW} \]

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What is the Solution to the LPI Problem Caused by Laser Hot Spots?

- The combined application of nonlinear optics (NLO), statistical optics (SO) and ultrashort pulse (USP) or ultrafast optics (UFO) together with judicious plasma choices will lead to the control LPI at will. This is non-trivial and not strictly plasma centric.

- You really have to understand speckle patterns, Gaussian random fields, multiscale wave-wave interaction dynamics and invent ways of modulating lasers on a sub-ps time scale & create diagnostics with large time-bandwidth products, synchronize overlapping laser beams, and then you can attempt LPI control at will.
The Experimental Program

• **Single Hot Spot Experiments on Jupiter** (with different \( f/\#s \)), going beyond Montgomery/Kline/Rousseau results. Measure via OUFTS and backscattered light with sub-ps time resolution but with a long record length (100s of ps) how plasmas self-organize in SBS rich and SRS rich conditions. Various regimes, with a without FIL, SCR, SDL, WDL, WCR, large \( k\lambda_D \), small \( k\lambda_D \), etc.

• **Witness and Capture plasma Self-Organization in action**: *More is different.* Wait long enough and/or have a large enough plasma and reductionism becomes wishful thinking. 1-2 ps teach you very little. Look beyond single pass gain.

• **Introduce STUD pulses** into the mix. Solve the excesses caused by RPP/SSD/intensity tails beyond the average intensity toy model (plane wave) picture. Jupiter, Z-Beamlet, Omega + Omega EP, **Enable \( 2\omega \) NIF**

• Start with the rattle plate method of generating inflexible STUD pulses as pioneered by LANL (Randy Johnson and David Montgomery). Shake down Jupiter’s readiness for such expt’s. Fast diagnostics, etc.

• **Move to SPA ACE flexible STUD pulse generation + CBET Control**
Theoretical and Computational Program

• Study single hot spot long time behavior with a multitude of models from fluid (pf3d) to Vlasov. Long time means avoid PIC categorically.

• Include multitude of waves and memory accumulation after 10’s to 100’s of SHS traversal times. Study both SRS and SBS in different regimes.

• Demonstrate conditions needed for self-organization beyond the single pass gain picture. More is different? Prove it!

• Move on to large beams in 2D and 3D. Establish how much self-organization is possible in situ over long times. Now compare to large spatial self-organization and re-amplification scenarios. Work with pf3d. Add STUD pulses.

• Extract the coherent overlap fractions of re-amplification for in situ or distributed settings. The “ε”s in the formulas below.

• Concentrate on SCR, WDL of SBS for the Mercedes program: SiO₂. Compare to He, CH, Ne. Add STUD pulses (w and w/o).
Simple Model Shows Differences btw Characteristics of WCR-WDL, WCR-SDL & SCR-WDL: Role of $f_{dc}$ in Fixed $T_{pulse}$ STUD Pulses

- In one $\Delta t$ interval with an `on' spike followed by off time, dictated by the duty cycle, $f_{dc}$, the statistically averaged power gain over a full beam can be estimated by this integral:

$$
\left< \exp\left[ 2G(1/\beta) \right] \right>_{\Delta t} = \exp\left[ -2\nu (1 - f_{dc}) \Delta t \right] \times \int_0^{NI_0} \exp\left[ 2 \alpha_{1/\beta} \left( f_{dc} \Delta t \right) I^{1/\beta} \right] \exp\left[-\frac{I}{I_0} \right] \frac{dI}{I_0}
$$

$$
\bar{\alpha} = 2 \alpha_{1/\beta} \Delta t I_{00}^{1/\beta} f_{dc}^{(1-1/\beta)} < 1
$$

$$
\mathcal{E} = I_{00} \Delta t = I_0 \Delta t_{on}
$$

$$
\left< \exp\left[ 2G(1/\beta) \right] \right>_{\Delta t} = \exp\left[ -2\nu (1 - f_{dc}) \Delta t \right] \times \int_0^N \exp\left[ \bar{\alpha} I^{1/\beta} \right] \exp[-I] \, dI
$$

$$
\left< \exp\left[ 2G(1/\beta) \right] \right>_{\Delta t} = \exp\left[ -2\nu (1 - f_{dc}) \Delta t \right] \times \sum_{n=0}^{\infty} \left( \frac{\bar{\alpha}^n}{n!} \right) \gamma \left( \frac{n}{\beta} + 1; N \right)
$$

- In SDL ($\beta=1$), no $f_{dc}$ dependence in temporal gain: Reduce Gain in WDL ($\beta=2$) and for SCR ($\beta=3$).
- $N_{spikes}$ consecutive spikes with HS scrambled give a total reflectivity as an incoherent sum. $x N_{spikes}$
- Otherwise, $\Delta t$ increases in the exponent and eventually the integral blows up, signaling NL and memory filled behavior.

$$
\left< \exp\left[ 2G(1) \right] \right>_{\Delta t} = \exp\left[ -2\nu (1 - f_{dc}) \Delta t \right] \times \frac{1 - \exp\left[ -\left(1 - \bar{\alpha} \right) N \right]}{1 - \bar{\alpha}}
$$
A Theoretical Model Captures These Anomalies Showing How STUD Pulses Lead to Taming and Control

Memory buildup in time and in space → More is different.

Triggers: Out of equilibrium kinetic effects, nonlinearity, absolute instability, localized modes, feedback loops

Fool the plasma and dissuade it from expecting same or hotter hot spots coming to the same place. Stop memory build-up, heal, democratize or homogenize gain in (x, t). TAME instabilities.

STUD pulses: \( l_{\text{snip}} = \frac{1}{2} \) makes G smaller. \( n_{\text{scram}} = 1 \) tames fastest. \( f_{dc} \) helps stack/intersperse beams and heal via damping between growth spurts.
Where Can STUD Pulse Design Do the Most Good in Reducing Reflectivity?
What Does Memory & Gain-Guided Growth Cones in Single (Diffraction Limited) Hot Spot SDL SBS Look Like?
What Do Memory, Self-Organization and Re-Amplification Look Like in SDL SBS for RPP?

