

# Quantum States of Matter in Strongly Correlated Electron Systems: Insight Through Advanced Computing

Thomas A. Maier  
Oak Ridge National Laboratory

SciDAC PI Meeting, July 2019

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This work was supported by the Scientific Discovery through Advanced Computing (SciDAC) program funded by U.S. Department of Energy, Office of Science, Advanced Scientific Computing Research and Basic Energy Sciences, Division of Materials Sciences and Engineering.



# Outline

1. **CompFUSE — Introduction**
2. **Some highlights**
3. **Bond-stretching phonons in  $\text{Ba}_{1-x}\text{K}_x\text{BiO}_3$**
4. **Two pairing domes in cuprates**
5. **Acknowledgments**

# CompFUSE Team

## Physics



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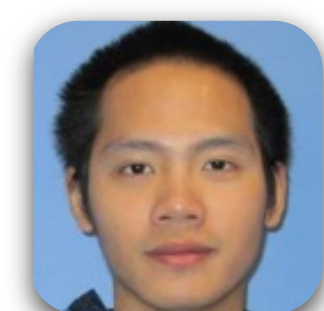
Peter Doak  
(ORNL)



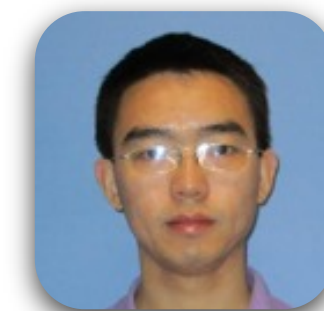
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(ORNL/UTK)



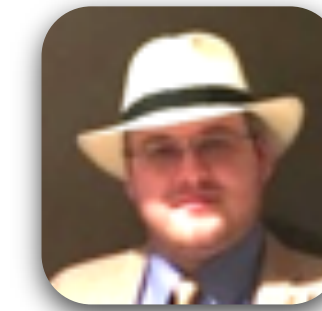
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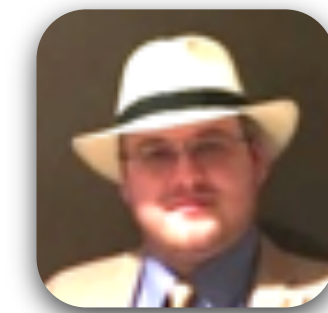
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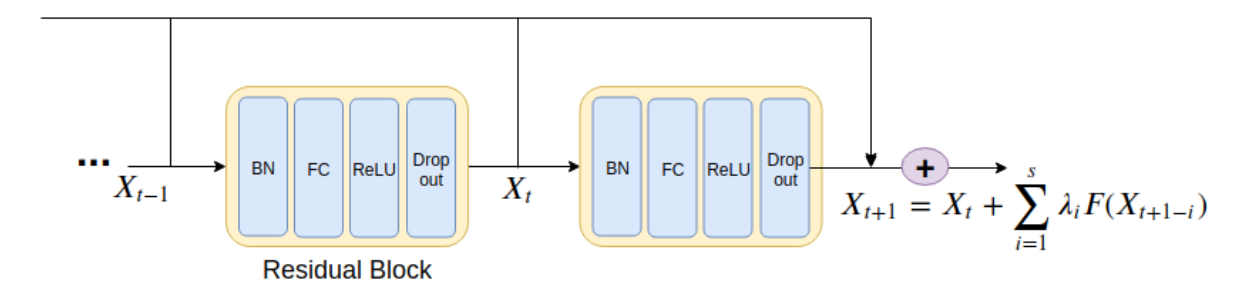
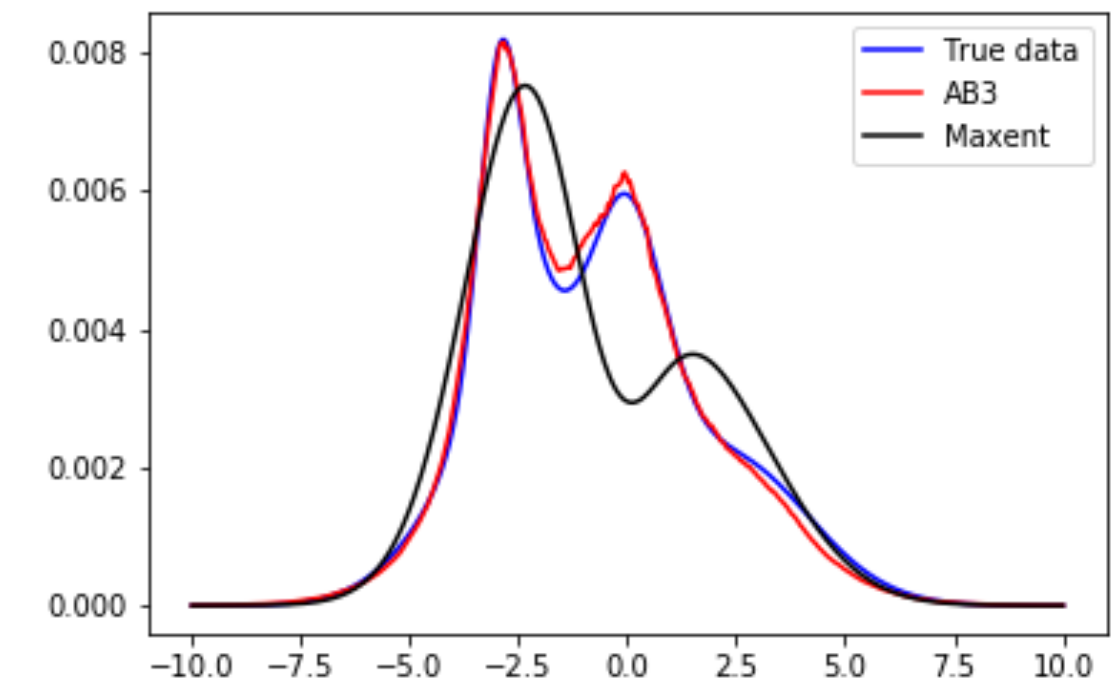
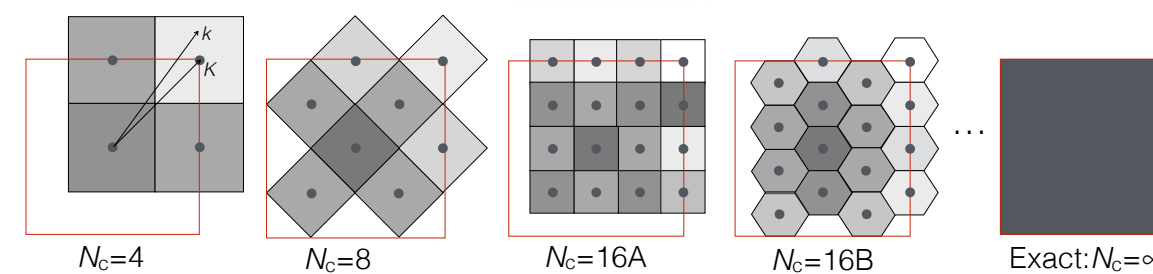
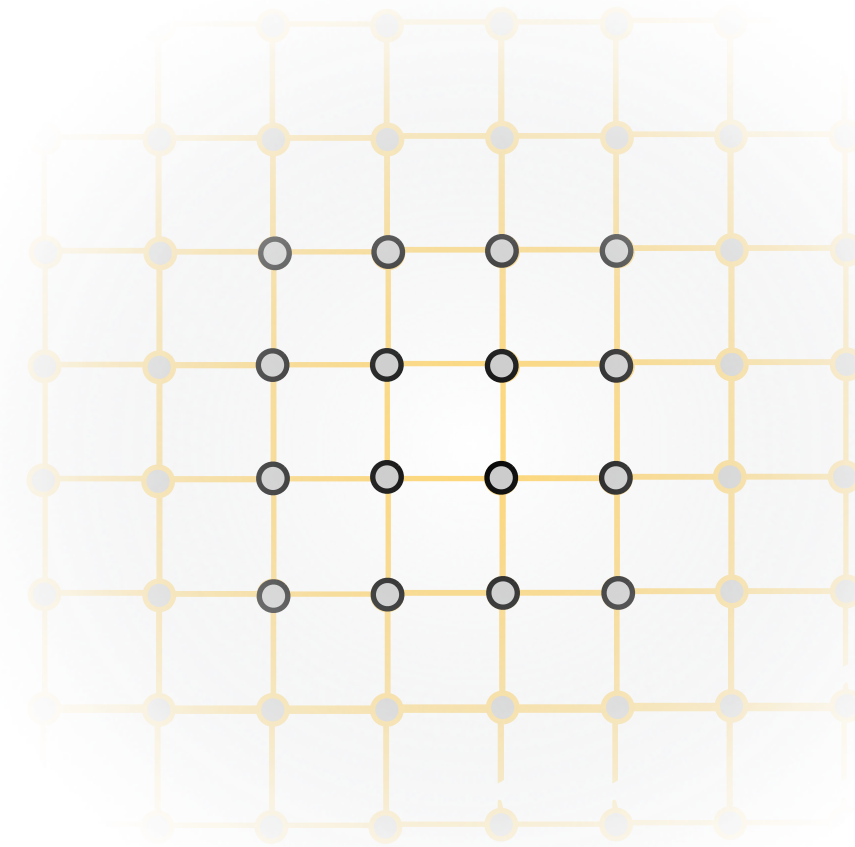
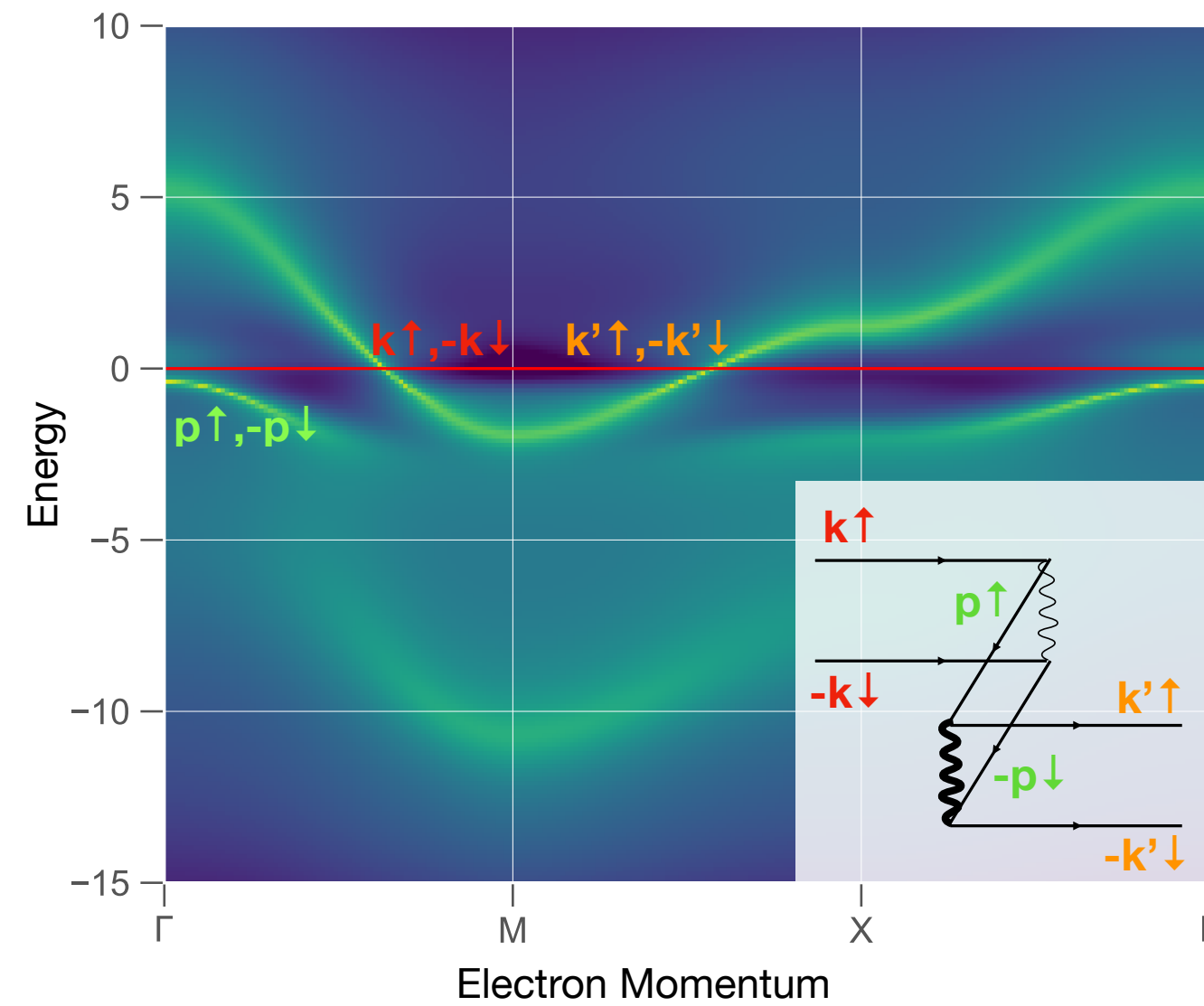
Matthew Bachstein  
(UTK)



