Dynamics of Rayleigh-Taylor driven flows at high Atwood numbers

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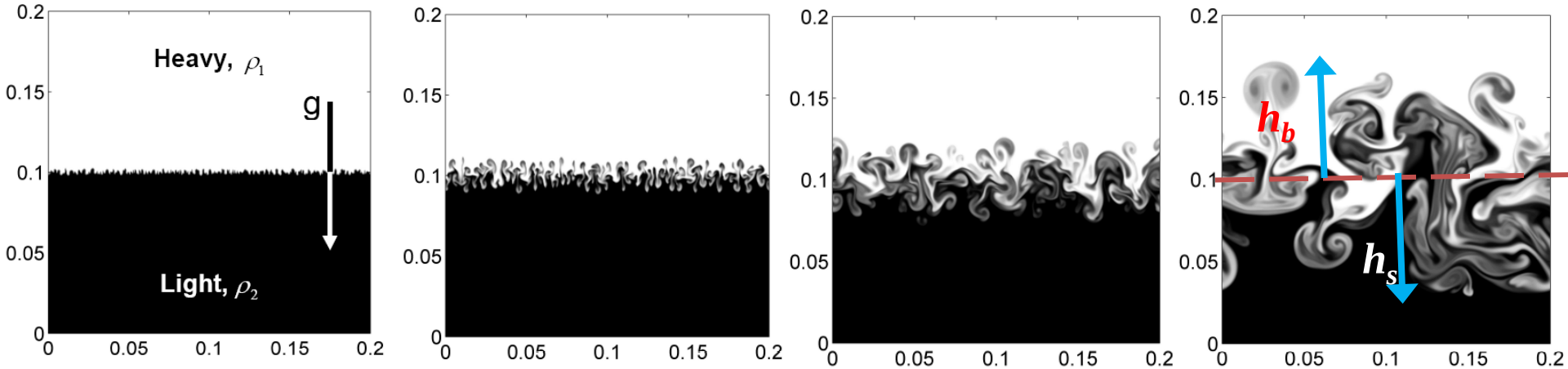
Major Updates

- PI moved from Texas A&M University to Georgia Tech in July 2014
- Facility moved along with the PI to the new place and it is now operational.
- 2 PhDs and 1 MS students graduated from the group last year
- Manuscript in Review/Preparation—3 (1 JFM, 2 PoF)
- Collaboration with LANL and LLNL
    - Thomas Finn (MS student) worked with Dr. Oleg Schilling
    - Bhanesh Akula (PhD student) worked closely with Dr. Malcolm Andrews
    - Mark Mikhaeil (PhD Student) will be spending summer at LANL with Dr. Nick Denissen

Rayleigh-Taylor Instability



Main non-dimensional number: Atwood#: $A_t = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$

Baroclinic generation of vorticity: $\frac{1}{\rho^2}\nabla p \times \nabla \rho$

Interface is unstable if: $\nabla p \bullet \nabla \rho < 0$

$$h_{b,s} = \alpha_{b,s} A_t g t^2$$

Reynolds number, $Re_{v'} = \frac{(h_s + h_b)\, v'}{\upsilon_{mix}}$

- Those accompanied by changes in composition, density, enthalpy, pressure, etc (e.g combustion, detonation, supernova)
- Mixing of fluids of different densities, where the mixing has a large influence on the velocity field (e.g, thermal convection, Rayleigh-Taylor Instability)
- Mixing of passive scalars (non-reacting species which produce no density changes and do not affect the dynamics)

Why do we care?

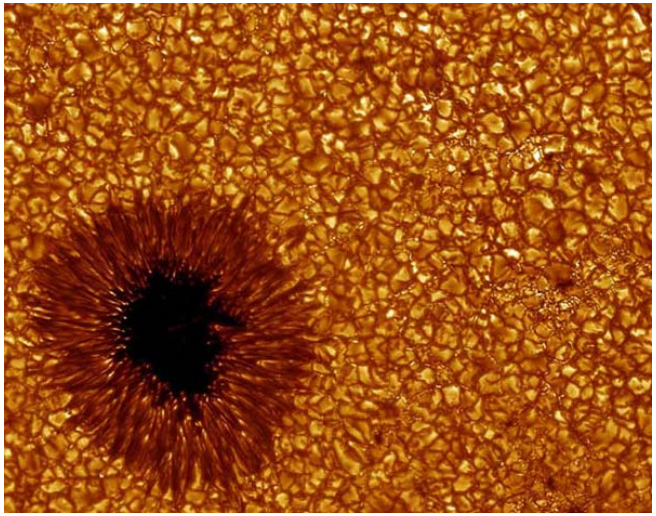
Fundamental fluid mechanics and low-energy-density applications

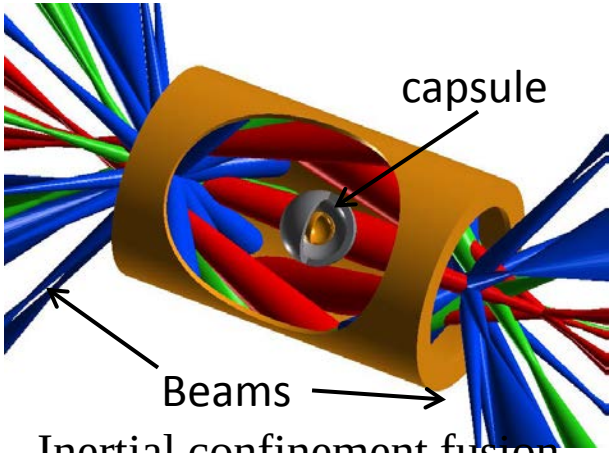
- Ocean mixed layer and stratified turbulence
- Effluent discharge into rivers and estuaries
- Atomization of droplets and sprays, Supersonic combustion
- Volcanic Eruptions (Plume Dynamics)

High-energy-density (extreme) applications

- Inertial Confinement Fusion (implosion, burning of DT fuel)
- Supernovae (explosion, thermonuclear flames)
- Interstellar turbulence (molecular clouds)

Turbulent Convection—Solar Granules
Ahlers Physics 2, 74, 2009





Inertial confinement fusion

Project Scope

- RT and Shear Measurements at low and High Atwood number
  - Effect of shear on RT mix development
  - Understand the mixing transition, if any
  - Velocity statistics
  - Density statistics

- Implementation of simultaneous density-velocity measurement system to obtain density-velocity statistics.

- **Provide benchmark data for verification of turbulence models**

- Understanding the nature of turbulence in multi-material density stratified flows.

- How do the frequencies and amplitudes of perturbations in the initial conditions impact the evolving morphology of the flow and subsequent mixing?

Vision: "Putting DESIGN into Variable-Density Turbulence"

Challenges of variable density turbulence in turbulence modeling

Reynolds Averaged Mean Momentum using the Boussinesq approximation (de-couple density and velocity) (Wolfgang & Rodi 1993),

$$\frac{\partial \overline{U_i}}{\partial t} + \overline{U_j}\frac{\partial \overline{U_i}}{\partial x_j} = \frac{-1}{\rho_o}\frac{\partial \overline{P}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\upsilon \frac{\partial \overline{U_i}}{\partial x_j} - \overline{u_i u_j}\right) + g_i \frac{\rho - \rho_o}{\rho_o}$$

For variable density turbulence, averaging of density coupled terms, results in correlations with density fluctuations (Chassaing *et al*. 2002)

$$\frac{\partial \overline{\rho}\,\overline{U_i}}{\partial t} + \frac{\partial \overline{\rho}\,\overline{U_i}\,\overline{U_j}}{\partial x_j} = \overline{\rho}\overline{F_i} - \frac{\partial \overline{P}}{\partial x_i} + \frac{\partial \overline{\tau_{ij}}}{\partial x_j} - \frac{\partial\left(\overline{\rho u_i' u_j'}\right)}{\partial x_j} - A_i$$

$$\tau_{ij} = \mu\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \frac{2}{3}\mu\frac{\partial U_k}{\partial x_k}\delta_{ij}$$

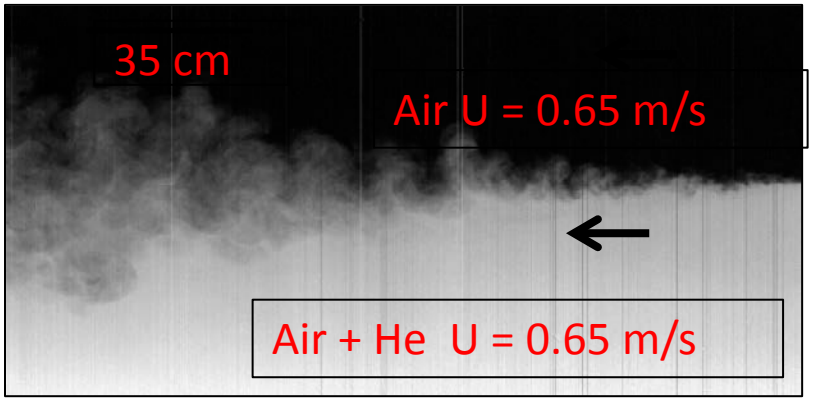
$$A_i = \frac{\partial \overline{\rho u_i'}}{\partial t} + \frac{\partial}{\partial x_j}\left(\overline{\rho u_i'}\,\overline{U_j} + \overline{\rho u_j'}\,\overline{U_i}\right)$$

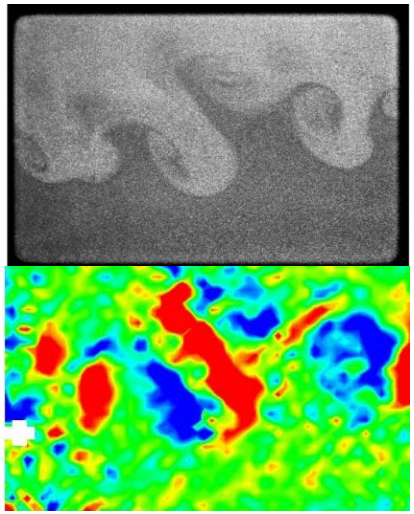
Introduction of density-velocity cross-correlations (turbulent mass flux) is a characteristic of variable density turbulence, terms need closure

STAM Lab
Shock Tube and Advanced Mixing Laboratory

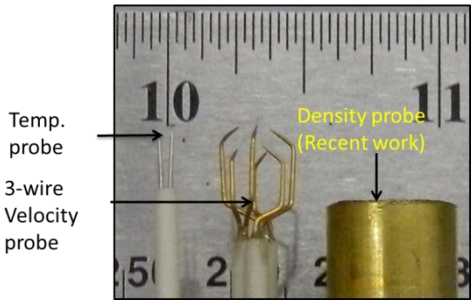
Experimental Setup-Gas Tunnel

- **Convective Setup**
  - Collection times up to 75 s
  - A_t up to 0.75
  - U up to 9 m/s
  - Multi-layer
- **Diagnostic Techniques**
  - Visualization (ethylene glycol)
  - 2D-PIV (glycerin)
  - Hot Wire Anemometry
    - Novel Density Probe directly measures fluid composition