**Pump Intensity**

**Backscattered Light Intensity**

**Density Fluctuations of IAWs**
Hot Spot Detection in action: for $G=2.14 \ \frac{dn}{n} =$, $t = 340$
Hot Spot Detection in action:
for G=2.14 \( \frac{dn}{n} = \), \( t = 340 \)
Statistical connections between amplification processes in various hot spots are captured via correlation functions in \((l, m, n, o, t)\) space

The **Complex Degree of Coherence (CDC)** of a complex field is the normalized version of the **Mutual Coherence Function (MCF)**. Here quantized onto an ordered hot spot lattice \((l, m)\)

\[
\overline{\rho}_{E_0, S} \left(l, m, l', m'; t \right) = \frac{\int_{\tau_{min}}^{\tau_{max}} E_{0,S} \left(l, m, \tau \right) E^*_{0,S} \left(l', m', t + \tau \right) d\tau}{\sqrt{\int_{\tau_{min}}^{\tau_{max}} \left| E_{0,S} \left(l, m, \tau \right) \right|^2 d\tau \int_{\tau_{min}}^{\tau_{max}} \left| E_{0,S} \left(l', m', \tau \right) \right|^2 d\tau}}
\]

Movies of MCF and CDC indicate whether initial pass tranches and final pass tranches evolve entirely differently negating the precepts that make up the universally help view that the independent hot spot model applies when in fact, many hot spots can reamplified previous gain sites in an RPP beam and reek havoc. (Think NIF/NIC)

This is what the STUD pulse program reverses.
Easy to see these correlations via two methods \((l, m, t)\) and \((l, n, t)\): \(I_0\) for \(G_{MNR} = 2.14\)
Easy to see these correlations via two methods \((l, m, t)\) and \((l, n, t)\): \(I_S\) for \(G_{MNR} = 2.14\)
Easy to see these correlations via two methods $(l, m, t)$ and $(l, n, t)$: $\delta n/n$ for $G_{\text{MNR}} = 2.14$
Mutual Coherence Function for $I_S$ for $G_{MNR} = 2.14$ and $\text{dn/n} \ 1e-9$
STUD Pulses Are Described by Specifying the Duty Cycle, $f_{dc\%}$, the Spatial Scrambling Rate, $n_{scram}$, and the ratio between three length scales: $L_{HS}$, $L_{INT}$ and $L_{spike}$.

$$l_{int} = \frac{L_{int}}{L_{HS}} \text{ and } l_{snip} = \frac{L_{spike}}{L_{HS}}.$$ 

Typical examples (to get used to the notation) are:

STUD 5010x1, 1:1:1/2. $l_{int} = 1$, $l_{snip} = 1/2$

STUD 2010x2, 1:1:1/4. $l_{int} = 1$, $l_{snip} = 1/4$

STUD 3010x1, 1:1:1/3. $l_{int} = 1$, $l_{snip} = 1/3$

STUD 6010x2, 1:2:1. $l_{int} = 2$, $l_{snip} = 1$

STUD 8010x1, 1:1:$\infty$. $l_{int} = 1$, $l_{snip} = \infty$
What Do STUD Pulses Look Like?

5010x∞ 1:1:1/2

Harmony Simulations

5010x1 1:1:1

STUD Pulses Program
SSAP 2-21-2018
A Comparison of $I_{\text{Pump}}$, $I_{\text{SBS}}$, $\delta n/n|_{\text{IAW}}$ for $\omega_{\text{IAW}} t = 20$

5010x16 1:1:1, 5010x16 1:1:1/2, 5010x8 1:1:1 & 5010x8 1:1:1/2. Notice Memory Loss, Democratization, Taming
A Comparison of $I_{\text{Pump}}, I_{\text{SBS}}, \delta n/n|_{\text{IAW}}$ for $\omega|_{\text{IAW}} t = 20$
for 5010x8 1:1:1, 5010x8 1:1:1/2, 5010x4 1:1:1 & 5010x4 1:1:1/2. Notice Memory Loss, Democratization, Taming.
A Comparison of $I_{\text{Pump}}$, $I_{\text{SBS}}$, $\delta n/n\big|_{\text{IAW}}$ for $\omega_{\text{IAW}} t = 20$ for 5010x4 1:1:1, 5010x4 1:1:1/2, 5010x2 1:1:1 & 5010x2 1:1:1/2. Notice Memory Loss, Democratization, Taming
A Comparison of $I_{\text{Pump}}$, $I_{\text{SBS}}$, $\delta n/n|_{\text{IAW}}$ for $\omega_{\text{IAW}}$ $t = 20$
for 5010x2 1:1:1, 5010x2 1:1:1/2, 5010x1 1:1:1 & 5010x1 1:1:1/2. Notice Memory Loss, Democratization, Taming.
A Comparison of $I_{\text{Pump}}$, $I_{\text{SBS}}$, $\delta n/n|_{\text{IAW}}$ at 4 times:

$\omega_{\text{IAW}} \ t = 10, 20, 40, 60$, for 5010x8 1:1:1
A Comparison of $I_{\text{Pump}}, I_{\text{SBS}}, \delta n/n|_{\text{IAW}}$ at 3 times: $\omega_{\text{IAW}} t = 10, 20, 40$, for $5010 \times 8$ 1:1:1/2
A Good Design Space for $T_{\text{pulse}}$ Fixed STUD Pulses
Is $2010-5010 \times 1, 1:1:1/2$ down to $1:1:1/4$
VPIC Simulations Demonstrate Strong SRBS Reduction Using STUD Pulses in NL Kinetic Regime
Jupiter Layout for Crossed Beam STUD Pulse Expts: Observing Weak to Strong Coupling Brillouin Interactions Directly via IAW Frequency Shifts via TS and Indirectly via FABS/TBD

Two interaction beams
527-nm, 700-ps, f/6 or f/12 with RPP
STUD or continuous

Backscatter energy streaked spectrum
(SBS, SRS)

Backscatter energy streaked spectrum
(SBS, SRS)

transmitted energy streaked imaging
(40x mag)
~ ps time resolution.

mm-scale
N\textsubscript{2} or N\textsubscript{2}H\textsubscript{2},
Ne, Ar
gas jet plasma

SiO\textsubscript{2} Aerogel
Foam
self-Thomson scattering (T\textsubscript{e}, n\textsubscript{e})

f/6
collection lens

Compare Random to Deterministic STUD Pulse sequences
Questions to Be Answered in Jupiter STUD Pulse Program, Proof of Principle Experiments

• **Prius Program:** Cross 2 STUD pulsed beams and monitor LPI driven by each as they overlap in time deterministically and randomly with narrow spikes and wider one from 1 to 20 ps in duration, with different off times testing 20% duty cycle to 50%. Do this in CH.