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# CompFUSE: Computational framework for unbiased studies of correlated electron systems



Advanced simulations of correlated quantum materials, including unconventional superconductors and quantum spin liquids.

Development of accelerated algorithms based on DQMC, DCA and DMRG and efficient implementations on leadership class supercomputers.

Development of new approaches for analytic continuation of QMC results to extract real time dynamics from imaginary time data.

**Focus:** Dynamics, excited states, finite temperatures, mechanisms



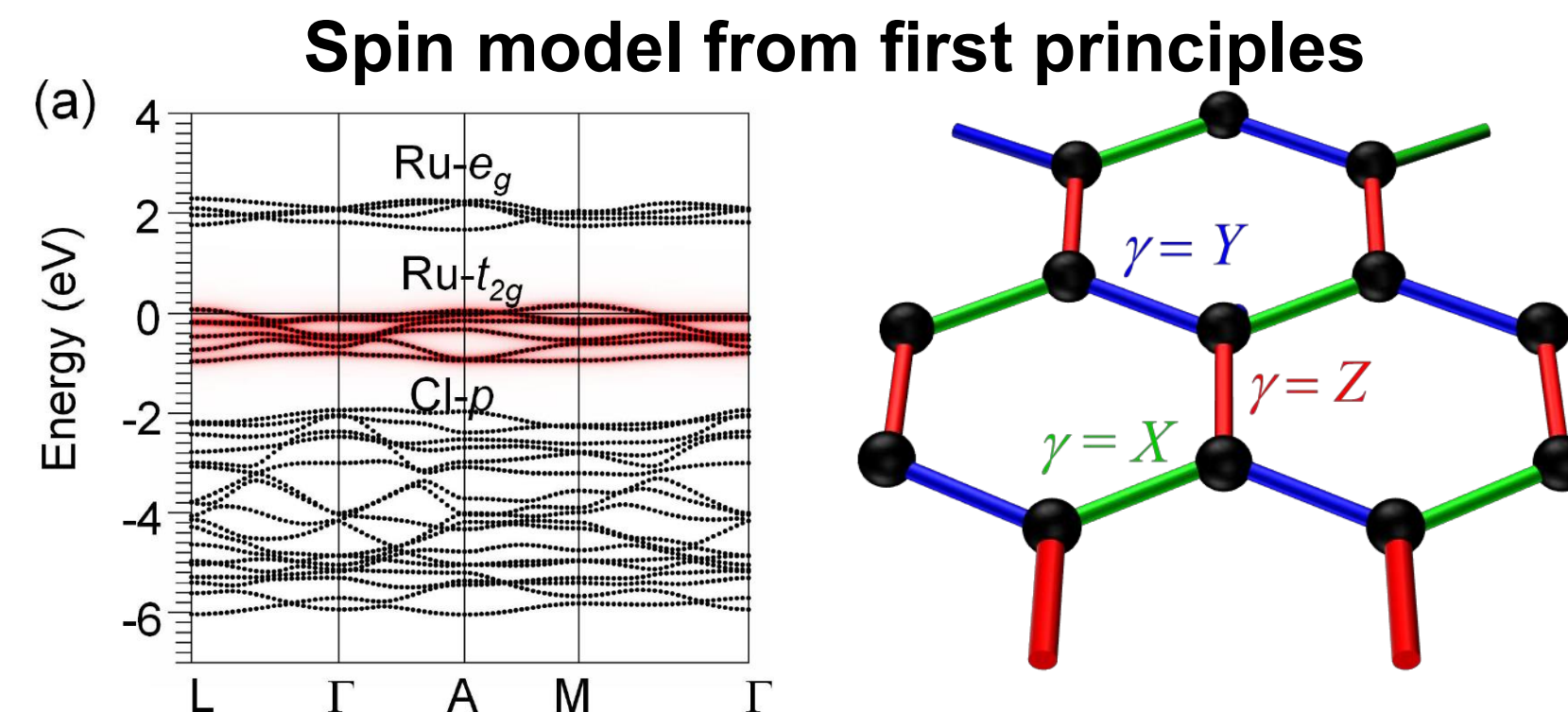
# Dynamical and thermal magnetic properties of $\alpha$ -RuCl<sub>3</sub>