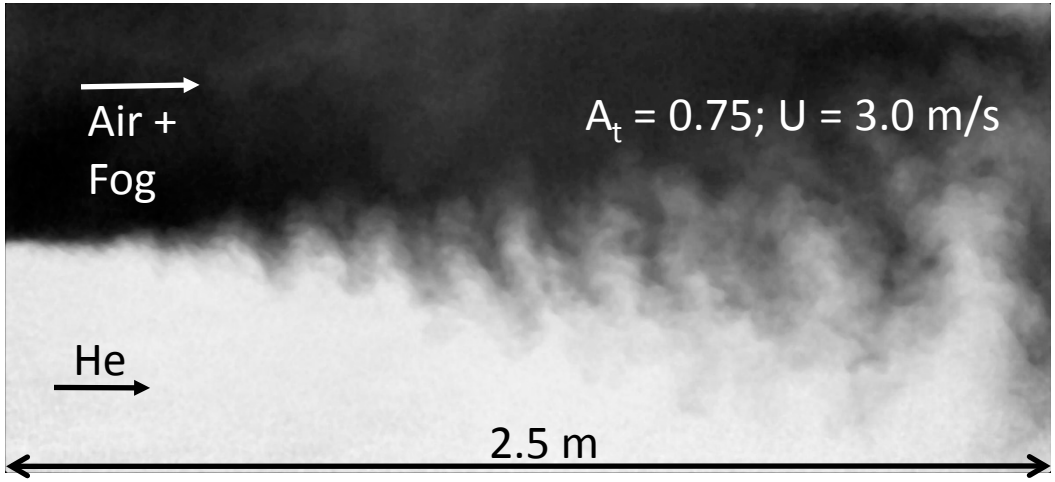


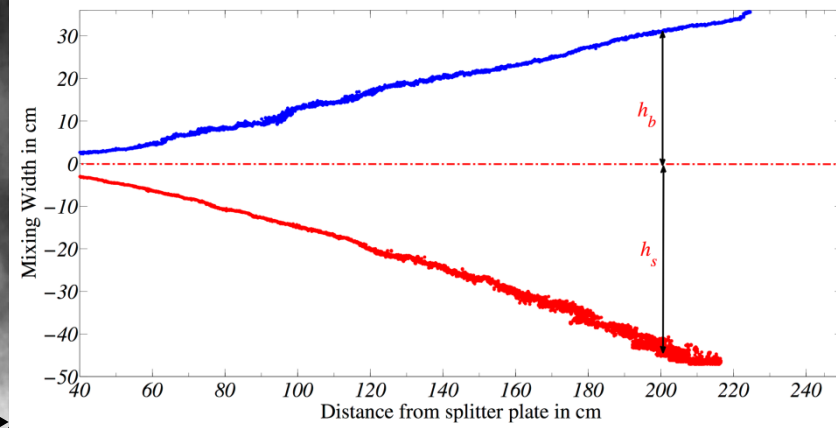


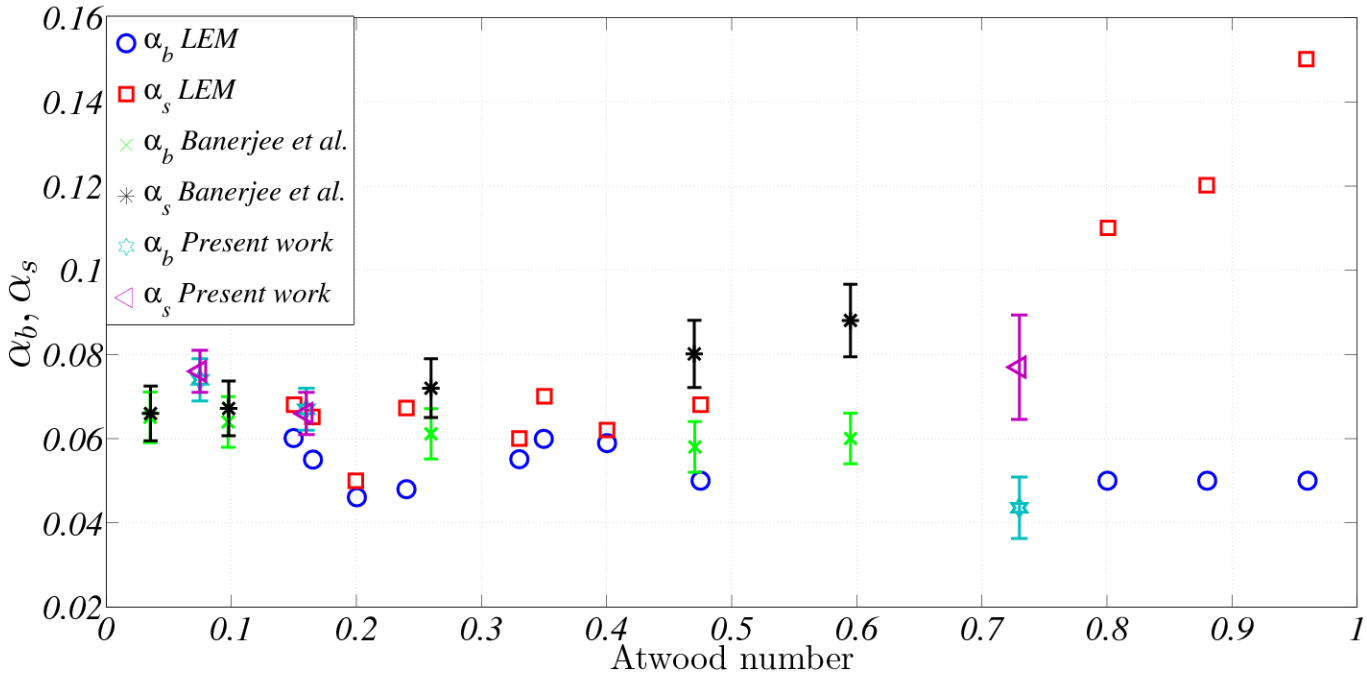
Hot wire Anemometry



Results: Atwood 0.75 Visualization experiments





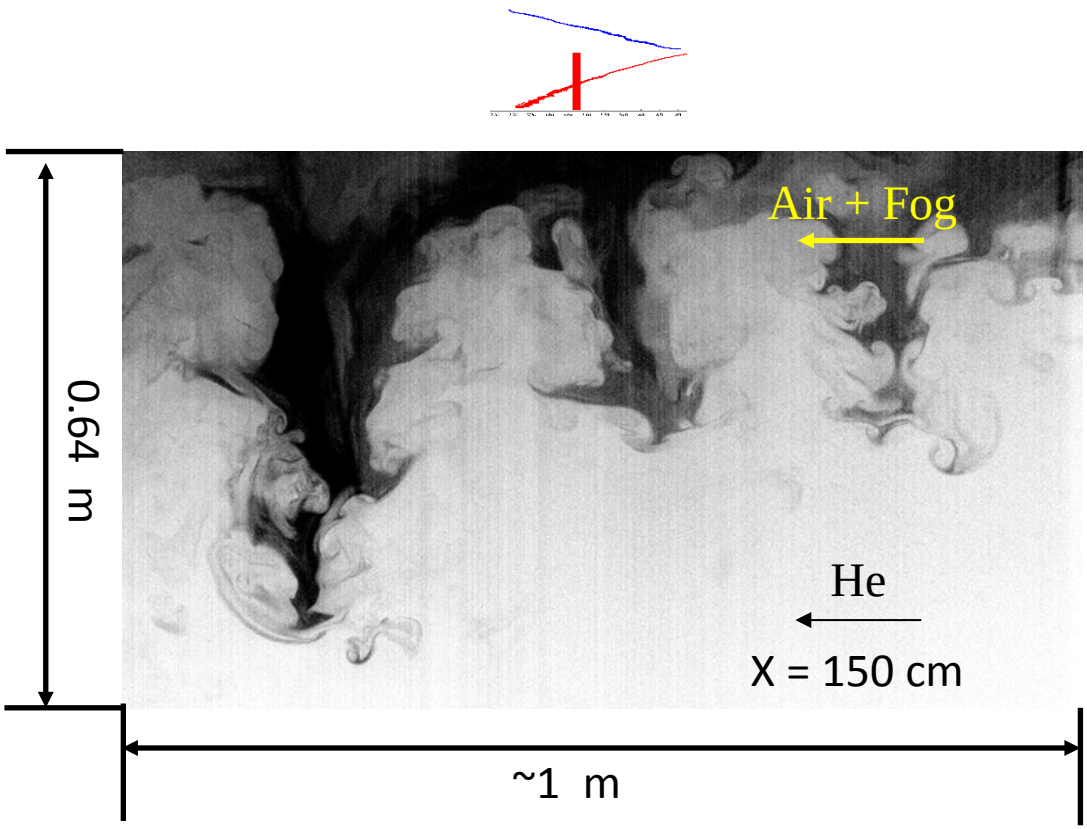


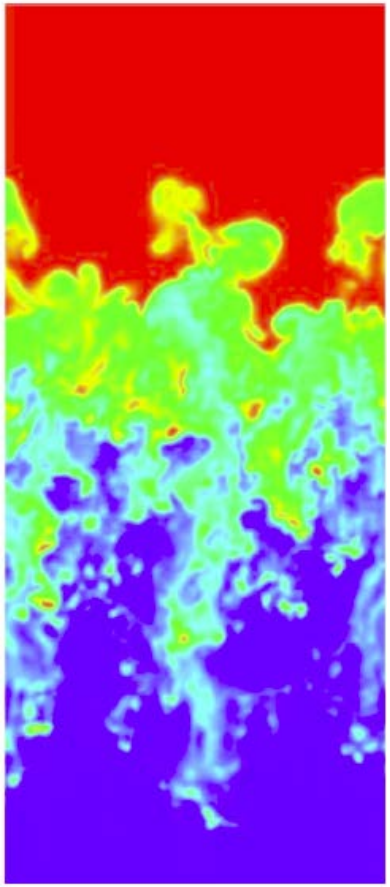
α_b	α_s	$\frac{\alpha_s}{\alpha_b}$
0.040	0.076	1.90

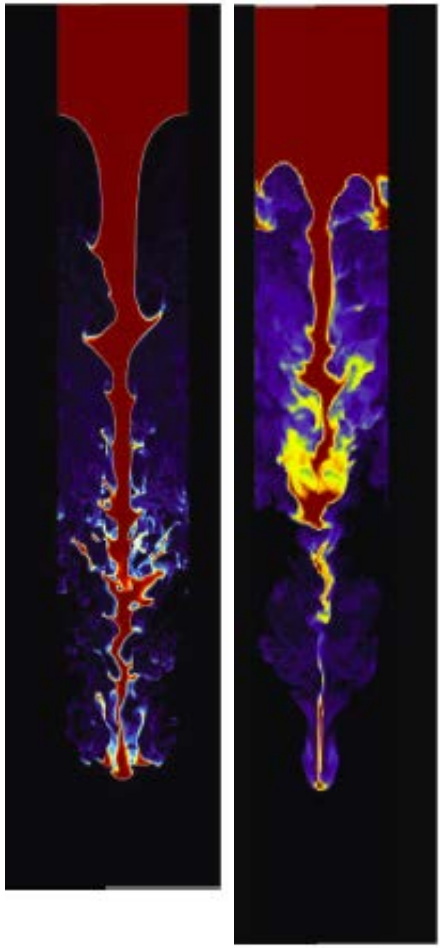
$$h_{b,s} = \alpha_{b,s} A_t g t^2$$

- Reported asymmetry (height ratio) close to the empirical correlation of Dimonte *et al.* (2000)