• **Mercedes Program:** Move to SiO$_2$ and the SCR of CBET and SBS. Directly measure the frequency shifts of IAW in the SCR near Mach -1 or not, in hot spots or not. Vary STUD pulse characteristics as above. Correlate LPI scattered light signatures to direct OUFTS.

• **In the Mercedes Program,** work with one STUD pulse beam alone, then one STUS pulse beam and one continuous beam crossing, and then two STUD pulse beams partially overlapping vs fully overlapping.

• **Examine scaling with f/# and RPP choices.** Compare to continuous beam behavior. Vary plasma composition and conditions.
How Would Findings on Jupiter Scale to NIF? (via Z-Beamlet or Ω-EP Blue)

• For NIF need single beam remediation in the foot (2ωₚ, SRS) and the peak (SRS, 2ωₚ, SBS) as well as multi-beam remediation (SRS, SBS, CBET, 2ωₚ) in the foot and the peak.

• These are entirely different plasma conditions (all unknown) whether low gas fill or high gas fill or intermediate gas fill.

• Confidence in random crossing fraction scaling of LPI and superiority with continuous beams overlapping will be established and should scale. It will be easy to test subsequently on NIF with a few quads equipped with STUD pulses.

• Deterministic synchronization solutions can be intimately coupled to plasma conditions which are very different between Trident and NIF.

• One step is to go to Omega EP Blue/Green Beams brought over to Omega and equipped with STUD pulses to do LPI experiments on Omega with STUD pulses en attendant NIF. Random synchronization is easiest scenario to test on Omega EP + Omega and for DD applications.
SBS & CBET Represent a Specific Example Where We Can Derive Such Equations

\[
\left( \frac{\partial}{\partial t} + \mathbf{u}(x) \cdot \nabla + \nu_{IAW} \right) \left( \frac{\partial}{\partial t} + \mathbf{u}(x) \cdot \nabla \right) - c_s^2 \nabla^2 \right) \frac{\delta n_e^{(n)}}{n_0} = \left( \frac{Z m_e}{M_I} \right) \nabla^2 \left( \mathbf{u}_0^{(n)} \cdot \mathbf{u}_s^{(n)} \right) + \left( c_s k_{IAW} \right)^2 \frac{\delta n_e^{(n-1)}}{n_0}
\]

\[
\frac{\partial^2}{\partial t^2} - c_s^2 \nabla^2 + \omega_p^2(x) \left( 1 - \frac{\nu_s}{\omega_s} \right) \right) \mathbf{u}_s^{(n)} = -\omega_p^2 \frac{\delta n_e^{(n)}}{n_0} \mathbf{u}_0^{(n)} + \omega_s^2 \mathbf{u}_s^{(n-1)}
\]

\[
\frac{\partial^2}{\partial t^2} - c_s^2 \nabla^2 + \omega_p^2(x) \left( 1 - \frac{\nu_0}{\omega_0} \right) \right) \mathbf{u}_0^{(n)} = -\omega_p^2 \frac{\delta n_e^{(n)}}{n_0} \mathbf{u}_s^{(n)} + \omega_0^2 \mathbf{u}_0^{(n-1)}
\]

Use electromagnetic units and extract the eikonal “fast” variables to leave the interaction physics living atop travelling waves.

\[
\frac{\delta n_e^{(n)}}{n_0}(\mathbf{x},t) = \alpha \frac{\delta \tilde{n}_e^{*}(n)}{n_0}(\mathbf{x},t) \exp \left[ -i \left( \int \mathbf{k}_{IAW} \cdot d\mathbf{x} - \omega_{IAW} t \right) \right]
\]

\[
\mathbf{u}_s^{(n)}(\mathbf{x},t) = \beta \mathbf{\hat{u}}_0 \tilde{\mathbf{u}}_s^{(n)}(\mathbf{x},t) \exp \left[ i \left( \int \mathbf{k}_s \cdot d\mathbf{x} - \omega_s t \right) \right]
\]

\[
\mathbf{u}_0^{(n)}(\mathbf{x},t) = \gamma \mathbf{\hat{u}}_0 \frac{\tilde{\mathbf{u}}_0^{(n)}}{2}(\mathbf{x},t) \exp \left[ i \left( \int \mathbf{k}_0 \cdot \mathbf{x} - \omega_0 t \right) \right] + cc.
\]
Including All Regimes in IAW Response for SBS Evolution in Inhomogeneous Flowing Plasma

\[
\left[ \left( \frac{\partial}{\partial t} + \frac{\vec{V}_{IAW}(\mathbf{x})}{2} + \vec{c}_s \left[ \mathbf{M}(\mathbf{x}) + \hat{\mathbf{k}}_{IAW} \right] \cdot \nabla \right) - \frac{i}{2\vec{c}_s} \left[ \left( \frac{\partial}{\partial t} + \frac{c_s}{c} \mathbf{M}(\mathbf{x}) \cdot \nabla \right)^2 - c_s^2 \nabla^2 \right] \right] \delta \tilde{n}_e^{(n)}(n) - \frac{i \gamma_0 e^{i \delta \omega t_s(n)} - i \vec{c}_s \delta \tilde{n}_e^{(n-1)}}{n_0} = \frac{1}{2} \mathbf{V}_s \cdot \nabla \tilde{u}_s^{(n)} - \frac{i \omega_s}{2} \tilde{u}_s^{(n-1)}
\]