## Failure of available spin models for $\alpha$ -RuCl<sub>3</sub>

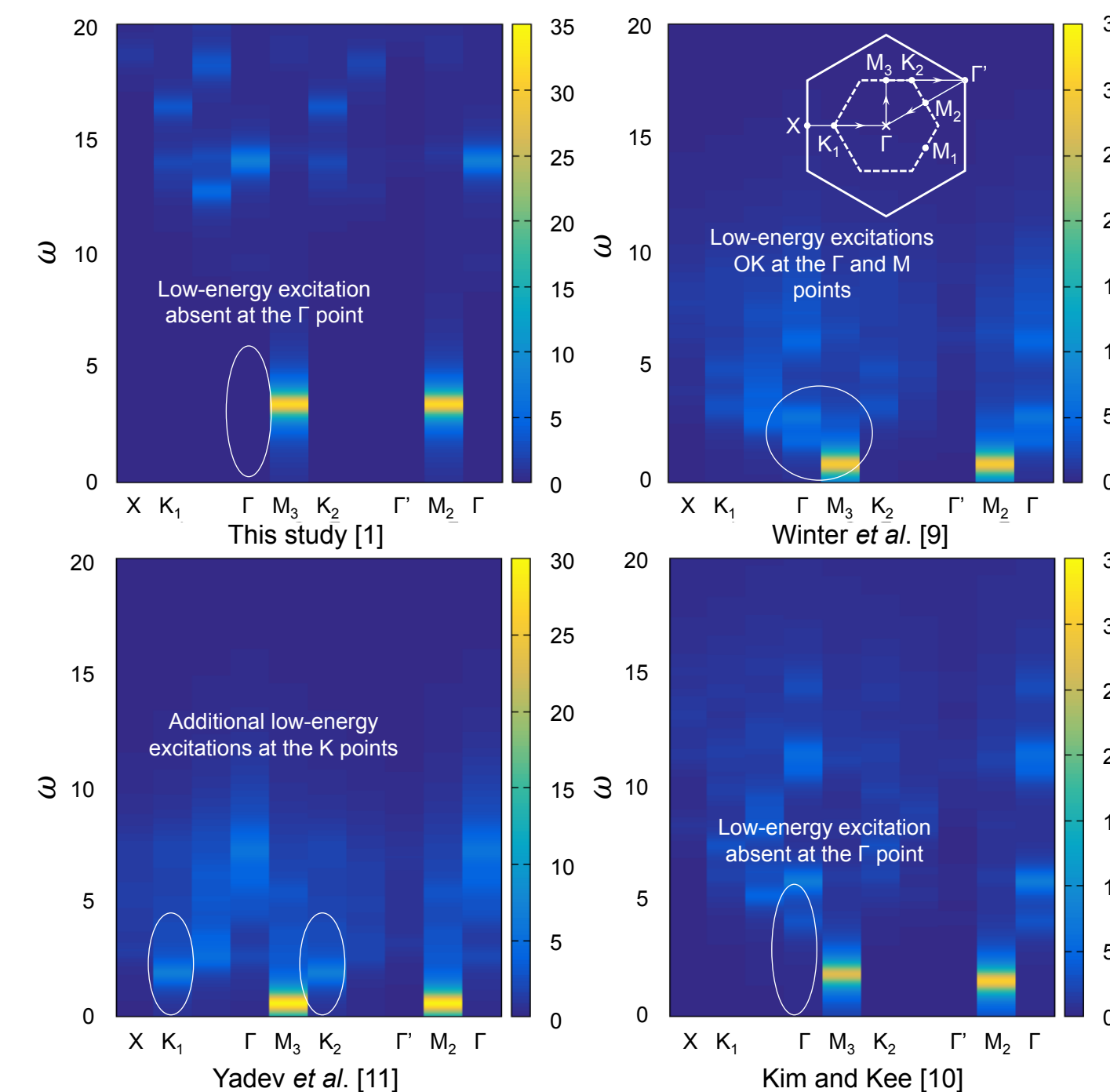
- **Derived a spin model for RuCl<sub>3</sub>** and studied several available models from the literature
- Using exact diagonalization based methods, calculated the **specific heat** and the **inelastic neutron scattering response**
- **All existing models fail to completely explain the experiments**

## Research Details

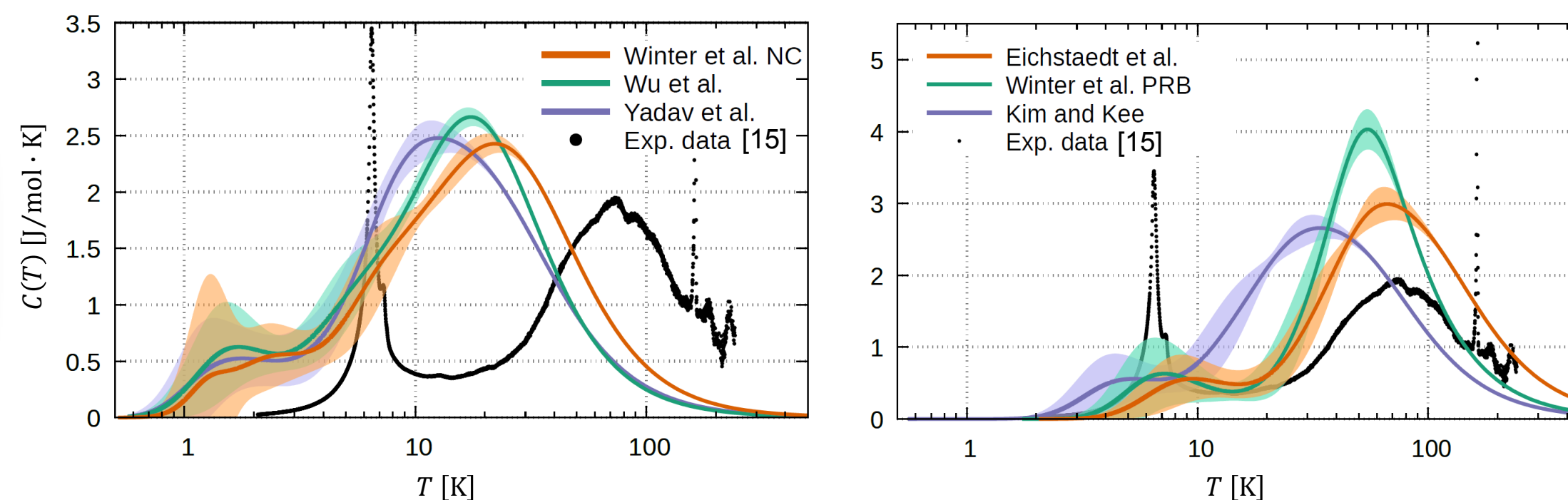
- DFT calculation of RuCl<sub>3</sub>, Wannier transformation to Ru-t<sub>2g</sub> states
- Interaction parameters from constrained Random Phase Approximation
- Spin-only model from perturbation theory