Results: Atwood 0.75 Planar Mie-Scattering Images





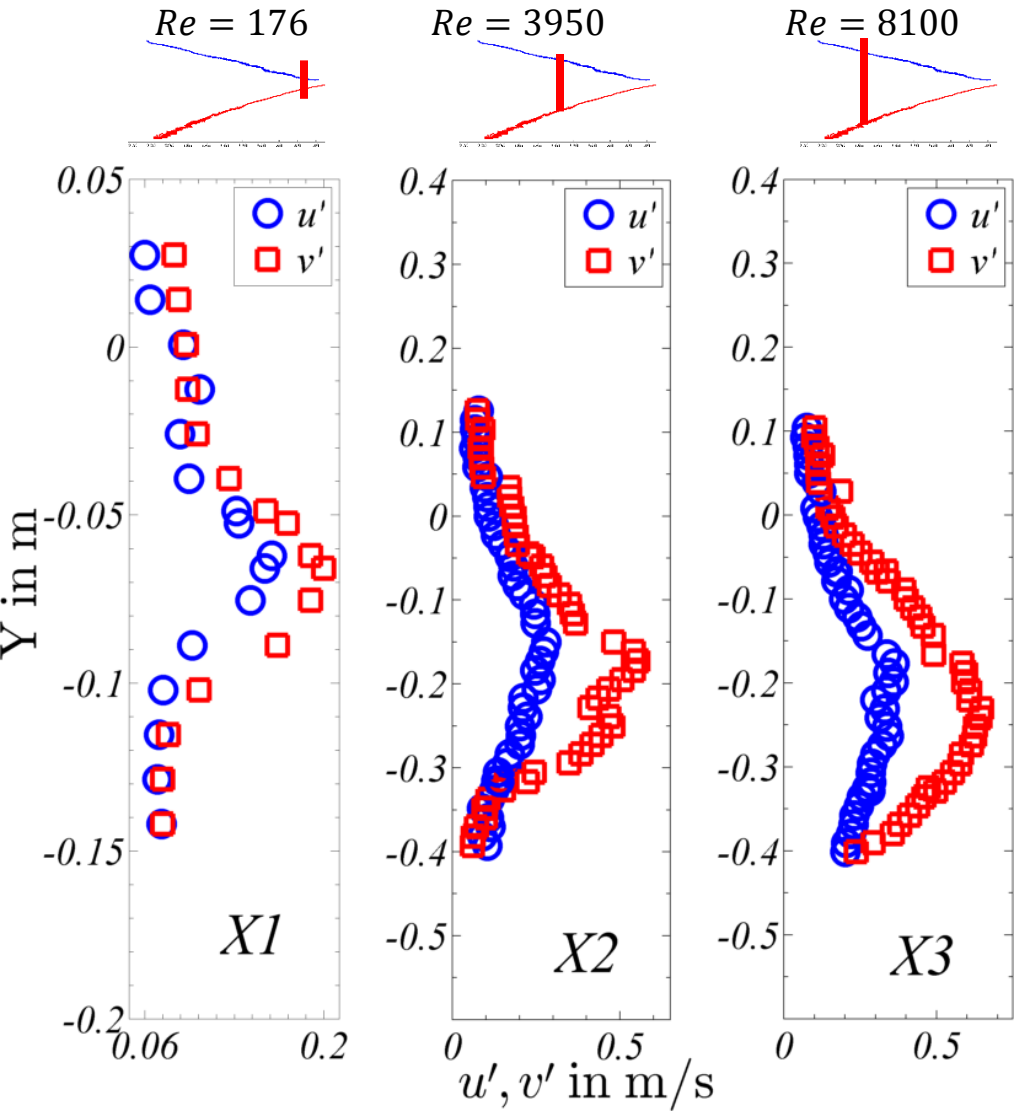


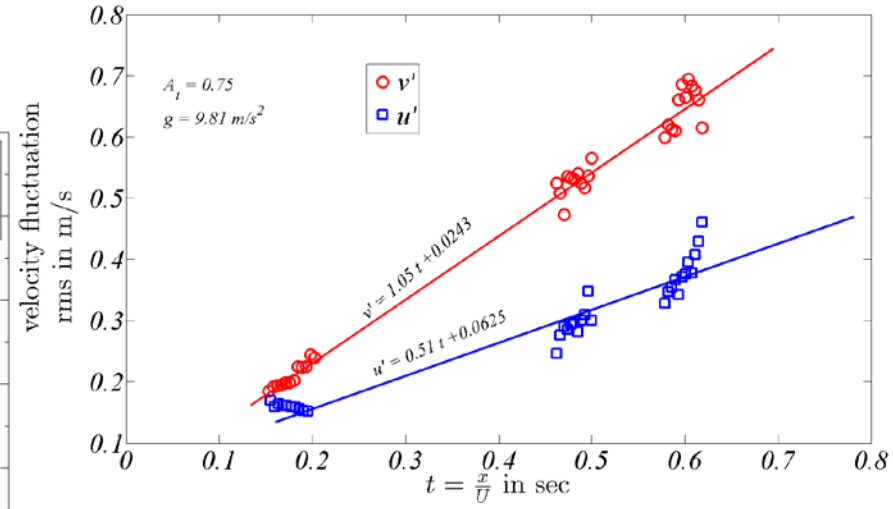
- Dendrite like structures are observed on spike
- Thinner spike and wider bubbles are observed

Burton., Phys Fluids 2011, “ultrahigh Atwood number RTI dynamics

Ramaprabhu *et al.*, Phys Fluids 2012, (late-time dynamics of the single-mode RTI

Velocity Statistics





$$Re_a = \sqrt{\frac{gA_t}{6}}\frac{2h_x{}^{3/2}}{\nu_{mix}}$$

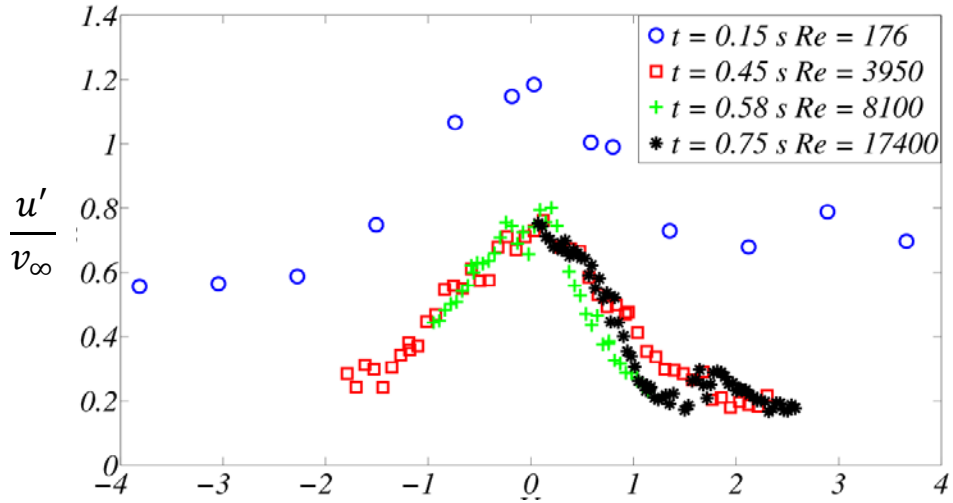
$$Re_b = \frac{h_x v_\infty}{\nu_{mix}}$$

$$Re_{v'} = \frac{(h_s + h_b)v'}{\nu_{mix}}, h_x = \frac{h_s + h_b}{2}.$$

	Re_a	Re_b	$Re_{v'}$
X1	135	154	176
X2	3640	4170	3950
X3	7920	9080	8100
X4	16850	19300	17380

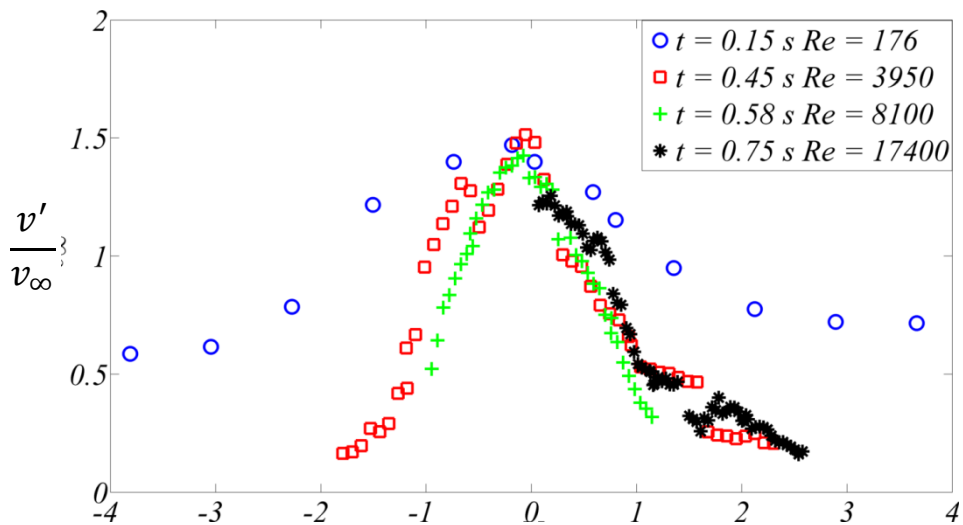
- u', v' Gaussian shaped profile across the mix layer
- The peak value location shifts towards the bottom side of the mix
- The slope of the line is proportional to growth rate constant α

Self Similarity at Late Times



Terminal bubble velocity $v_\infty = 0.7\sqrt{\frac{A_t g h_x}{2}}$

Reynolds number $Re = \frac{(h_s + h_b)v'}{\nu_{mix}}$



- Collapse at late development times past 0.45 s
- $\frac{v'}{v_\infty} \approx 1.5$

Density Statistics

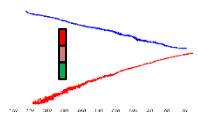
$$f_{v,1} = \lim_{T\to\infty} \frac{1}{T}\int_0^T \frac{\rho-\rho_2}{\Delta\rho}\,dt$$

$$f_{v,2} = 1 - f_{v,1}$$

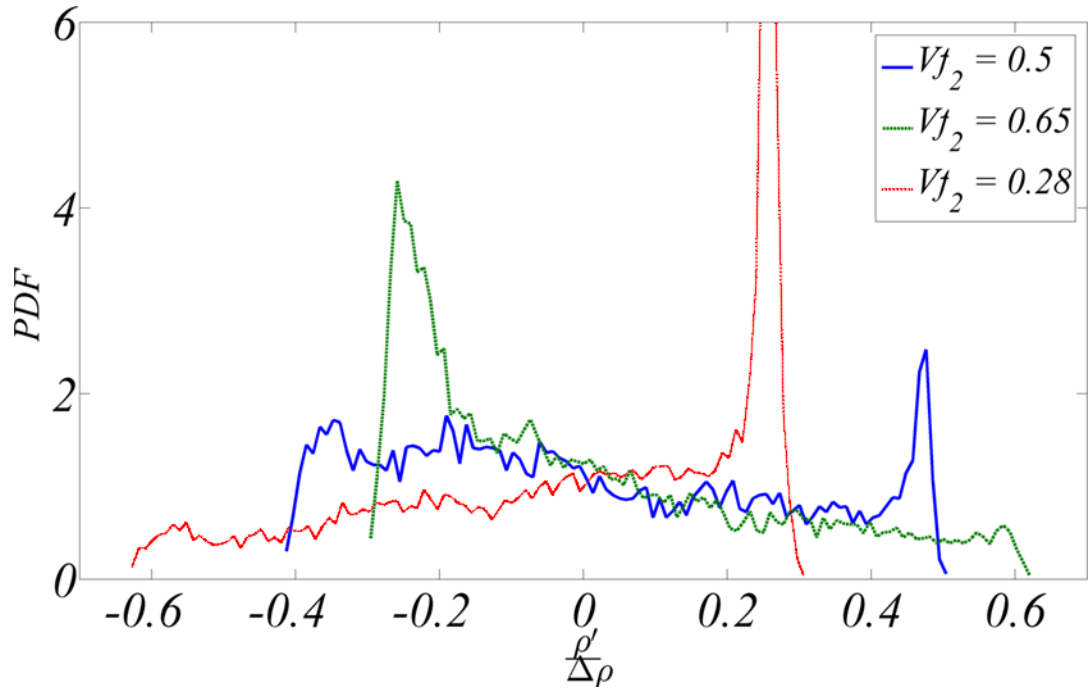
$$B_0 = \lim_{T\to\infty} \frac{1}{T}\int_0^T \frac{(\rho-\bar{\rho})^2}{(\Delta\rho)^2}\,dt$$

$$B_2 = f_{v,1} f_{v,2}$$

$$\theta = 1 - \frac{B_0}{B_2}$$



$Re = 8100$



Y/H	$\bar{f}_2$	B_0	B_2	θ
0.42	0.28	0.067	0.227	0.71
0.14	0.44	0.077	0.250	0.70
0	0.50	0.70	0.247	0.71
-0.14	0.56	0.068	0.240	0.72
-0.28	0.65	0.051	0.220	0.76

- At centerline, two small peaks at edges of distribution, flat center distribution
- Skewed distributions away from the center
- θ close to self-similar at low A_t