\[
\vec{V}_{IAW}(\mathbf{x}) = \left[ \left( \frac{\vec{V}_{IAW}}{c_s k_{IAW}} \right) \vec{c}_s - 2i \vec{c}_s \left[ (\mathbf{x} - \mathbf{x}_0) \cdot \nabla \mathbf{M}(\mathbf{x}) \right] \hat{\mathbf{k}}_{IAW} \right]
\]

\[
\vec{V}_s(\mathbf{x}) = \frac{\omega_s^2}{\omega_0^2} \left[ \left( \frac{\vec{V}_s}{\omega_0} \right) + 2i \left( (\mathbf{x} - \mathbf{x}_0) \cdot \nabla \ln(\omega_0^2(\mathbf{x})) \right) \right]
\]

\[
\gamma_{WCR-WDL} = \left[ \gamma_0^2 - \frac{1}{2} \left( \frac{\vec{V}_{IAW}}{2 c_s k_{IAW}} \right) \left( \frac{\vec{V}_s}{2 \omega_s} - \frac{\delta \omega}{4} \right)^{-1} \right] - \left( \frac{\vec{V}_{IAW}}{2 c_s k_{IAW}} \right) - \left( \frac{\vec{V}_s}{2 \omega_s} \right)
\]

\[
\gamma_{WCR-SDL} = \left[ \gamma_0^2 \left( \frac{\vec{V}_{IAW}}{2 c_s k_{IAW}} \right) \right] - \left( \frac{\vec{V}_s}{2 \omega_s} \right)
\]

\[
\gamma_{SCR-WDL} = \cos \left( \frac{\pi}{6} \right) \left[ \vec{c}_s \left[ \gamma_0^2 - \frac{1}{2} \left( \frac{\vec{V}_{IAW}}{2 c_s k_{IAW}} \right) \left( \frac{\vec{V}_s}{2 \omega_s} - \frac{\delta \omega}{4} \right) \right] \right]^{1/3}
\]

\[
\frac{\gamma_0^2}{\omega_0^2} = \frac{1}{8} \left( \frac{Z m_e}{M_i} \right) \left( \frac{u_0}{c} \right)^2 \left( \frac{\omega_p^2}{\omega_0^2} \right) \left( \frac{k_{IAW}}{k_0} \right)^2 \left( \frac{c_s/c}{k_{IAW}/k_0} \right)
\]

\[
\frac{\gamma_0^2}{\omega_0^2} = 4.1835 \times 10^{-8} \left( \frac{2 Z}{A} \right)^{1/2} \left[ \frac{n_e/n_i}{0.1} \right] \left[ 2 \sin(\theta_s/2) \right] \frac{I_{14,W/cm^2} \lambda_{0.351 \mu m}^2}{\sqrt{T_{e,keV}}}
\]

\[
\left( \frac{u_0}{c} \right)^2 = 8.9712 \times 10^{-6} I_{14,W/cm^2} \lambda_{0.351 \mu m}^2
\]

\[
\left( \frac{c_s}{c} \right) = 7.3 \times 10^{-4} \left( \frac{2 Z}{A} \right)^{1/2} \sqrt{T_{e,keV}}
\]

\[
\vec{c}_s = \left[ (c_s/c)(k_{IAW}/k_0) \right] = 7.3 \times 10^{-4} \left( \frac{2 Z}{A} \right)^{1/2} \sqrt{T_{e,keV}} \left[ 2 \sin(\theta_s/2) \right]
\]

\[
\omega_0 \Delta t_{ps} = \frac{5.3702 \times 10^3}{\lambda_{0.351 \mu m}} \Delta t_{ps}
\]

STUD Pulses Program
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WKB Solution of the Steady State MNR Model for Convective Gain from WDL to SDL: $G_{\text{MNR,MAX}}$ and $L_{\text{INT}}$

\[ L_1 a_1 = \left( v_1 + V_1 \frac{\partial}{\partial z} \right) a_1 = \gamma_0 a_2^* \exp[i \Delta \phi] \]

\[ L_2 a_2^* = \left( v_2 + V_2 \frac{\partial}{\partial z} \right) a_2^* = \gamma_0^* a_1 \exp[-i \Delta \phi] \]

\[ \Delta \phi(z) = \int [k_0 - k_1 - k_2] dz' \approx \kappa'_{z=z_{\text{PPMP}}} (z - z_{\text{PPMP}}) \]

\[ \begin{bmatrix} L_2 - iV_2 \left( \frac{\partial \Delta \phi}{\partial z} \right) \end{bmatrix} L_1 a_1 - |\gamma_0|^2 a_1 = 0 \]

\[ \begin{bmatrix} L_1 + iV_1 \left( \frac{\partial \Delta \phi}{\partial z} \right) \end{bmatrix} L_2 a_2^* - |\gamma_0|^2 a_2^* = 0 \]

\[ \xi = \langle K \rangle (z - z_{\text{PPMP}}); \quad \bar{V}_{1,2} = \langle K \rangle V_{1,2}; \quad \bar{V}_{1,2} = \frac{V_{1,2}}{\bar{V}_{1,2}}; \quad \bar{\kappa}' = \frac{\kappa'}{\langle K \rangle^2} \]

\[ |\bar{\gamma}_0|^2 = \frac{\gamma_0^2}{\bar{V}_1 \bar{V}_2}; \quad |\bar{\gamma}_0|^2 = |\bar{\gamma}_0|^2 \left( 1 - \frac{\bar{V}_1 \bar{V}_2}{|\bar{\gamma}_0|^2} \right) \]

\[ G_{\text{MNR,MAX}} = \frac{2\pi |\gamma_0|^2 \left( 1 - \frac{v_1 V_2}{|\gamma_0|^2} \right)}{\left( \frac{\partial \Delta \phi}{\partial z} \right)|V_1 V_2|} \equiv 2\langle K \rangle L_{\text{INT}}; \quad \bar{v}_+ = (\bar{v}_1 + \bar{v}_2); \quad \bar{v}_+ = \bar{v}_+ - i \left( \frac{\partial \Delta \phi}{\partial \xi} \right) \]