## Inelastic neutron scattering



## Magnetic specific heat



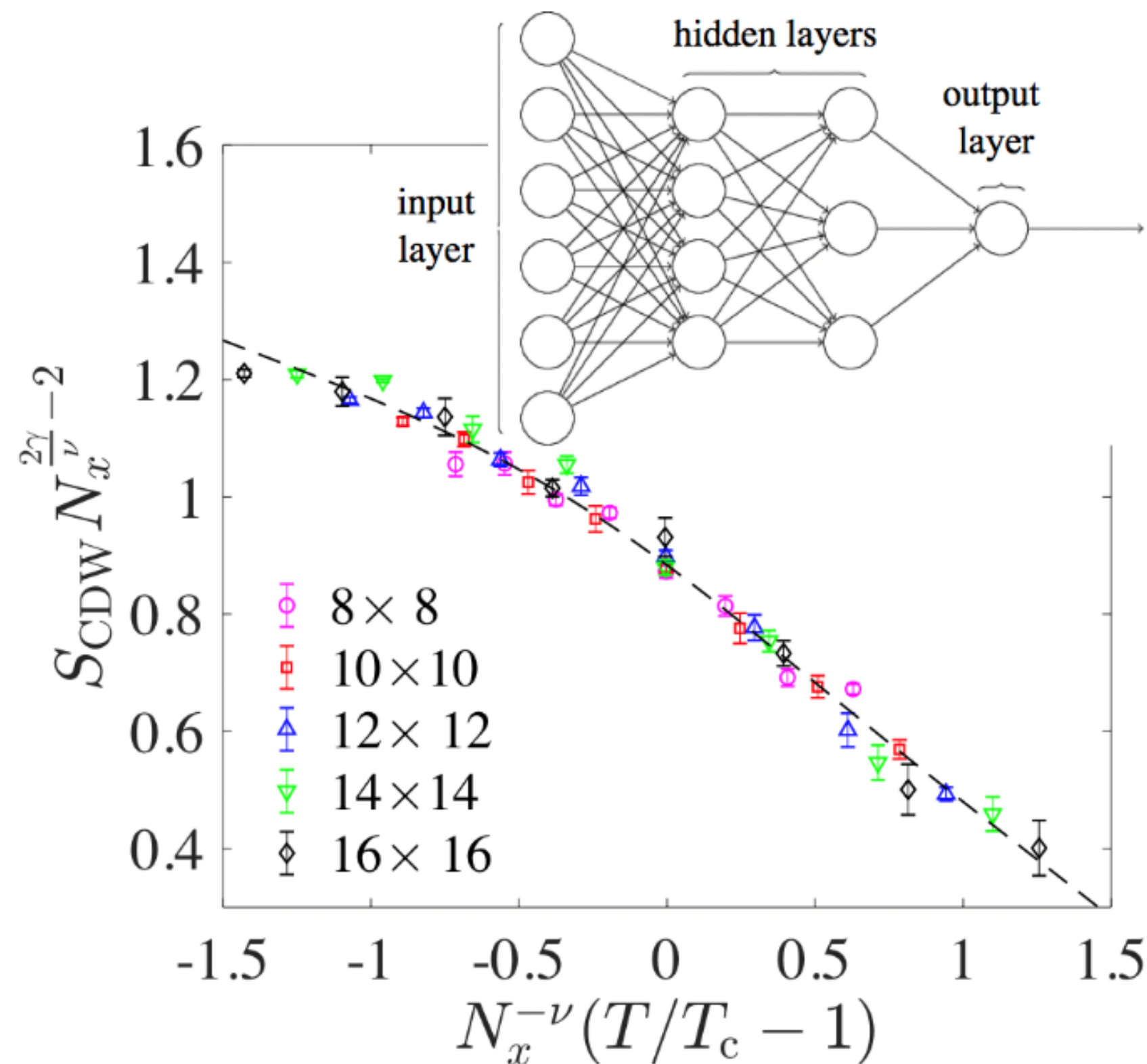
For details see poster by Satoshi Okamoto



*Eichstaedt et al., arXiv:1904.01523*  
*Laurell & Okamoto, arXiv:1906.07579*



# AI to accelerate determinantal lattice quantum Monte Carlo simulations



Thermodynamic scaling analysis of the **charge density wave structure factor** of the half-filled two-dimensional Holstein model in proximity to the metal-to-CDW insulator transition  $T_c$

## Neural networks to speed up DQMC

- **Artificial neural networks (ANNs)** can **predict** with near-perfect accuracy **moves in quantum Monte Carlo (QMC)** simulations of many-body Hamiltonians and obtain an order of magnitude reduction in the run time.
- Demonstration that machines can learn to perform efficient QMC simulations—without an underlying physics model and given only limited information about the configuration space—means the **method can be easily generalized even to other challenging models, such as the Fermi-Hubbard model.**

## Research Details

- ANNs were designed to predict the acceptance probabilities of local and global moves in determinantal QMC simulations of the Holstein model.
- This development in artificial intelligence (AI) granted access to large systems at low-temperatures, overcame long autocorrelation times for the model, and facilitated a thermodynamic scaling analysis.