Turbulent kinetic energy in buoyancy-driven turbulence

- Shear production: $$-\overline{\rho u_i'' u_j''}\,\frac{\partial \tilde{U}_i}{\partial x_j} \rightarrow 0$$ (Chassaing *et al.* 2002)

Where Favre averaging is used, $$\tilde{F} = \frac{\overline{\rho F}}{\overline{\rho}}$$

$$\overline{u_i''} = \frac{-\overline{\rho u_i'}}{\overline{\rho}}$$

* No turbulent kinetic energy production from mean velocity gradients, fundamentally different than other turbulent flows

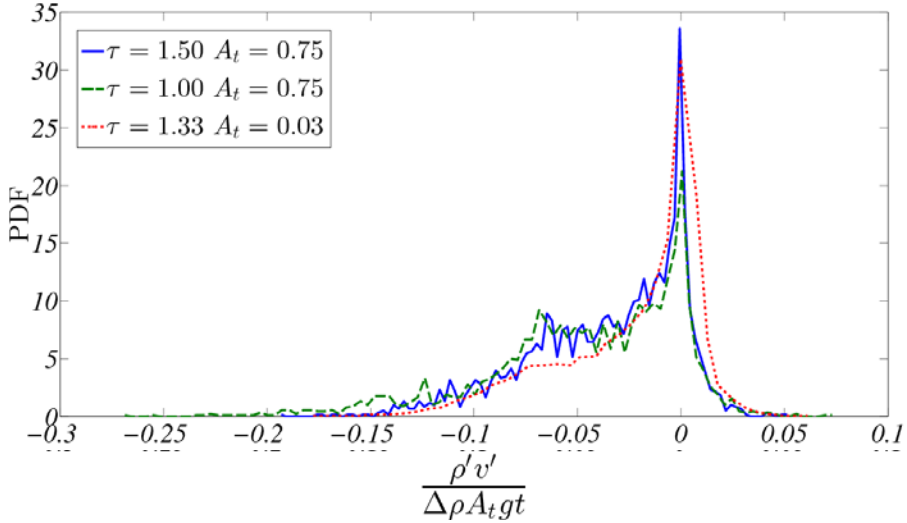
- Buoyancy production: $$-\frac{\partial \overline{P}\,\overline{u_i''}}{\partial x_i} = -\left(\overline{u_i''}\frac{\partial \overline{P}}{\partial x_i} + \overline{P}\frac{\partial \overline{u_i''}}{\partial x_i}\right) = \overline{u_i''}\frac{\partial \overline{P}}{\partial x_i} = \frac{-\overline{\rho u_i'}}{\overline{\rho}}\frac{\partial \overline{P}}{\partial x_i}$$

$\rightarrow 0$, incompressible

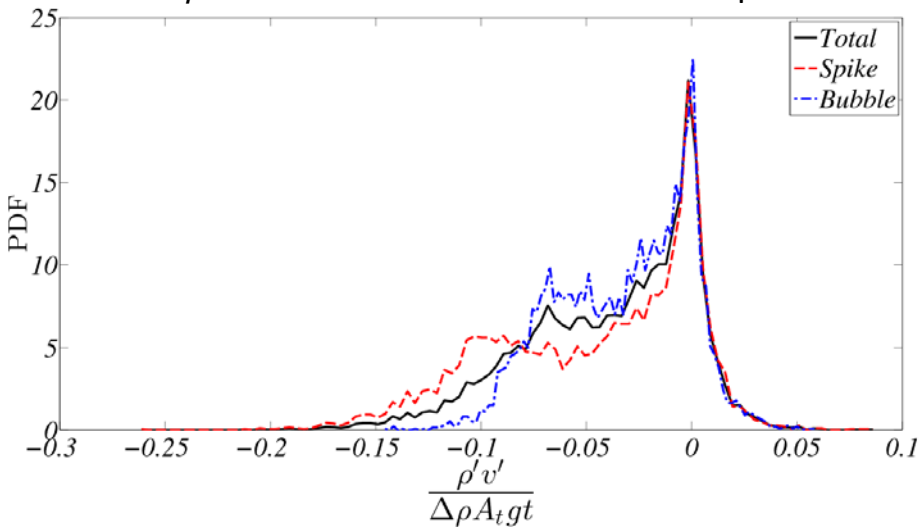
Dominant production term, acts only in the direction of the acceleration where there is a mean pressure gradient, i.e. $\overline{\rho' v'}$ is the dominant turbulent mass flux

Turbulent Mass Flux

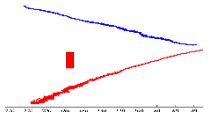
$\rho' v'$ at varying A_t and τ



$\rho' v'$ variation between bubble and spike



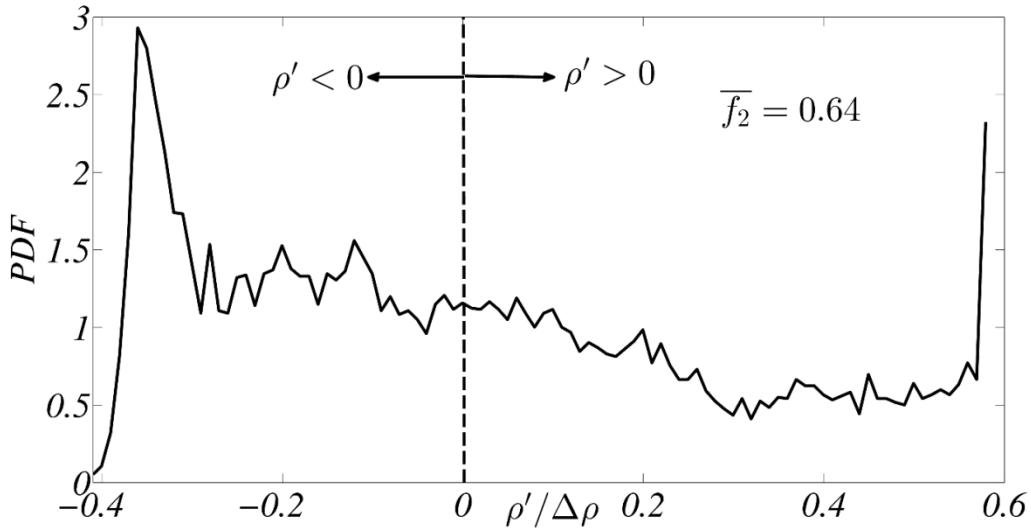
Statistic	Based on v' (− Spike, + Bubble)		
	Spike	Bubble	Spike/Bubble
$\overline{\rho' v'}$	-0.18	-0.13	1.384

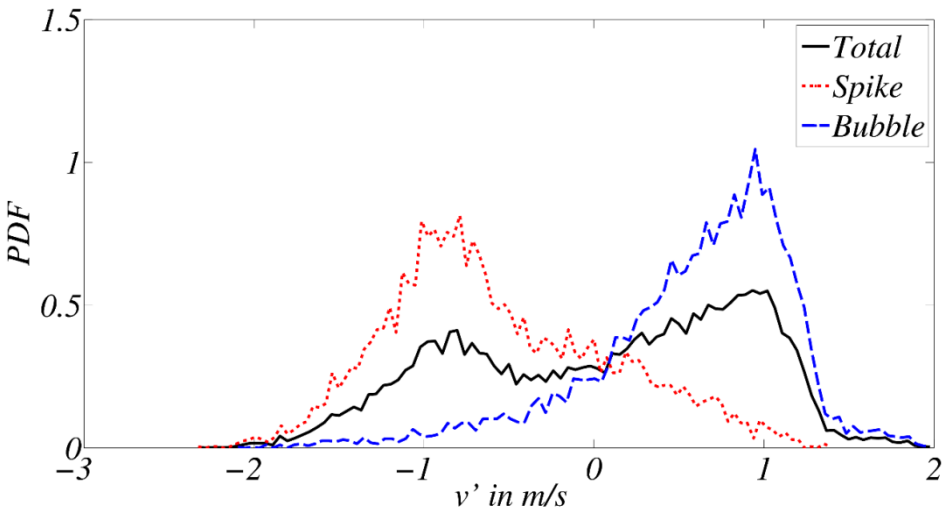


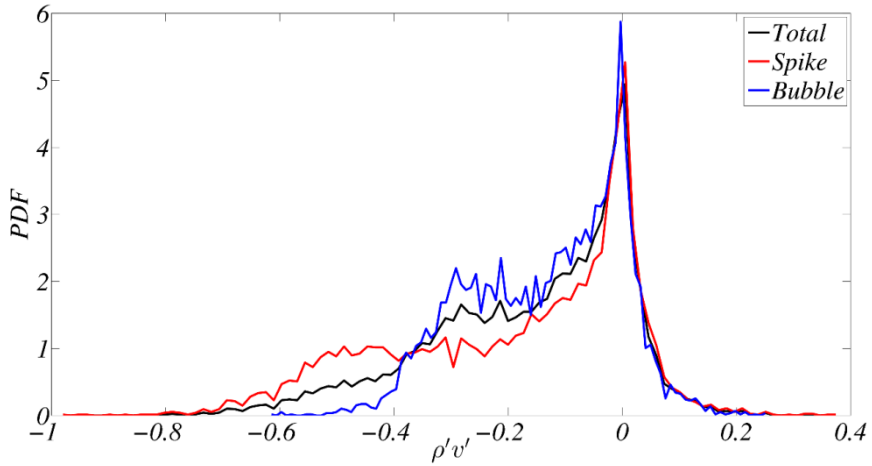
$Re = 8100$ $k = k_{max}$

- Overall behavior similar to low A_t experiments
- Asymmetry of average turbulent mass flux ($\overline{\rho' v'}$)
- Spike dominates production
- Differences in behavior between bubble and spike