\[ \begin{bmatrix} \frac{\partial^2}{\partial \xi^2} + \bar{v}_+ \frac{\partial}{\partial \xi} + \bar{v}_1 (\bar{v}_+ - \bar{v}_1) - |\bar{\gamma}_0|^2 \end{bmatrix} a_1 = 0; \quad a_1 = \psi \exp \left[ -\frac{1}{2} \int \bar{v}_+ d\xi' \right] \]
(2 of 3) WKB Solution of the Steady State MNR Model for Convective Gain from WDL to SDL: $G_{\text{MNR,MAX}}$ and $L_{\text{INT}}$

\[
\left[ \frac{\partial^2}{\partial \xi^2} + \bar{V}_+ \frac{\partial}{\partial \xi} + \bar{V}_1 \left( \bar{V}_+ - \bar{V}_1 \right) - |\bar{\gamma}_0|^2 \right] a_1 = 0; \quad a_1 = \psi \exp \left[ -\frac{1}{2} \int \bar{V}_+ d \xi' \right]
\]

\[
\psi'' - \left[ |\bar{\gamma}_0|^2 - \bar{V}_1 \left( \bar{V}_+ - \bar{V}_1 \right) + \frac{\bar{V}_+^2}{4} + \frac{\bar{V}_+'}{2} \right] \psi = 0
\]

\[
\psi'' - Q(\xi) \psi = 0; \quad a_1 \sim \exp \left[ \int \left( \sqrt{Q} - \frac{1}{2} \bar{V}_+ \right) d \xi' \right] \equiv \exp \left[ \int S(\xi') d \xi' \right]
\]

\[
S = \left( \sqrt{Q} - \frac{1}{2} \bar{V}_+ \right); \quad Q = \left[ |\bar{\gamma}_0|^2 - \bar{V}_1 \left( \bar{V}_+ - \bar{V}_1 \right) + \frac{\bar{V}_+^2}{4} + \frac{\bar{V}_+'}{2} \right]
\]

For a linear density profile, this becomes:

\[
S^{(\text{LIN})} \approx \sqrt{\left[ |\bar{\gamma}_0|^2 + \frac{\left( \bar{V}_+ - i \kappa' \xi \right)^2}{4} \right] - \frac{1}{2} \left( \bar{V}_+ - i \kappa' \xi \right)}
\]

\[
|\bar{\gamma}_0|^2 = |\bar{\gamma}_0|^2 \left( 1 - \frac{\bar{V}_1 \bar{V}_2}{|\bar{\gamma}_0|^2} \right); \quad \kappa' = \frac{\kappa'}{\langle K \rangle^2}; \quad \xi = \langle K \rangle (z - z_{\text{PPMP}}); \quad \bar{V}_{1,2} = \langle K \rangle V_{1,2}; \quad \bar{V}_{1,2} = \frac{V_{1,2}}{V_{1,2}}; \quad \bar{V}_+ = (\bar{V}_1 + \bar{V}_2)
\]

\[
G = \int S^{(\text{LIN})} d \xi = \int \left( \sqrt{\left| \bar{\gamma}_0 \right|^2 - \frac{u^2}{4}} + \frac{iu}{2 \kappa'} \right) du; \quad (\bar{V}_+ - i \kappa' \xi) \equiv - i u
\]

Our job is to calculate the real part of the integral of $S$, which is the gain of the convective amplifier within WKB, and to measure the width of the region that gives rise to significant gain.
Our job is to calculate the real part of the integral of the gain of the convective amplifier within WKB, and to measure the width of the region that gives rise to significant gain.

For a linear density profile, this becomes:

\[
G_{\text{MNR,MAX}}^{(\text{LIN})} = \frac{2\pi |\gamma_0|^2 \left(1 - \nu_1 \nu_2 / |\gamma_0|^2 \right)}{|\kappa' V_1 V_2|} \equiv 2 \langle K \rangle L_{\text{INT}}
\]

The strong damping limit is when \( x \to \infty \), and the \( \pm ix \) terms in the integrand of \( I(x) \) cancel, and the next order term is \(-i/2x\). Its real part in fact becomes the SDL gain exponent once integrated. In the weak damping limit, the dominant factor of this integral, i.e., the one that does not depend on damping at all, in an integral which runs between -1 and 1, and is equal to 1.

\[
L_{\text{INT}}^{(\text{WDL})} = 2 \frac{2 G_{\text{MNR,MAX}}^{(\text{LIN})}}{\pi \kappa'}
\]

\[
L_{\text{INT}}^{(\text{SDL})} = 2 \frac{\left(\frac{V_1 + V_2}{V_1 V_2}\right)}{\kappa'}
\]

• For a related look at the \((\nu_1 \nu_2)\) damping story, see E. A. Williams, PoF B3, 1504 (1991).
Besides the Gain Exponent, A Crucial Quantity Is the Gain Length Which Varies between SDL, WDL, SCL, WCL, SRS, SBS

\[ L_{\text{INH}}^{(WDL)} = \left[ \frac{2}{V_2} \left( \frac{dk_2}{dz} \right) \right] \left[ \frac{\gamma_0}{\omega_0} \left( \frac{V_2}{V_1} \right) \right] \]

\[ L_{\text{INH}}^{(SDL)} = \left[ \frac{2}{V_2} \left( \frac{dk_2}{dz} \right) \right] \left[ \frac{V_2}{\omega_0} \right] \]

For Raman Backscattering, in particular:

\[ \left[ \frac{2}{V_2} \left( \frac{dk_2}{dz} \right) \right]_{\text{SRBS}} = 4L_n \]

\[ \frac{\gamma_0}{\omega_0}_{\text{SRBS}} = \frac{1}{2} \frac{v_{\text{osc}}}{c} = 4.267 \times 10^{-3} \sqrt{I_{14}} \lambda_{0, \mu m} \]

\[ \left( \frac{V_2}{\sqrt{V_1}} \right)_{\text{SRBS}} = 7.66 \times 10^{-2} \sqrt{T_{e, \text{keV}}} \sqrt{\left( 1 - \frac{n}{n_c} + \sqrt{1 - 2 \frac{n}{n_c}} \right)^{3/4}} \left( \frac{n}{n_c} \right)^{3/4} \]

\[ k_{\text{EPW}} \lambda_D = 4.42 \times 10^{-2} \sqrt{T_{e, \text{keV}}} \left( \sqrt{1 - \frac{n}{n_c} + \sqrt{1 - 2 \frac{n}{n_c}}} \right) \sqrt{\frac{n}{n_c}} \]