For details see  
poster by  
Steve Johnston



*Li et al., arXiv:1905.10430*



# Analytic continuation of noisy data using multistep neural network

## New data-driven framework

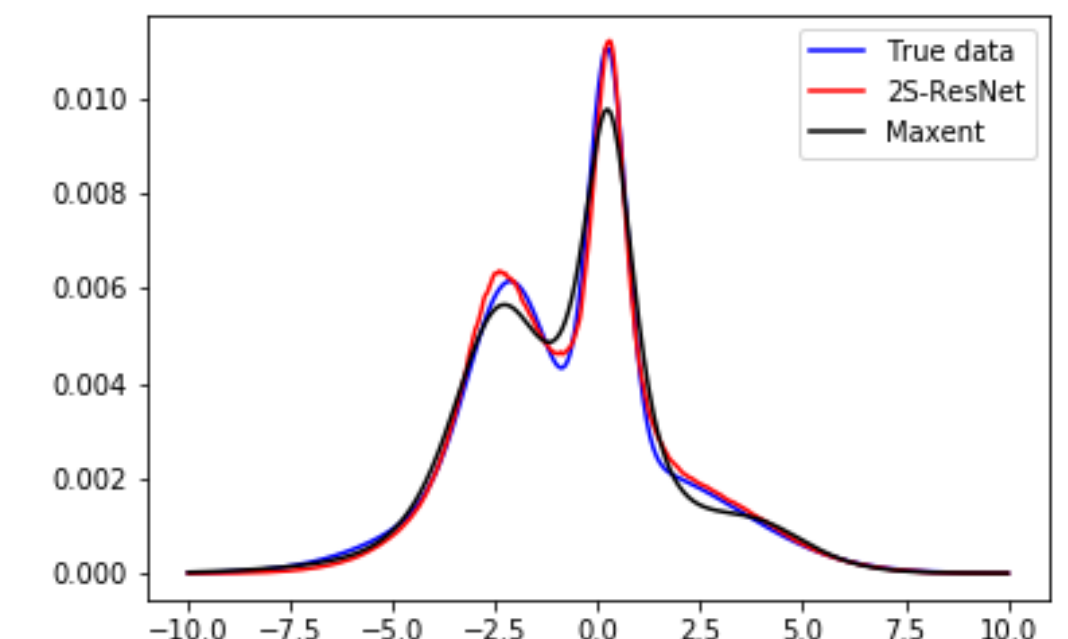
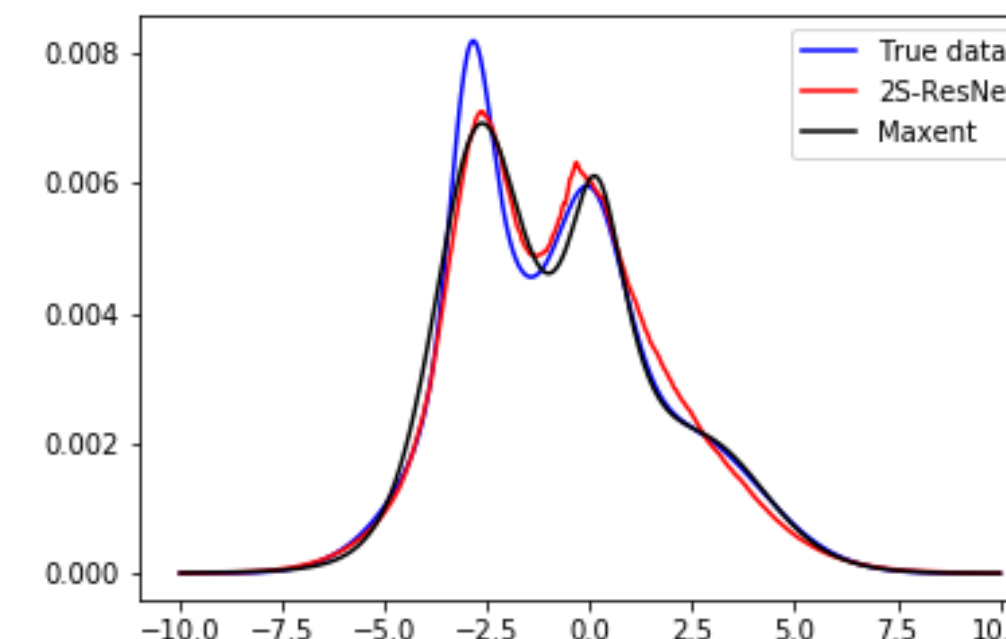
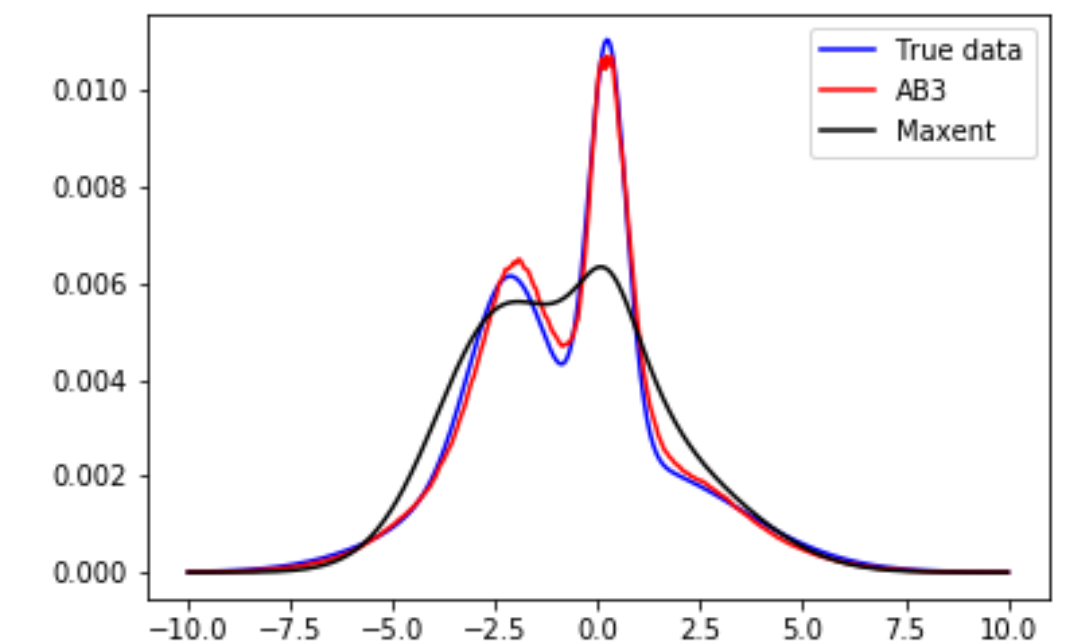
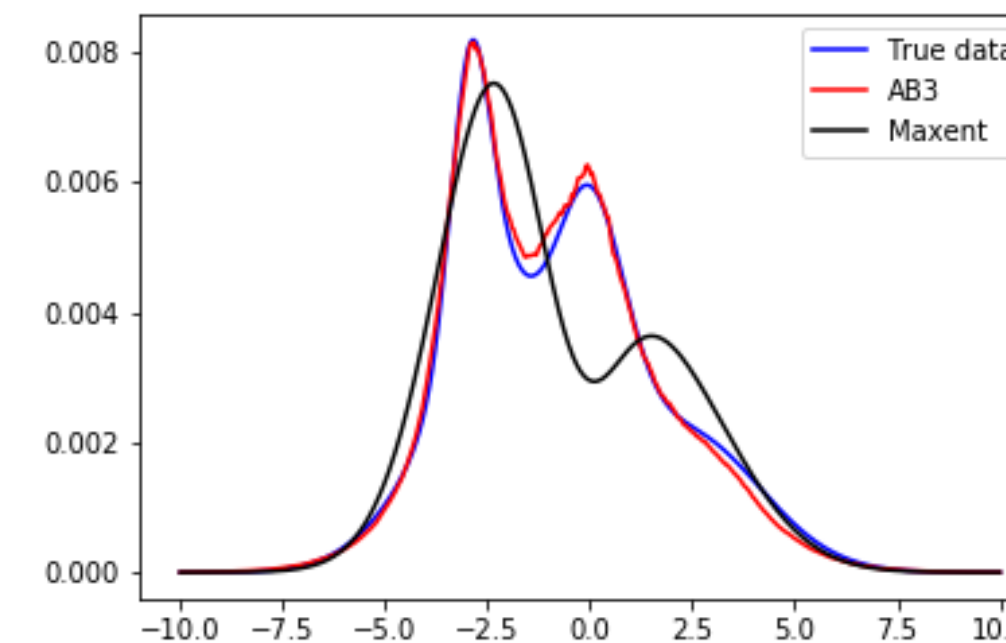
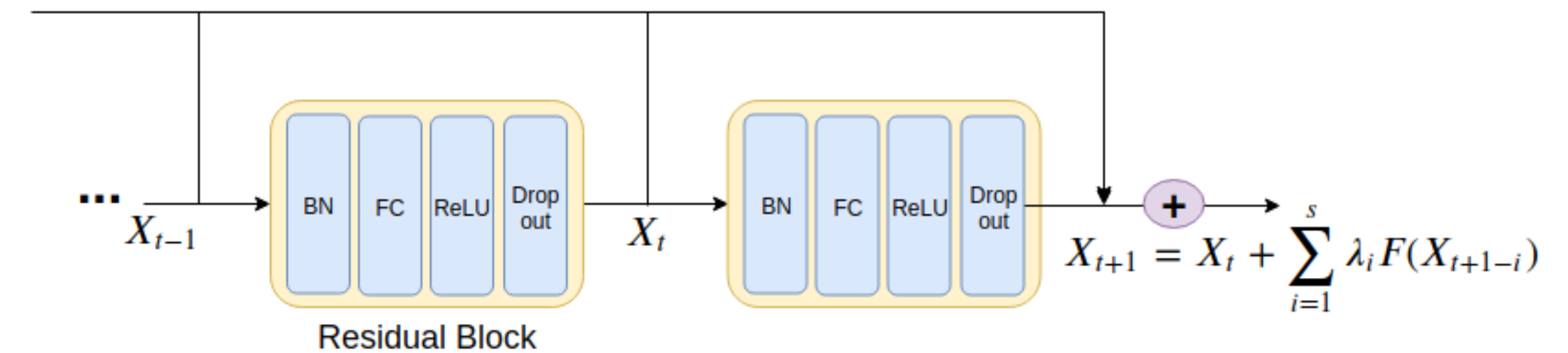
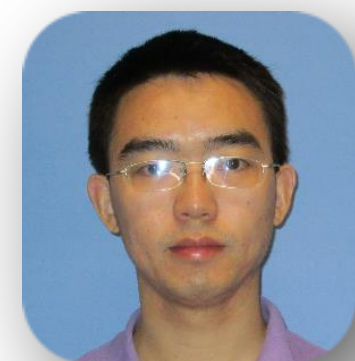
$$G(\tau) = \int K(\tau, \omega) A(\omega) d\omega; \quad K(\tau, \omega) = \frac{e^{-\omega\tau}}{1 + e^{-\beta\omega}}$$

- Use *novel linear multi-step residual neural network (ResNet)* to learn the inverse of the kernel
- Generate training data set as a sum of R uncorrelated Gaussians
- 100k training data set, 1000 samples for validation and testing

## Compared to Maximum Entropy

- 2-step ResNet gives similar results for high-quality data, and much better results at high noise levels.