Conditional Statistics

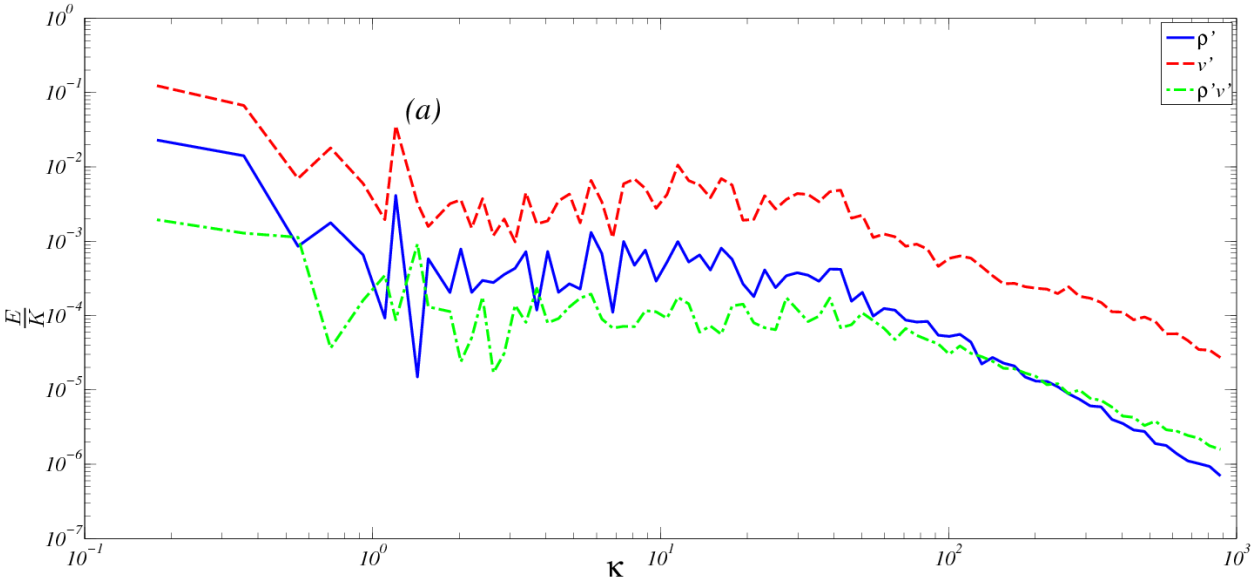


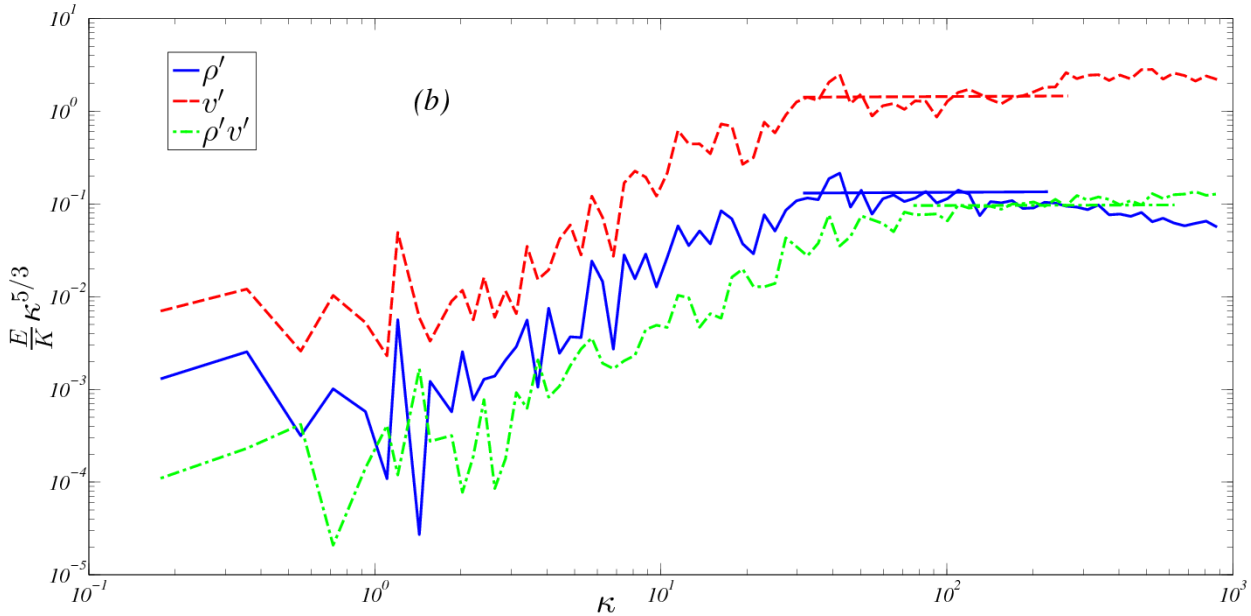




Asymmetric growth of the mixing layer results in v' *p.d.f.*'s which are not mirrored; rather more of the heavy fluid is entrained in the rising bubble than light fluid into the falling spike

Spectra of Density Measurements





ρ' and v' Spectra follows Kolmogorov inertial range 5/3 scaling for 1 decade. $\rho' v'$ does not show such scaling. Non-isotropic turbulence.

FLAG and BHR

FLAG Features

- Arbitrary Lagrangian-Eulerian (ALE), Fully unstructured
- Mixed material zones, Multi-physics

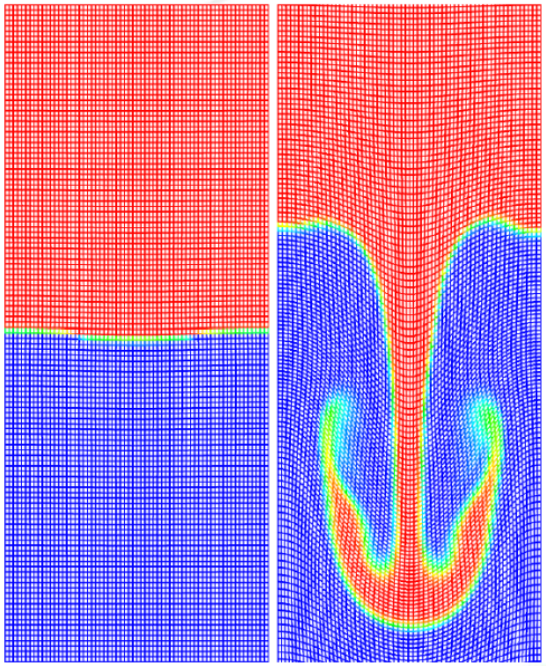
BHR Turbulence Closure Model

- Favre-averaged Navier-Stokes plus transport equations for $\tilde{R}_{ij}$, a_i, b, L_T, L_D

1D Simulations (1024 Zones)

Parameter	Value
ρ_1	3.0 (At = 0.5) or 7.0 kg/m^3(At = 0.75)
ρ_2	1.0 kg/m^3
γ	1.66
P_0	791 Pa
T_0	300 K
Size	2 m
Δz	1.95 mm

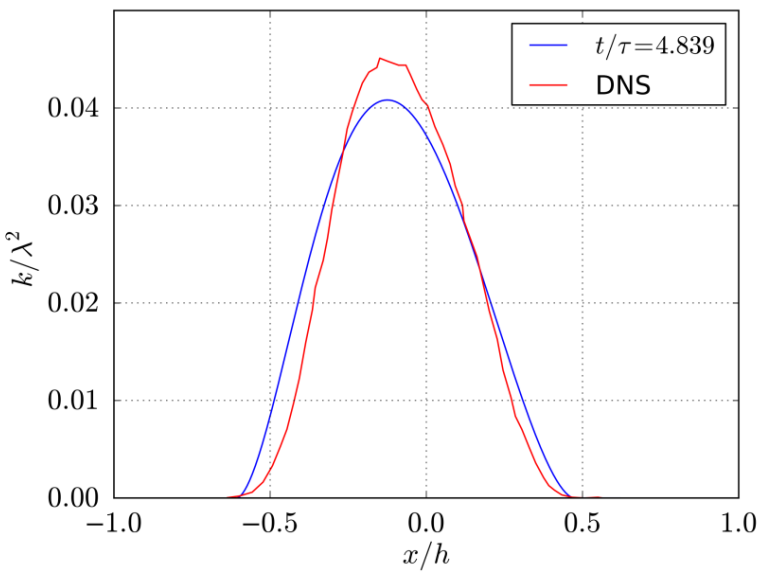
RM Instability Growth

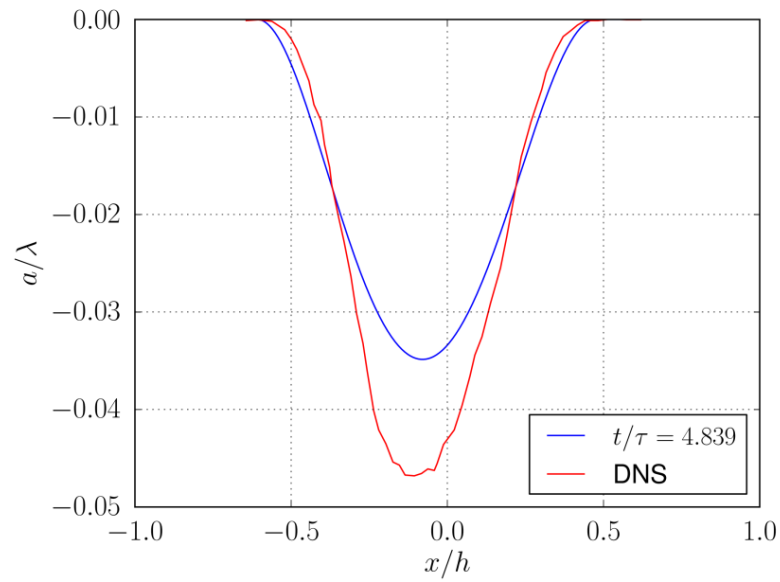


Turbulence Statistic Profiles

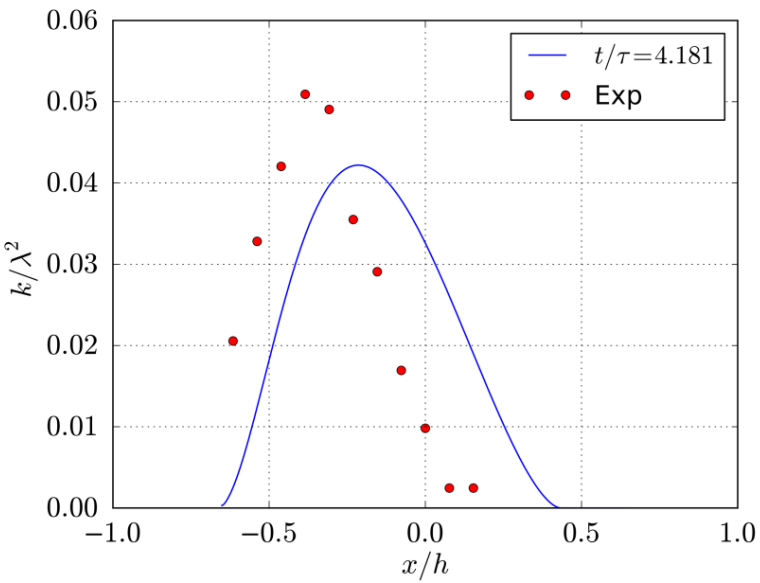
$A = 0.5$

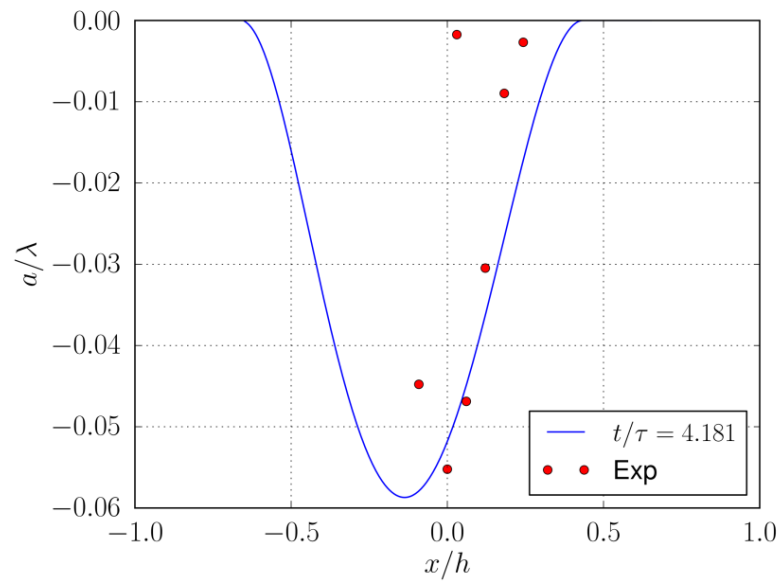
(Cabot and Cook, 2006) (Livescu et al.,2010)