Design STUD pulses so that:

\[ L_{\text{SPIKE}} < L_{\text{INH}} < L_{\text{HS}} \]

\[ L_{\text{HS}} \approx 4 f^2 \lambda_0 \]

\[ L_{\text{SPIKE}} = t_{\text{spike}} \times V_g, \text{scatt} \]

\[ G_{\text{SRBS(LIN)}}^{\text{SRBS(LIN)}} = \frac{4 \pi \left( \frac{\gamma_0}{\omega_0} \right)^2 \left( \frac{2\pi L_n}{\lambda_0} \right)}{\left[ \left( 1 - \frac{2}{n_c} \right) - \frac{n}{n_c} \right] - \frac{n}{n_c}} \]

\[ \left( \frac{V_2}{\omega_2} \left( k_{\text{EPW}} \lambda_D > 0.25 \right) \right)_{\text{SRBS}} = -0.23 \times 2.2 \left( k_{\text{EPW}} \lambda_D \right) - 6.6 \left( k_{\text{EPW}} \lambda_D \right)^2 + 6.8 \left( k_{\text{EPW}} \lambda_D \right)^3 - 3.9 \left( k_{\text{EPW}} \lambda_D \right)^4 + 0.96 \left( k_{\text{EPW}} \lambda_D \right)^5 \]
Further Details on SRBS & a Generalized Interaction Length that Goes from WDL to SDL (1 of 2)

$$G_{\text{MAX}}^{(\text{LIN})} = \frac{2\pi|\gamma_0|^4}{|\kappa' V_1 V_2|} \left(1 - \frac{V_1 V_2}{|\gamma_0|^2}\right) = 2 \left(K^{\text{LIN}}\right) L_{\text{INT}}^{(\text{LIN})}$$

$$L_{\text{INT}} = \sqrt{\left(L_{\text{INT}}^{(\text{WDL})}\right)^2 + \left(L_{\text{INT}}^{(\text{SDL})}\right)^2}$$

$$L_{\text{INT}}^{(\text{WDL})} = 2 \frac{2G_{\text{MAX}}^{(\text{LIN})}}{\pi|\kappa'|}$$

$$L_{\text{INT}}^{(\text{SDL})} = 2 \frac{|V_2|}{V_2 \kappa'}$$

$$V_1 = \sqrt{\frac{1 - \frac{n}{n_c}}{1 - \frac{n}{n_c}}}$$

$$V_2 = \left[\frac{3v_{th}^2}{c^2}\right] \left[\sqrt{\epsilon} + \sqrt{1 - \frac{n}{n_c}}\right]$$

$$\frac{c^2 \kappa'}{\omega_0} = \frac{2 \left(V_2/c\right) \left(o_0 L_n/c\right)}{2 \left(V_2/c\right) \left(o_0 L_n/c\right)}$$

$$\frac{\gamma_0^2}{\omega_0} = [4.267 \times 10^{-3}]^2 I_{14} \lambda_{n,\mu m}^2$$

$$G_{\text{MAX}}^{(\text{LIN})} = \frac{1.4188 \times I_{14} \lambda_{n,\mu m} L_{n,100\mu m}}{\sqrt{\left(\frac{\sqrt{1 - \frac{n}{n_c}}}{\sqrt{n_c}}\right)^2 \left(\frac{n}{n_c}\right)}}$$

$$\left(\frac{V_2}{\kappa_{\text{EPW}} \lambda_D > 0.25}\right)_{\text{SRBS}} = -0.23 + 2.2 \left(k_{\text{EPW}} \lambda_D\right) - 6.6 \left(k_{\text{EPW}} \lambda_D\right)^2$$

$$+ 6.8 \left(k_{\text{EPW}} \lambda_D\right)^3 - 3.9 \left(k_{\text{EPW}} \lambda_D\right)^4 + 0.96 \left(k_{\text{EPW}} \lambda_D\right)^5$$

$$L_{\text{INT}} = 4 L_n \left[\alpha^2 I_{14} + \left(v_2 \left(T_e,\text{keV}\right) / \omega_2\right)^2\right]^{1/2}$$

So as the intensity increases, $L_n$ must be decreased to keep $L_{\text{INT}}$ fixed and $\approx 4 f^2 \lambda_0$.

For fixed density and temperature, this represents a rule between $L_n$ and $I_{14}$:

$$\frac{L_n}{\lambda_0} = \left[\frac{f/\#}{\alpha^2 I_{14} + \left(v_2 \left(T_e,\text{keV}\right) / \omega_2\right)^2}\right]^{1/2}$$

This is the criterion with which to keep $L_{\text{INT}} = L_{\text{HS}}$, as the intensity of the laser is varied for fixed Landau damping rate, $v_2$.

$$\alpha^2 = \frac{1.4188 \lambda_{0,\mu m} L_{n,100 \mu m}}{(n/n_c)} \left(\frac{V_2}{V_1}\right)$$
Further Details on SRBS & a Generalized Interaction Length that Goes from WDL to SDL (2 of 2)

\[ \left| \frac{\gamma_0}{\omega_0} \right|^2 = [4.267 \times 10^{-3}]^2 I_{14} \lambda_{0,\mu m}^2 \]

\[ G_{\text{MAX}}^{(\text{LIN})} = \frac{1.4188 \times I_{14} \lambda_{0,\mu m} L_{n,100\mu m}}{\left[ \left( \sqrt{\frac{n}{n_c}} - 2 \sqrt{\frac{n}{n_c}} \right) \sqrt{\frac{n}{n_c}} \right]} \]