For details see  
poster by  
Xuping Xie



Xie et al., arXiv:1905.10430



# Accelerating DCA++ on Summit

## DCA++ code

- Two kernels: (1) Coarse-graining of momentum space to map lattice problem onto effective cluster problem, (2) *quantum Monte Carlo cluster (QMC) solver*
- *Two-level parallelization (MPI internode + std::threads intranode)*

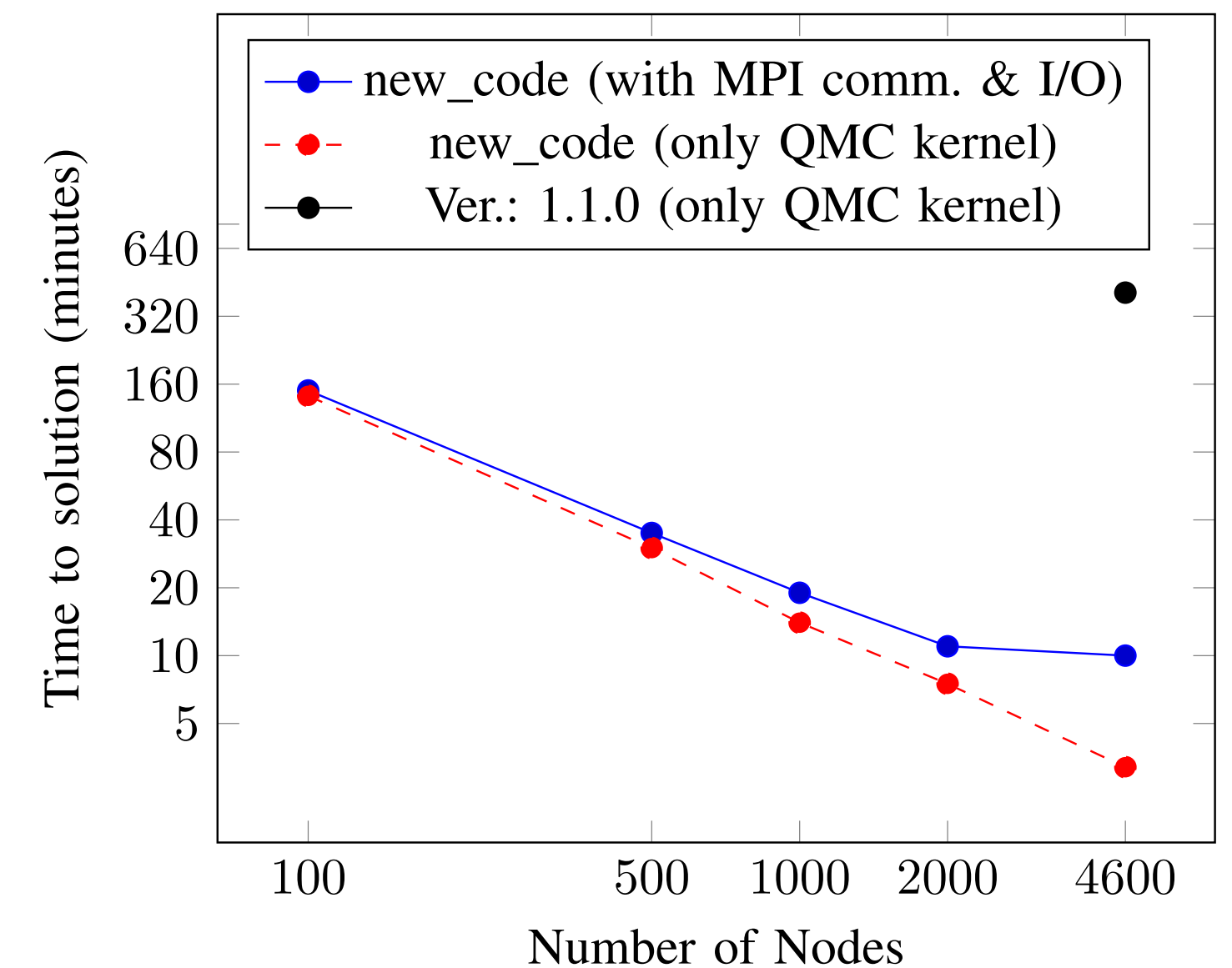
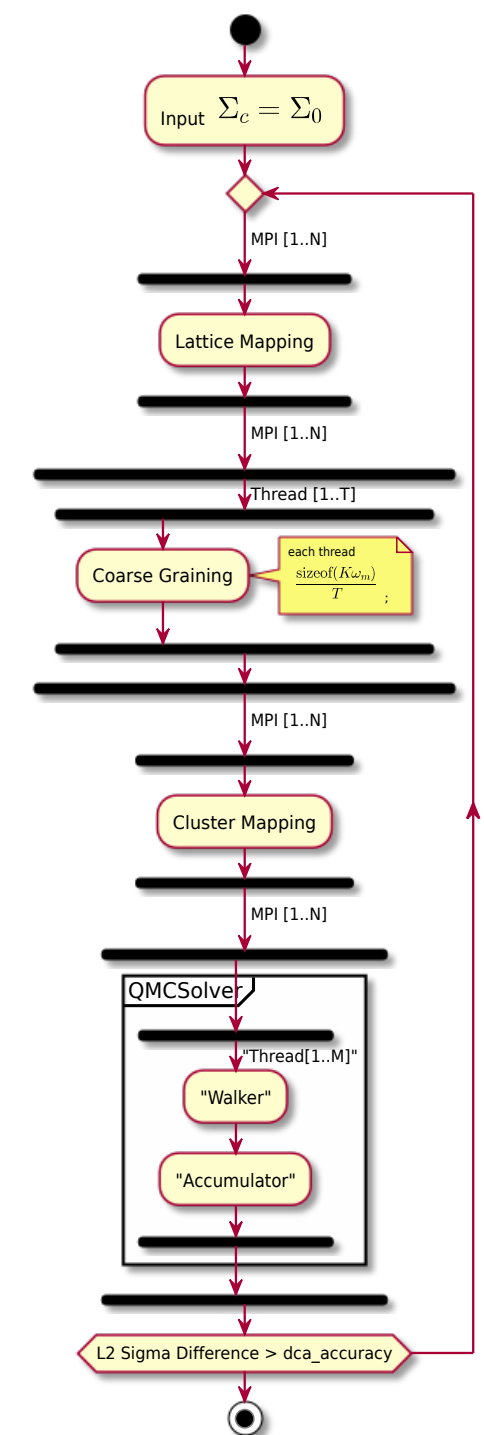
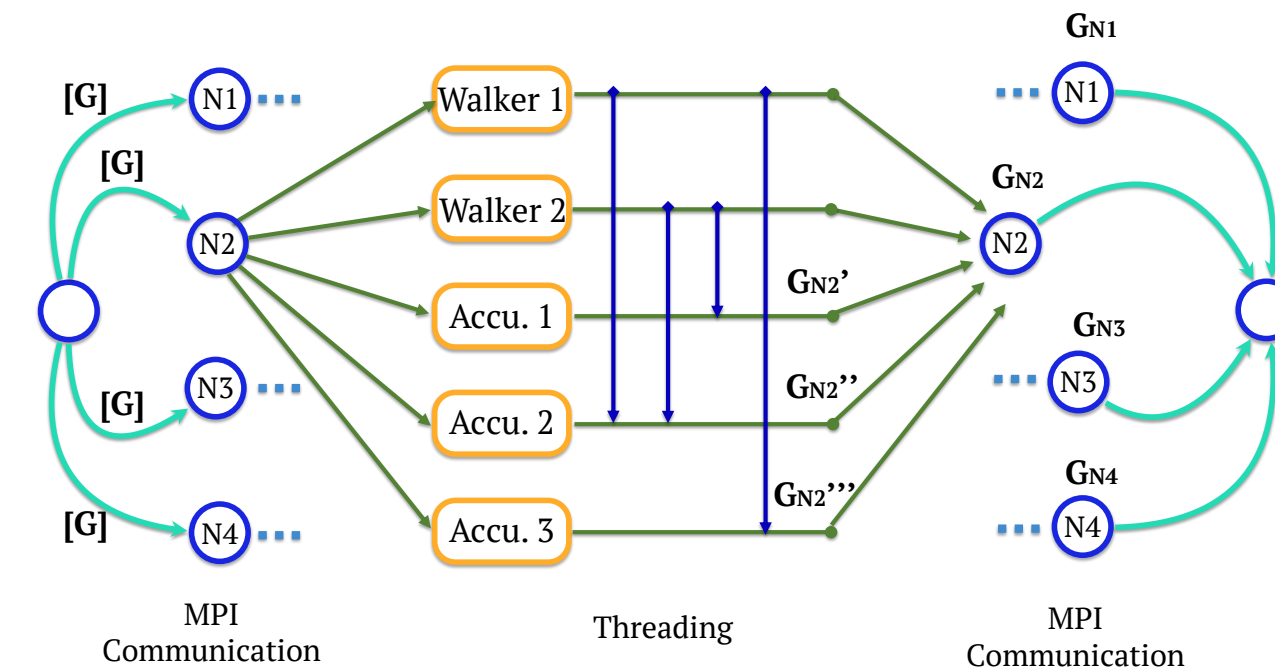
## Several optimizations