$A = 0.75$

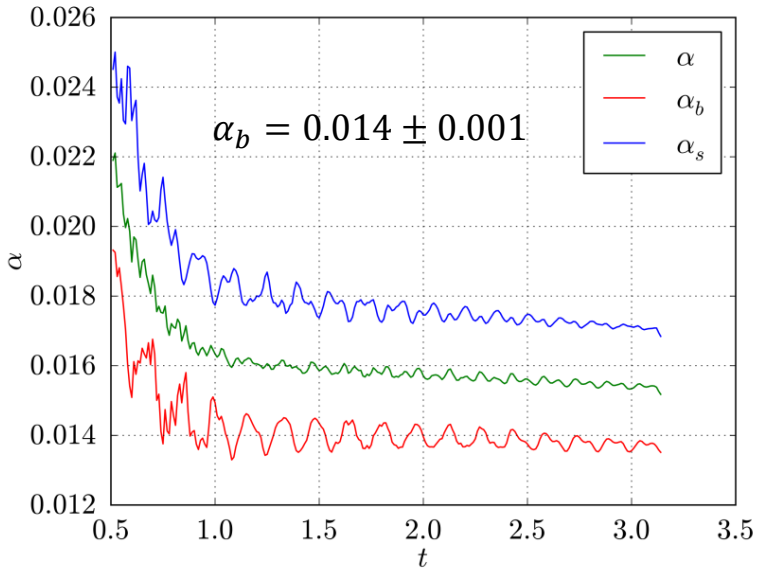


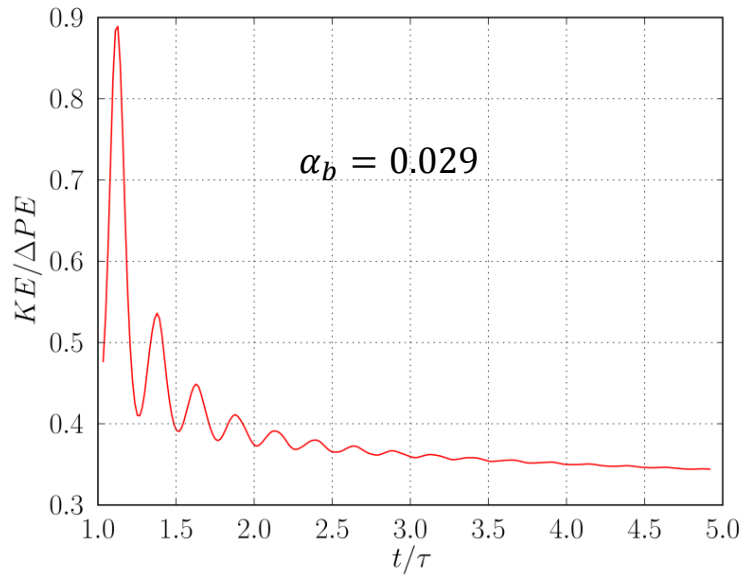


Growth Rate and Energy Balance

$A = 0.5$

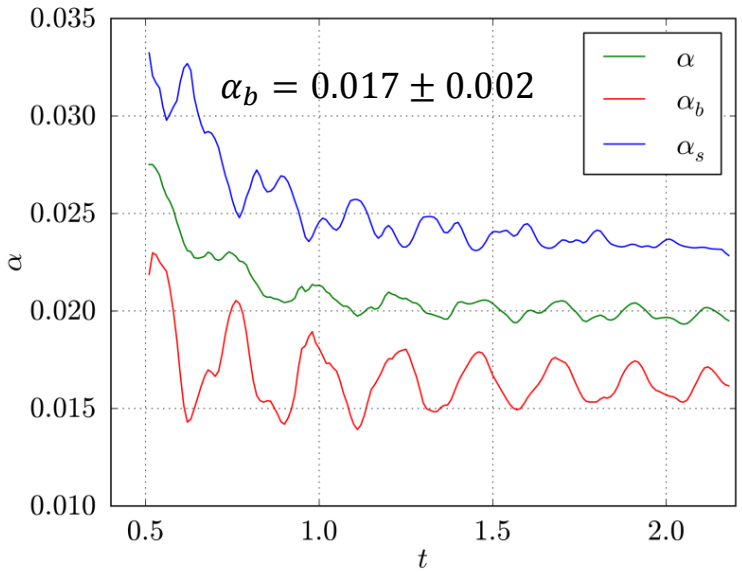
$\frac{\alpha_s}{\alpha_b} = 1.27$

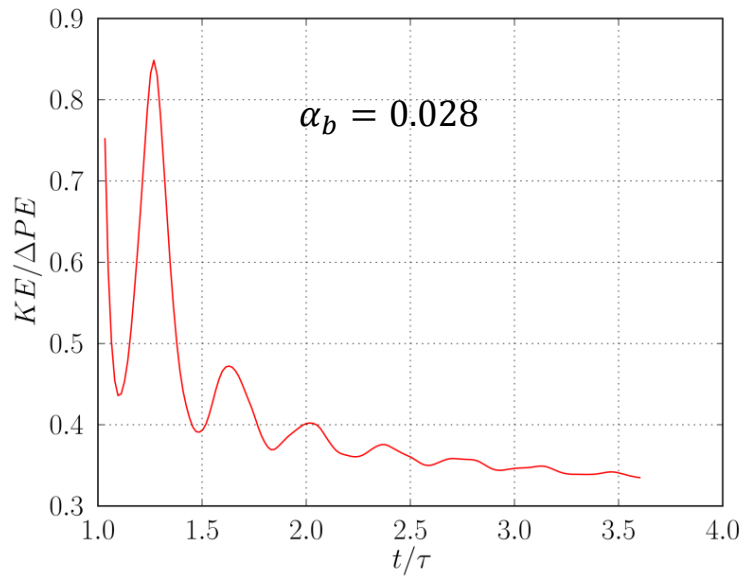




$A = 0.75$

$\frac{\alpha_s}{\alpha_b} = 1.50$



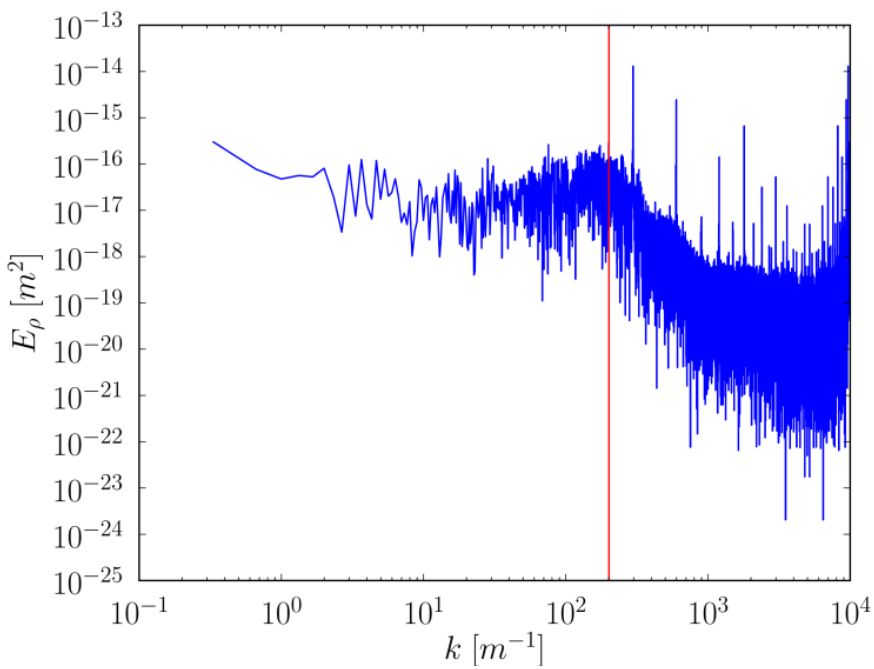


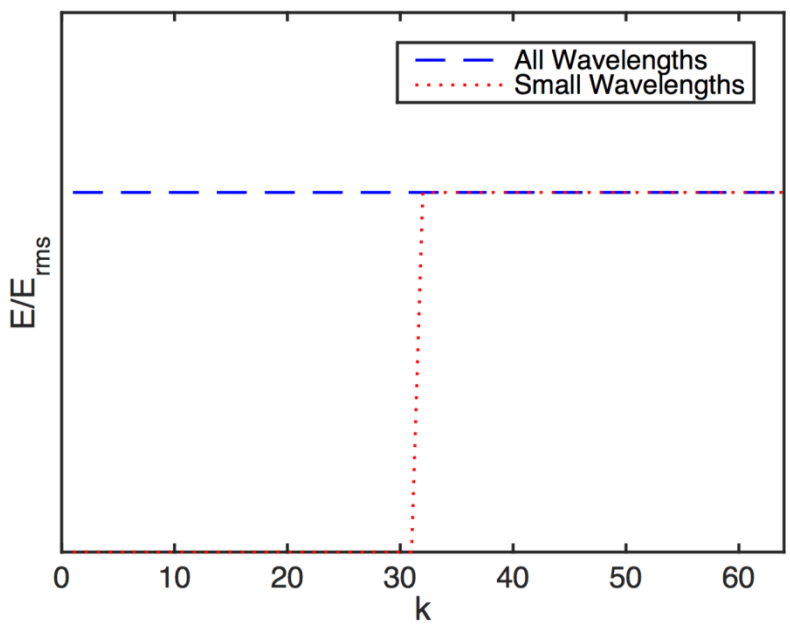
3D Simulations

- 2 Cases
- 4.2 million zones
- Hydrostatic pressure gradient
- Non-diffuse interface
- Initial condition based on Gas Tunnel hotwire measurements

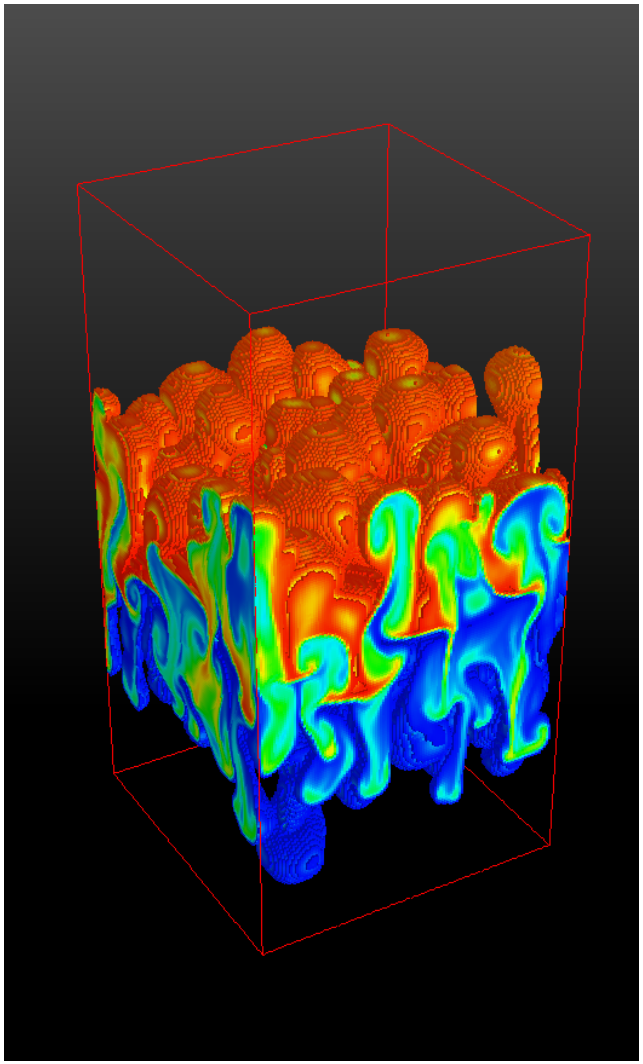
Case A	Modes 1 – 64
Case B	Modes 32 – 64