\[ \left( \frac{V_2 \left( k_{\text{SRBS}} \lambda_D > 0.25 \right)}{\omega_2} \right)_{\text{SRBS}} = -0.23 + 2.2 \left( k_{\text{SRBS}} \lambda_D \right) - 6.6 \left( k_{\text{SRBS}} \lambda_D \right)^2 + 6.8 \left( k_{\text{SRBS}} \lambda_D \right)^3 - 3.9 \left( k_{\text{SRBS}} \lambda_D \right)^4 + 0.96 \left( k_{\text{SRBS}} \lambda_D \right)^5 \]

\[ L_{\text{INT}} = 4 L_n \left[ \alpha^2 I_{14} + \left( v_2 \left( T_{e,\text{keV}} \right) / \omega_2 \right)^2 \right] \]

\[ \frac{L_n}{\lambda_0} = \left[ \left( \frac{f/\#}{\alpha^2 I_{14} + \left( v_2 \left( T_{e,\text{keV}} \right) / \omega_2 \right)^2} \right) \right]^{1/2} \]

\[ \alpha^2 = \frac{1.4188 \lambda_{0,\mu m} L_{n,100\mu m}}{(n/n_c)} \left( \frac{V_2}{V_1} \right) \]
Besides the Gain Exponent, A Crucial Quantity Is the Gain Length Which Varies between SDL, WDL, SCL, WCL, SRS, SBS

\[ L^{(WDL)}_{INT} = \left[ \frac{2}{\left( \frac{V_2}{\omega_2} \right) \left( \frac{dk_2}{dz} \right)} \right] \left[ \frac{\left( \frac{\gamma_0}{\omega_0} \right)}{\left( \frac{V_2}{\sqrt{V_1}} \right)} \right] \]

\[ L^{(SDL)}_{INT} = \left[ \frac{2}{\left( \frac{V_2}{\omega_2} \right) \left( \frac{dk_2}{dz} \right)} \right] \left[ \frac{\left( \frac{V_2}{\omega_0} \right)}{\left( \frac{V_2}{\omega_2} \right)} \right] \]

Design STUD pulses so that:

\[ L_{spike} < L_{HS} < L_{INT} \]

\[ L_{HS} \sim 4 f^2 \lambda_0 \]

\[ L_{spike} = t_{spike} \times V_{g, scatt} \]

For Brillouin Backscattering, in the weak coupling and strong damping limit in particular:

\[ L^{SDL}_{INT,SBBS,100} = 0.2 \left[ \frac{1 + M(0)}{M(0)} \right] L_{V,100} \left( \frac{V_{LAW}}{0.1 \omega_{LAW}} \right) \]

\[ \frac{\gamma_0}{\omega_0}_{SBBS} = \frac{1}{2} \left( \frac{Z m_e}{M_I} \right)^{1/2} \left( \frac{n_e}{n_c} \right)^{1/2} \left( \frac{k_{LAW}}{\omega_0/c} \right) \left( \frac{v_0/c}{(\omega_{LAW} \omega_s) / \omega_0^2} \right) \]

\[ \frac{\gamma_0}{\omega_0}_{SBBS} = 2.19 \times 10^{-3} \varepsilon^{1/4} \left( \frac{Z}{A} \right)^{1/2} \left( \frac{n_e}{n_c} \right)^{1/2} \frac{\sqrt{T_{e,keV}}}{T_{e,keV}} \]

\[ G^{SBBS}_{MNR} = 1.46 \left( \frac{n_e}{n_c} \right) I_{14} \lambda_{0,\mu m}^2 \frac{2 \pi L_{V,100}}{M(0) \lambda_{0,\mu m}} \]
A Hierarchy of Scales and Models Dictate the Additional Physics Needed to Properly Model SBS or SRS in Different Regimes of Operation

• **WDL:** \( \nu_{IAW} / \gamma_0 << 1 \Rightarrow G \sim \gamma_0^2/\kappa'V_1V_2 \) but also, the possibility of an absolute instability.

• **SDL:** \( \nu_{IAW} / \gamma_0 >> 1 \Rightarrow G \sim \gamma_0^2/\kappa'V_1V_2 \) without absolute instabilities.

• **WCL:** \( \gamma_0/\omega_{IAW} << 1 \Rightarrow G \sim \gamma_0^2/\kappa'V_1V_2 \) easy to violate this limit in hot spots.

• **SCL:** \( \gamma_0/\omega_{IAW} >> 1 \Rightarrow G \sim \gamma_0^{2/3} + \) laser intensity dependent IAW frequency shifts.
  Most alarmingly, allows multiple resonances in an inhomogeneous flow profile.

• **Pump Depletion** or w/o PD: Clamp Gain to Reflectivity < 1 values or allow arbitrarily large growth or need to model IAW nonlinearity.

• **Self Focusing** or w/o SF: SCL & FIL in nonuniform flow lead to nonstationarity: No longer GRF. Prominent tails develop. New regimes of statistical behavior.

• **Single Beam vs Overlapped Beams:** Also possible to get off the GRF reservation. Without Gaussian Random Fields, the theoretical arsenal shrinks considerably.
Speckle Statistics: Gaussian Random Fields Basics

Rice’s Lemma: 
\[ M_u^V = \int \delta[\nabla A(x)] \ 1_{A(x) \geq u} \ |\det \nabla \nabla A(x)| \ 1_{\nabla \nabla A(x) < 0} \ dx \]

\[ \left< M_u^{3D} \right> \approx 56 \times L_{\text{Beam,um}} \approx 56,000 \text{per mm} \]

\[
\left< M_u^{3D} \right> = \frac{\pi^{3/2} \sqrt{5} V_{\text{tot}}}{27 \rho_c^2 z_c} \left[ \left( \frac{u}{I_0} \right)^{3/2} - \frac{3}{10} \left( \frac{u}{I_0} \right)^{1/2} \right] \exp\left[ - \frac{u}{I_0} \right]
\]

\[
R \propto \int_{N(m)}^{\infty} d\left< M_u \right> \int_0^{\infty} d\alpha u z_c \exp\left[ \alpha u z_c \right] \times P_u(z_c)
\]
What Do Structured or Speckled RPP/DPP/CPP Laser Beams Look Like?
We Can Detect the Hot Spots and Classify Their Properties (Sizes, Orientations, Peaks, Fluxes)
Once You Detect the Hot Spots You Can Classify Their Properties & Find Correlations