- *Extensive profiling* (in collaboration with RAPIDS institute)
- Coarse-graining: Improved multi-threading and intra-process scheduling
- **QMC solver:** (1) Improved asynchronous CPU-GPU communication  
(2) *GPU support for measurement accumulation of observables*  
(3) Improved (direct) walker - accumulator communication  
(4) Dynamic intra-node workload distribution  
(5) Overlapping computation and communication  
(6) *Mixed precision* – Double precision (walker), single precision (two-particle vertex accumulation)

## Speedup & Performance on Summit

- ~ 100 x speedup
- ~ 75 PFlops

*Chatterjee et al., submitted to PACT '19*





# Accelerating DMRG++ for dynamics and finite temperatures

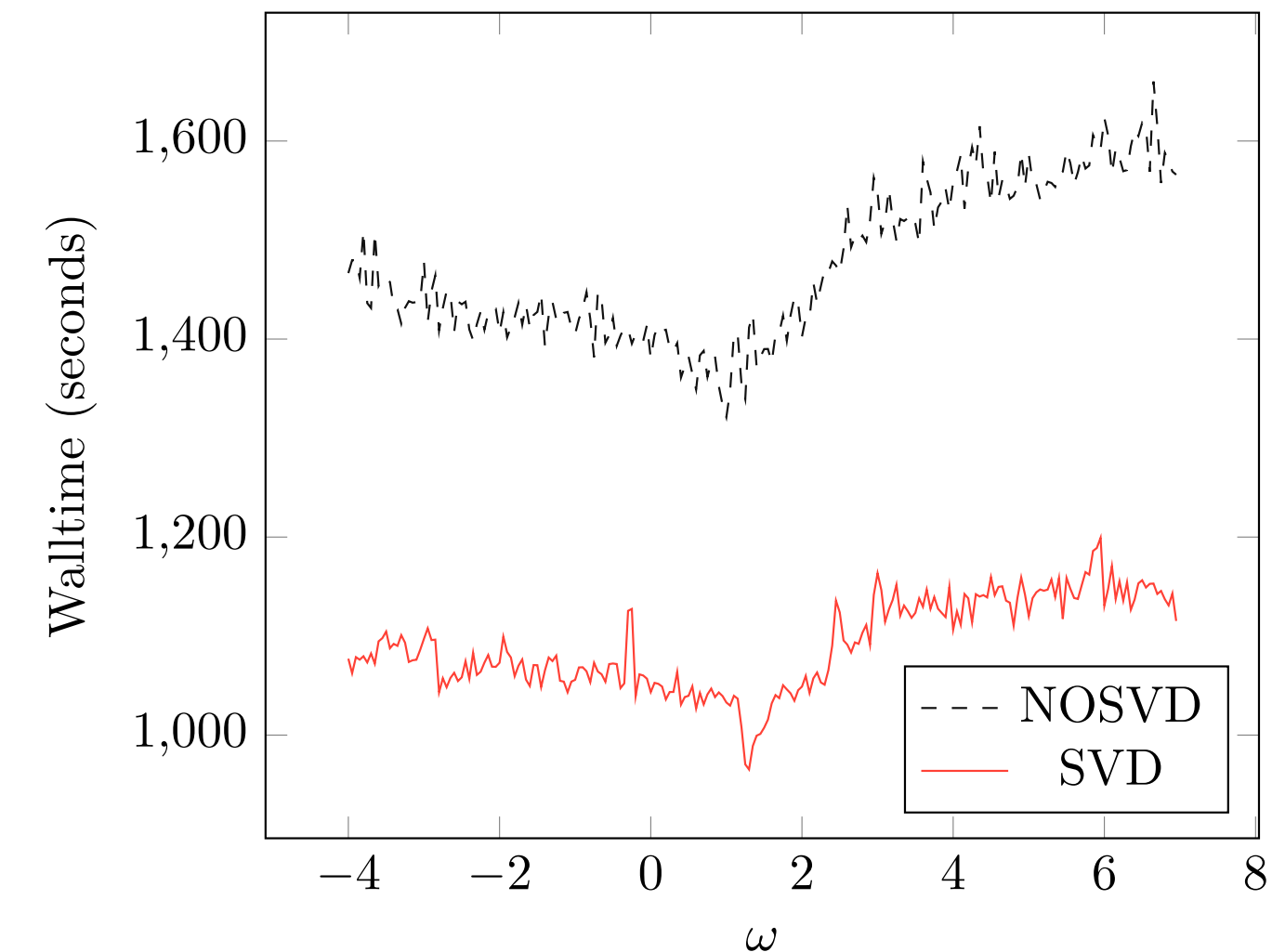
## Targeting multiple states in DMRG

- **Multiple states** are needed in density matrix matrix for **finite temperature calculations and dynamics**
- Replacing the density matrix by a **singular value decomposition (SVD) increases performance** significantly and enables multiple targeting

## Optimized GPU support for Summit

- Use of **multiple GPUs** for larger problems (more states)
- Use of **atomic updates in MAGMA** library to take advantage of symmetry of Hamiltonian to reduce HBM storage on GPU
- Calculation of spectral functions is embarrassingly parallel and thus can take full advantage of parallelization on Summit, significantly increasing frequency resolution