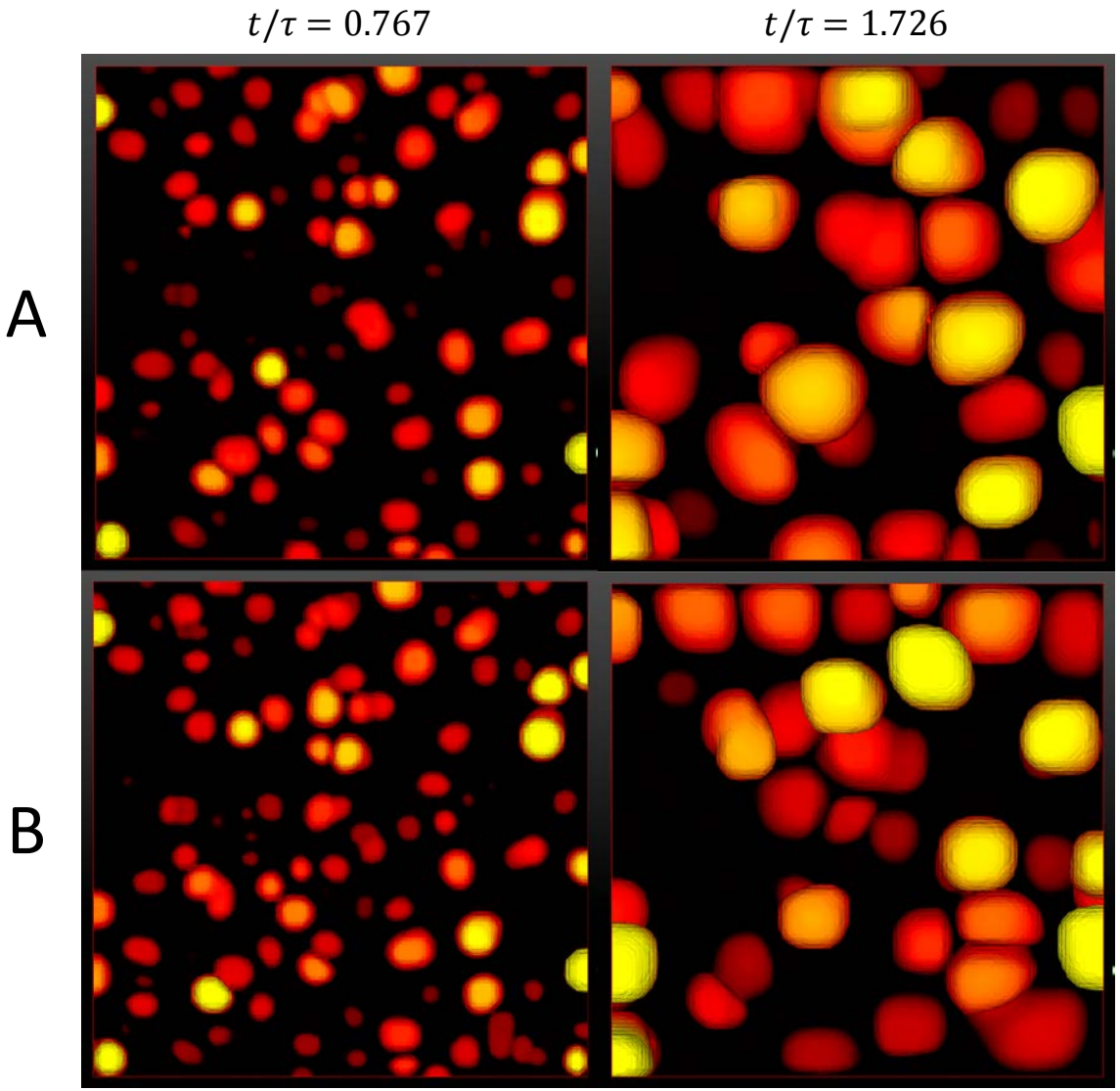
What is the effect of large wavelengths in IC?





Qualitative Results

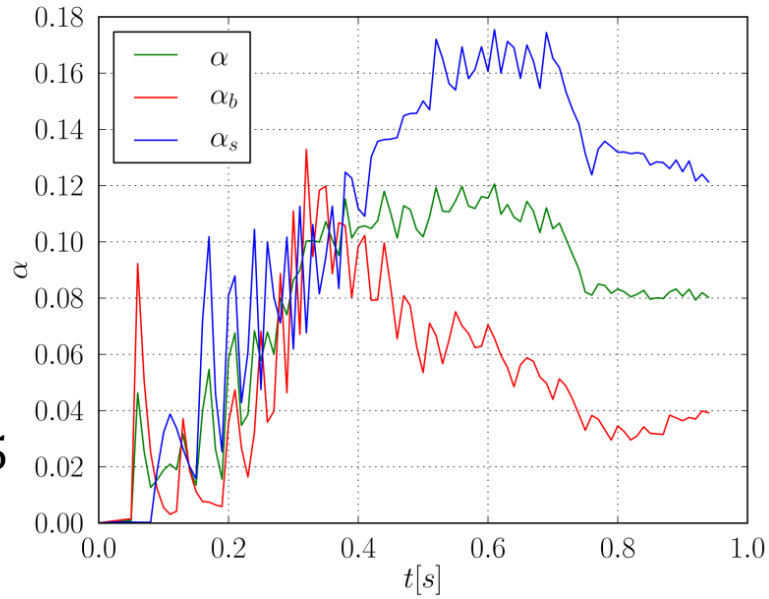


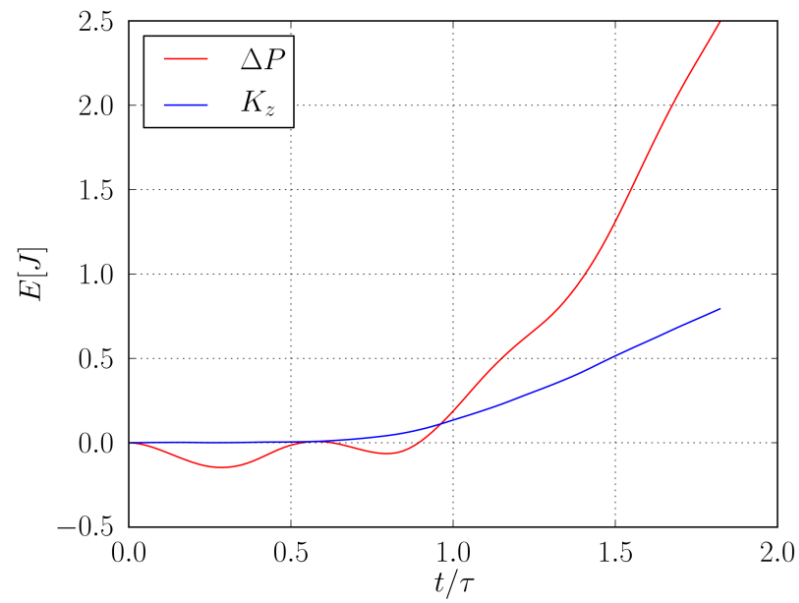


Quantitative Results

A

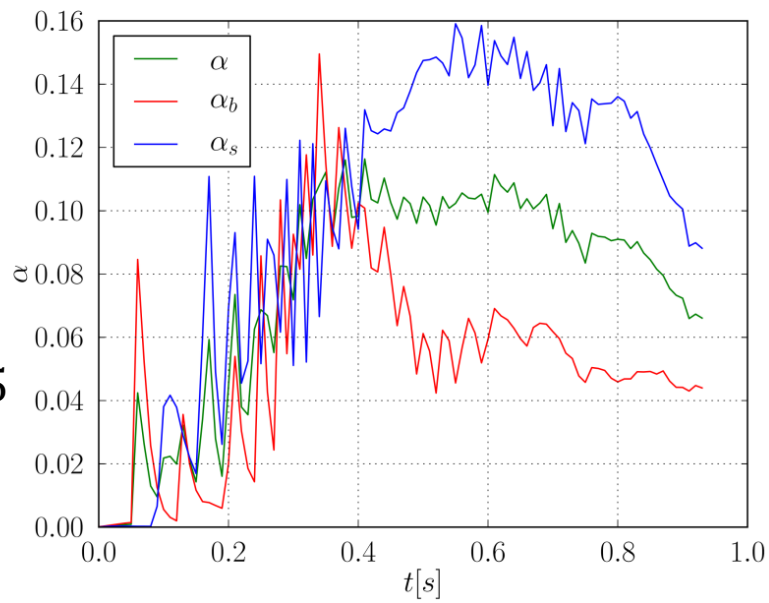
$\alpha_b = 0.04$
$\alpha = 0.08$
$K_z/\Delta P = 0.35$

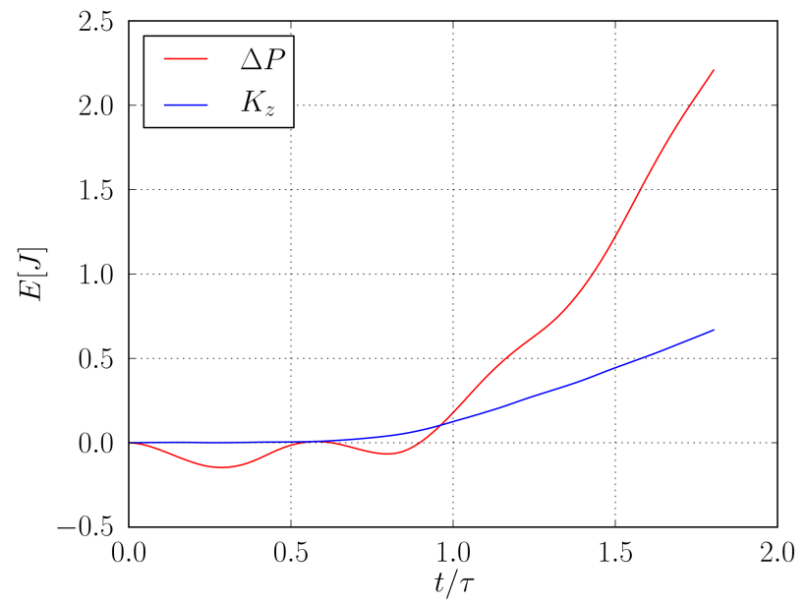




B

$\alpha_b = 0.04$
$\alpha = 0.06$
$K_z/\Delta P = 0.35$





Final Remarks

Experimental Summary

- For the first time, simultaneous density/velocity statistics are measured at A_t = 0.75 in miscible fluids
- Growth Rate Parameter, $\alpha_b \approx 0.04$, Mixing layer grows asymmetrically, with $\frac{h_s}{h_b} \approx 1.9$
- Conditional Statistics show contribution of spike ~40% larger than bubble

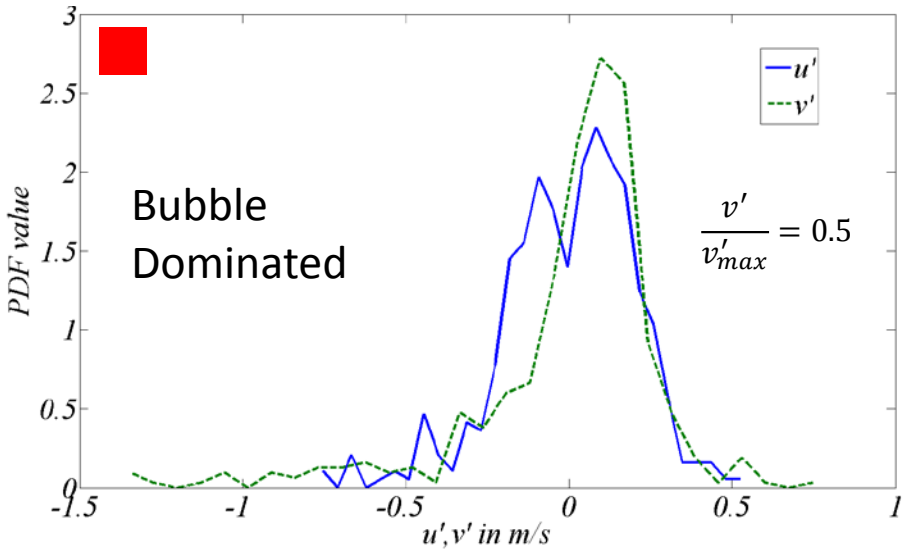
Simulation Summary

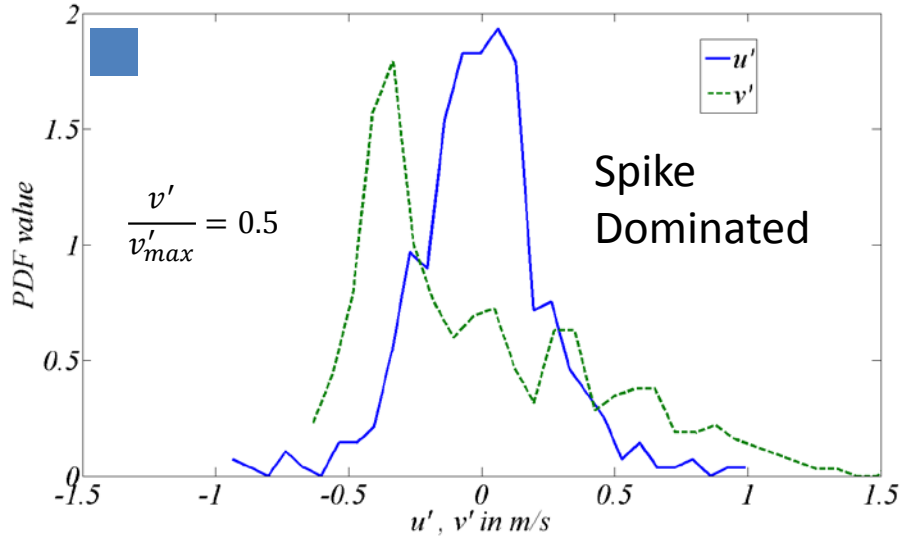
- FLAG and BHR can provide insights without high cost of resources
- Growth rates less than DNS and experimental expectations
- Density ratio effects on asymmetry: D_α = 0.19