Integrated Fluxes in Each Detected Hot Spot

Classification of Hot Spot Statistics

- **Flux**
- **Width of Detected Hot Spot**
- **Length of Detected Hot Spot**

Peak Value

Flux

Integrated Fluxes in Each Detected Hot Spot

Classification of Hot Spot Statistics

- **FwhmB**
Two Realizations of Sections of f/20 Beams
Statistical Properties of Two Independent Realizations of RPP f/20 Beams

Overlap of Hot Spot Peak Locations

Comparison of Length and Width Distribution of Hot Spots in Two Separate RPP Laser Profiles
The Geometry of NIF Quads: Top View, 4 Cones: Inner (Red 23°, Orange 30°) & Outer (Dark 44° & Light Green 50°)

48 Quads in Total: 16 Outer and 8 Inner Per Hemisphere

FIG. 1. Left: A schematic of a NIF ignition Hohlraum with the approximate dimensions, showing the inner and outer beam cones entering the Hohlraum through the two laser entrance holes. Right: The specific angles of the NIF beam quads, which are color coded: inner quads orange (θ = 30°) and red (θ = 23.5°), outer quads light (θ = 50°) and dark green (θ = 44.5°), where θ is the polar angle.
SRS and SBS in the Strong Damping Limit Driven by STUD Pulses

$$\overline{L}_1 a_1 = \left( \frac{\partial}{\partial t} + V_1 \frac{\partial}{\partial z} + i \beta_1 \nabla_{\perp}^2 + V_1 - \frac{|\gamma_0|^2 |a_0|^2}{\overline{\nu_2}} \right) a_1 = \frac{\gamma_0 a_0 e^{i\varphi}}{\overline{\nu_2}} S_2^* + S_1$$

$$\overline{L}_0 a_0 = \left( \frac{\partial}{\partial t} - V_0 \frac{\partial}{\partial z} - i \beta_2 \nabla_{\perp}^2 + V_0 - \frac{|\gamma_0|^2 |a_1|^2}{\overline{\nu_2}} \right) a_0 = \frac{\gamma_0^* a_1 e^{-i\varphi}}{\overline{\nu_2}} S_2 + S_0$$

$$\overline{\nu_2} = \nu_2 - i \nu_2 \left[ k_0 (z) - k_1 (z) - k_2 (z) \right]$$

$$S_0(z,t;x_\perp) = \sum_{i=1}^{N_{HS}} f_z \left( \frac{z - z_{C,i}}{z_{W,i}} \right) \sum_{j=1}^{N_{spikes}} S_{0,j}^{(i)} \left( \frac{t - t_{C,j}}{t_{W,j}} \right)$$

Action flux conservation:

$$\left( \frac{\partial}{\partial t} + V_1 \frac{\partial}{\partial z} + V_1 \right) |a_1|^2 = \left( \frac{\partial}{\partial t} - V_0 \frac{\partial}{\partial z} + V_0 \right) |a_0|^2 = \frac{2|\gamma_0|^2 |a_0|^2 |a_1|^2}{\nu_2^2 + V_2^2 \left( k_2' \right)^2} + 4 \text{Re} \left[ \frac{\gamma_0 a_0 a_1^* S_2^* e^{i\varphi} \left( \nu_2 - i \nu_2 k_2' \right)}{\left[ \nu_2^2 + V_2^2 \left( k_2' \right)^2 \right]} \right]$$

Transform into a frame moving with the STUD pulse SPIKES and do integration over pulses as integrals over space (z).

Then average over transverse distributions which reflect the hot spot exponential intensity statistics.
The Gain Exponent of SRS or SBS in the Strong Damping Limit is Made Up of Individual Elements of this Form:

\[ L_{HS} \left( 4 f^2 \lambda_0 \right) : L_{INT} \left( \frac{V_2}{V_2 \kappa'} \right) : L_{\text{Spike}(i)} \left( V_1 \tau_{\text{Spike}(i)} \right) \]

The smallest of these three lengths will dictate the individual HS’s contribution to the overall gain during each spike of a STUD pulse train.

\[ \tilde{G}^{(i)}(z) = \left[ \frac{2 |\gamma_0^2_{\text{Ave}} | a_0^{(i)}|^2}{V_1 V_2 \kappa'} \right] \left( 1 - \frac{V_1 V_2}{|\gamma_0^2_{\text{MAX}}|} \right) \times \left[ \tan^{-1} \left( \frac{z - z_{PPMP}}{L_{\text{INT}}} \right) - \tan^{-1} \left( \frac{z_R - z_{PPMP}}{L_{\text{INT}}} \right) \right] \]

\[ \tilde{G}^{(i)}(z) \bigg|_{\text{Largest Possible Gain}} = 2\pi \left[ \frac{|\gamma_0^2_{\text{Ave}} | a_0^{(i)}|^2}{V_1 V_2 \kappa'} \right] \left( 1 - \frac{V_1 V_2}{|\gamma_0^2_{\text{MAX}}|} \right) \]

\[ \tilde{G}^{(i)}(z) \bigg|_{\text{small Gain}} = \left[ \frac{2 |\gamma_0^2_{\text{Ave}} | a_0^{(i)}|^2}{(V_1 V_2 + V_2 V_1)} \right] \left( 1 - \frac{V_1 V_2}{|\gamma_0^2_{\text{MAX}}|} \right) \times |(z - z_R)| \]

\[ S_0(z, t; x_\perp) = \sum_{i=1}^{N_{HS}} f_z \left( \frac{z - z_{C,i}}{z_{W,i}} \right) \sum_{j=1}^{N_{\text{spikes}}} S^{(i)}_{0,j} \left( \frac{t - t_{C,j}}{t_{W,j}} \right) \]

\[ L_{\text{INT}}^{(SDL)} = \frac{2 \left( \frac{V_1}{V_1} + \frac{V_2}{V_2} \right)}{\frac{d}{dz} \left( k_0 - k_1 - k_2 \right)} \bigg|_{z=z_{PPMP}} \]
Looking at the IAW on a Linear vs Log Scale for an RPP Beam @ 3 Times + the Pump