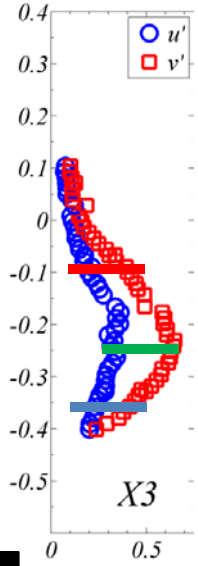
Future Work

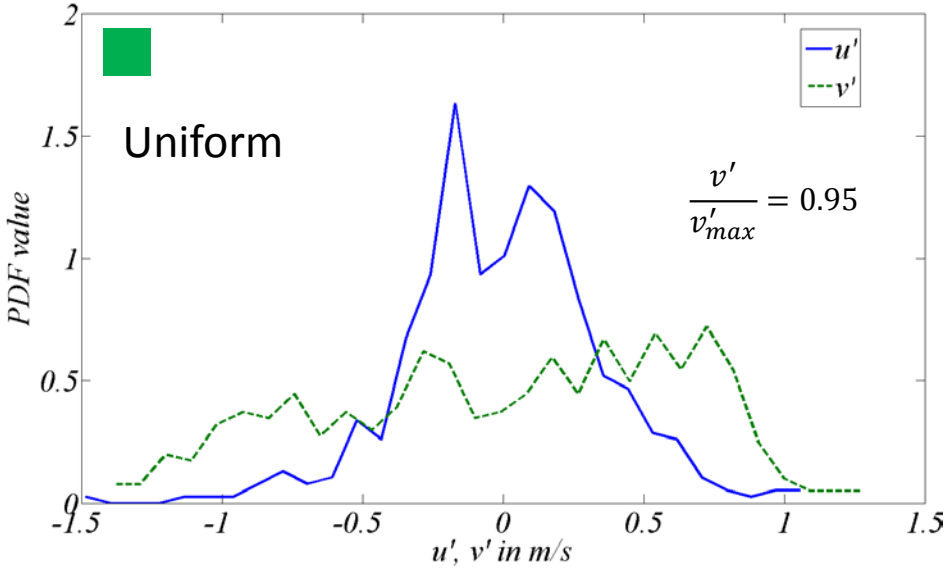
- Collecting density field with PLIF while simultaneously collecting velocity field with PIV
- IC effects on BHR Results
- Multi-layer experiments and simulations

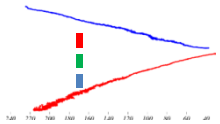
Velocity Fluctuation: PDF





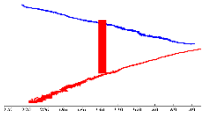






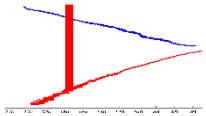
$Re = 8100$

PIV Anisotropy

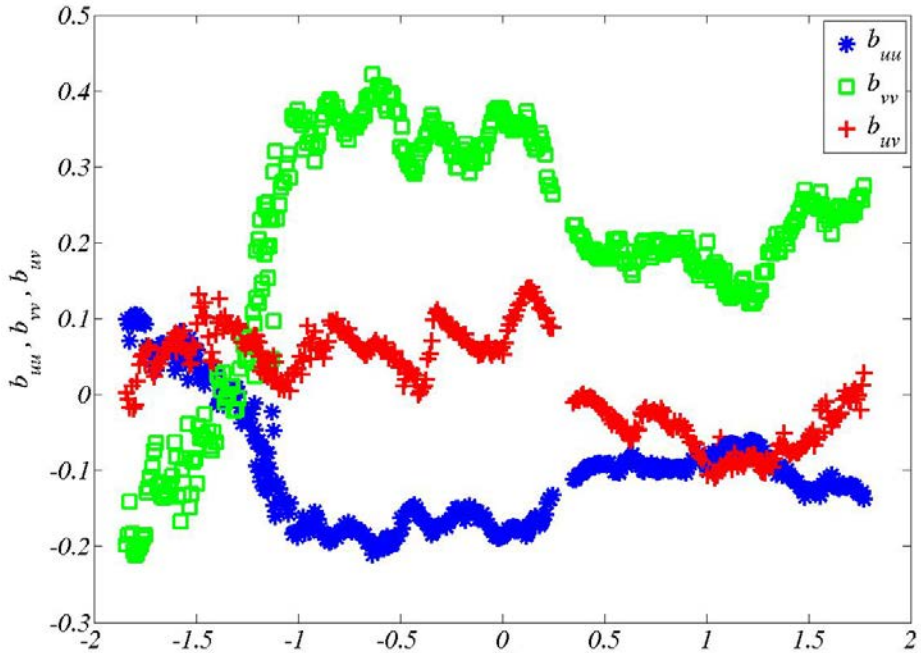


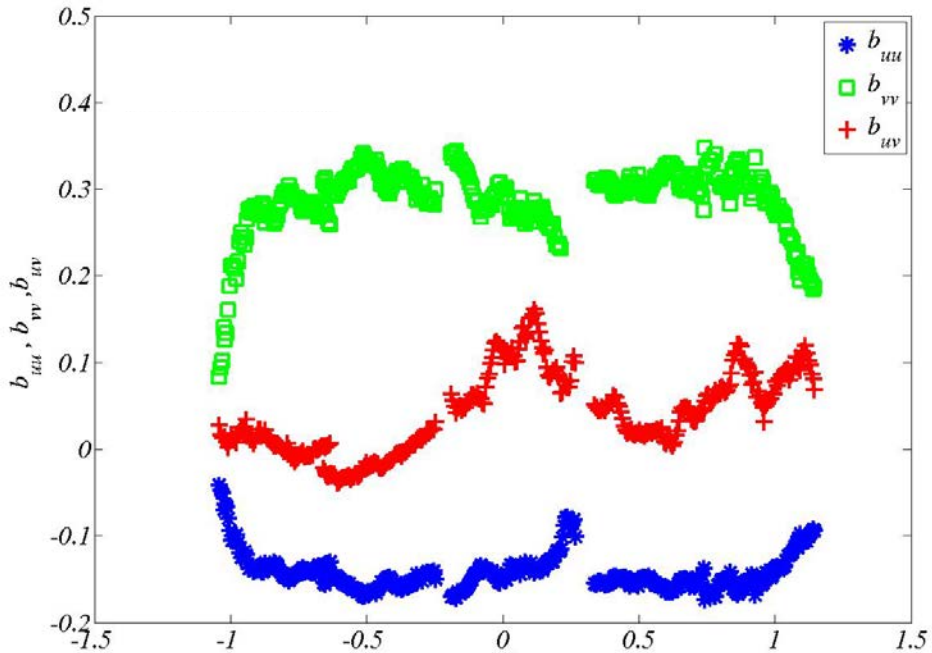
$Re = 3950$

$$b_{ij} = \frac{\overline{u_i u_j}}{\overline{u_k u_k}} - \frac{\delta_{ij}}{3}$$



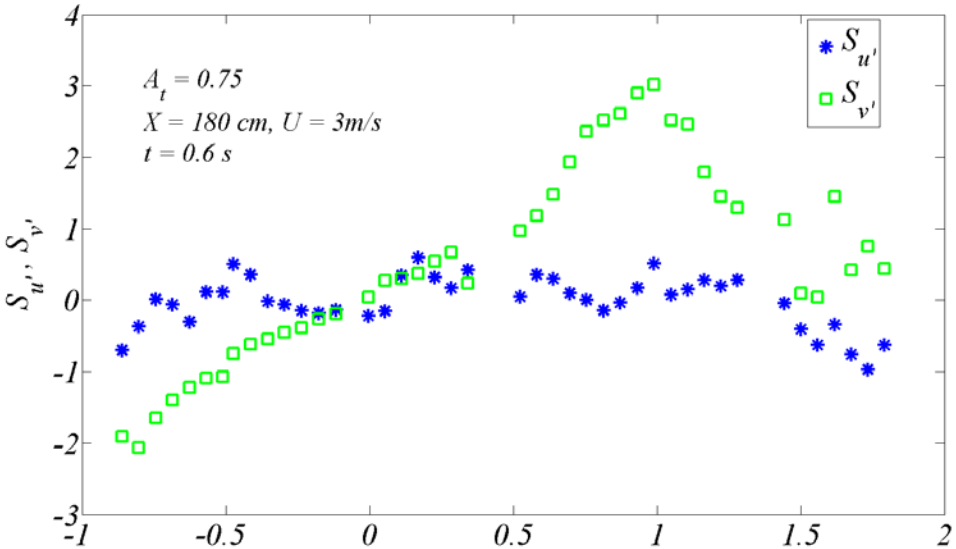
$Re = 8100$

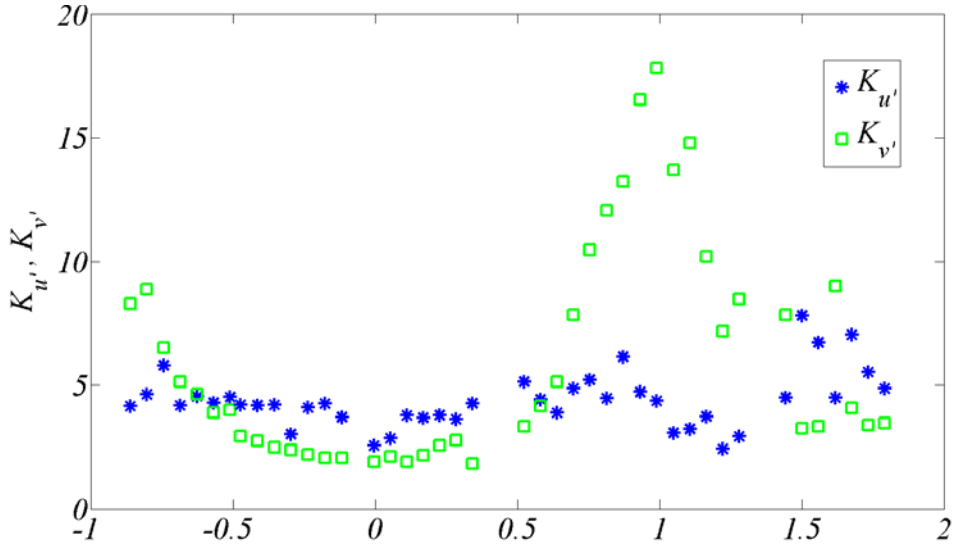


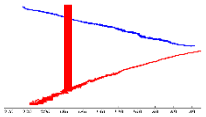


- Span-wise fluctuations assumed to be equal to stream-wise fluctuations
- Dominance of vertical velocity fluctuations in turbulent kinetic energy
- $b_{uv} \approx 0$

Skewness and Kurtosis







$Re = 8100$

$$S_{u'} = \frac{\overline{u'^3}}{\sigma_u^3}$$

Skewness

$$K_{u'} = \frac{\overline{u'^4}}{\sigma_u^4}$$

Kurtosis

- For Gaussian Distribution
  - S = 0
  - K = 3
- Gaussian like distribution of u' across the mix
- Dramatic skewing of v'