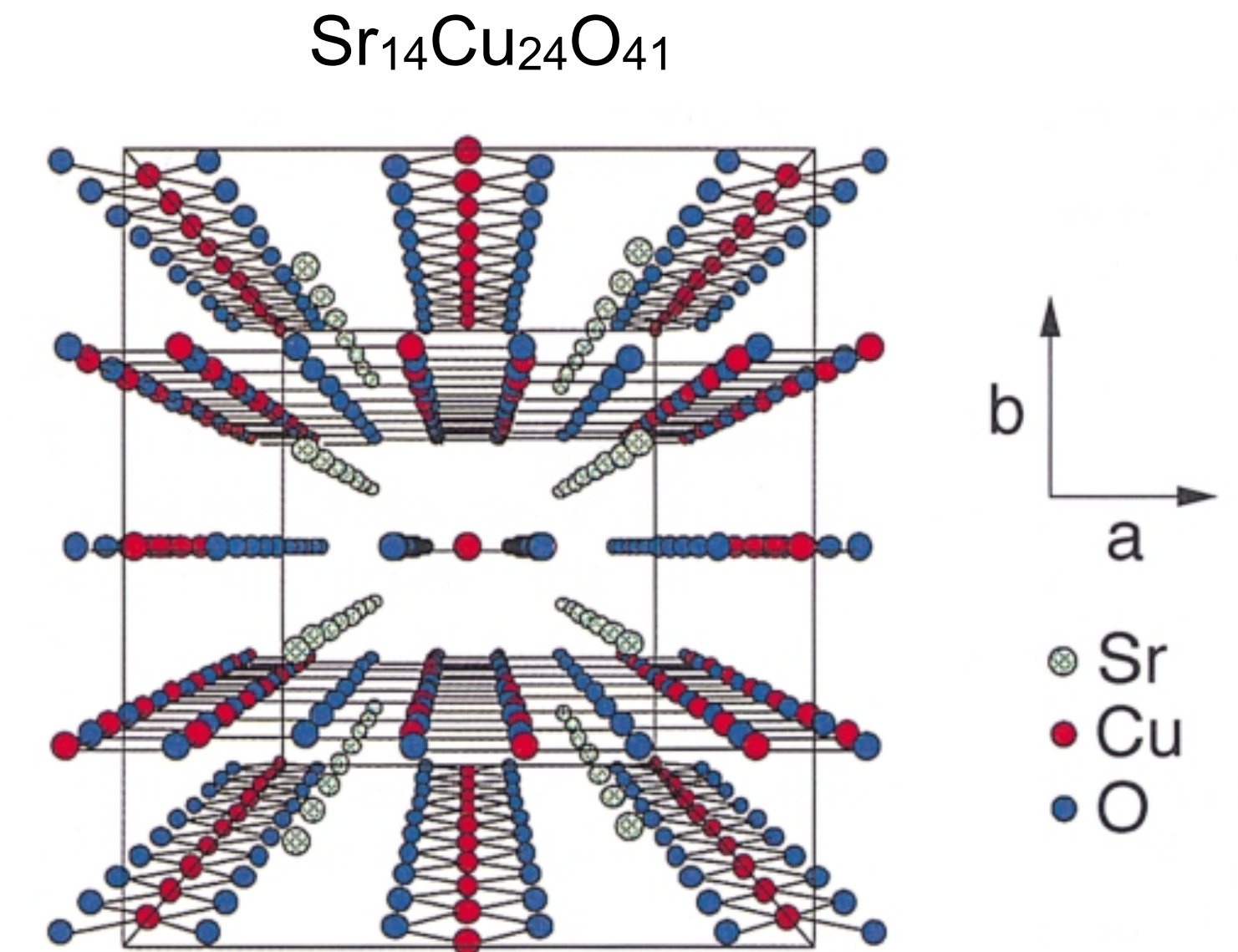
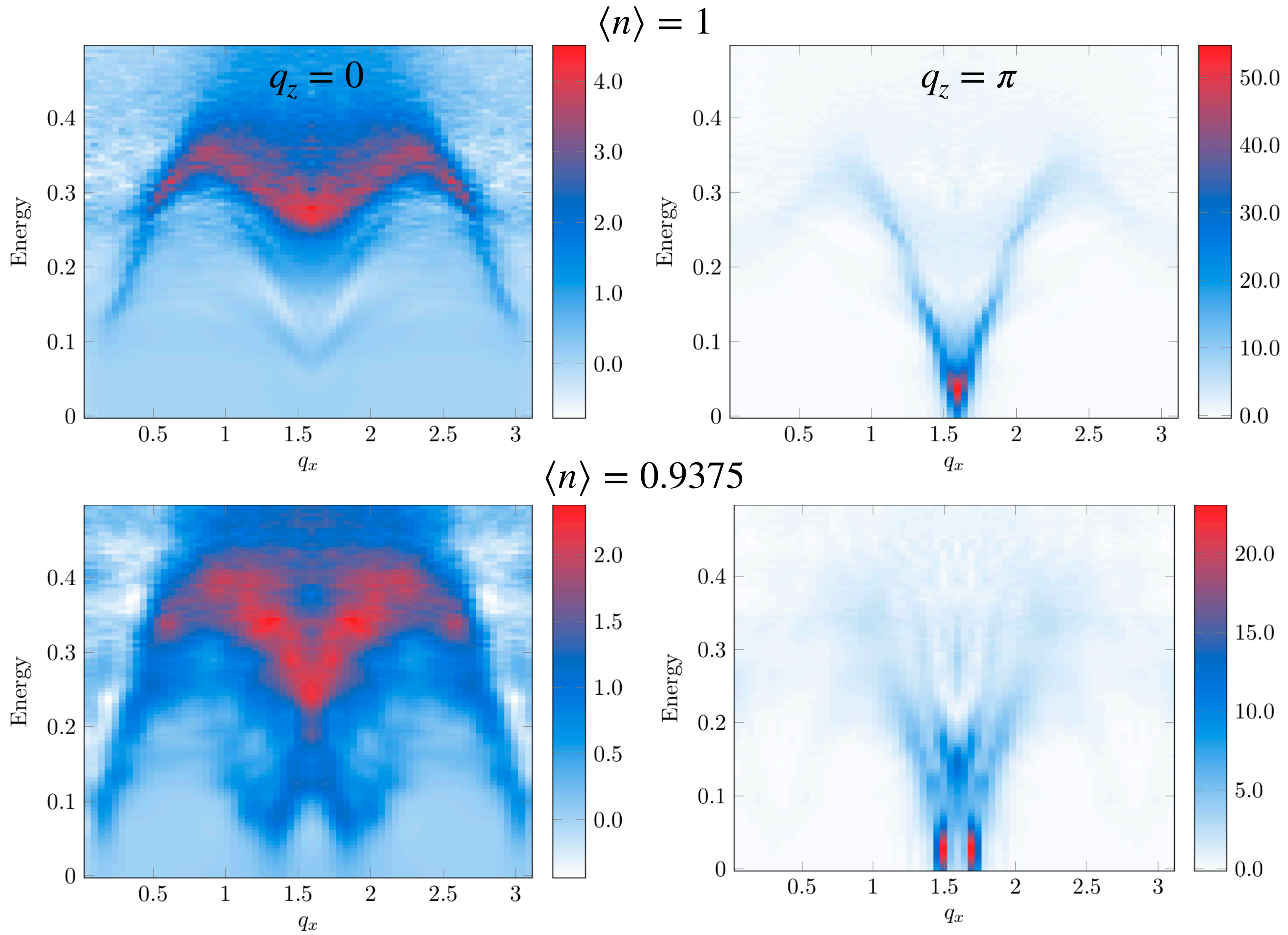
*D’Azevedo et al., submitted to Phys. Rev. E (2018)*



| $\omega$ | GPU        |           | CPU        |           |
|----------|------------|-----------|------------|-----------|
|          | fullRunSVD | fullRunDM | fullRunSVD | fullRunDM |
| GS       | 191        | 231       | 193        | 241       |
| -2       | 1073       | 1406      | 1690       | 1969      |
| 0        | 1043       | 1380      | 1710       | 1947      |
| 2        | 1048       | 1402      | 1757       | 2014      |

Run-time in seconds on a Volta GPU on x86 Linux workstation. Note all SVD performed on CPU.

# DMRG: $S(Q, \omega)$ for telephone ladder cuprates



*From Matsuda et al., Riken Review '99*

**Parameters:**  
 2-leg Hubbard ladder;  $64 \times 2$ ;  
 intermediate to strong coupling



# Outline

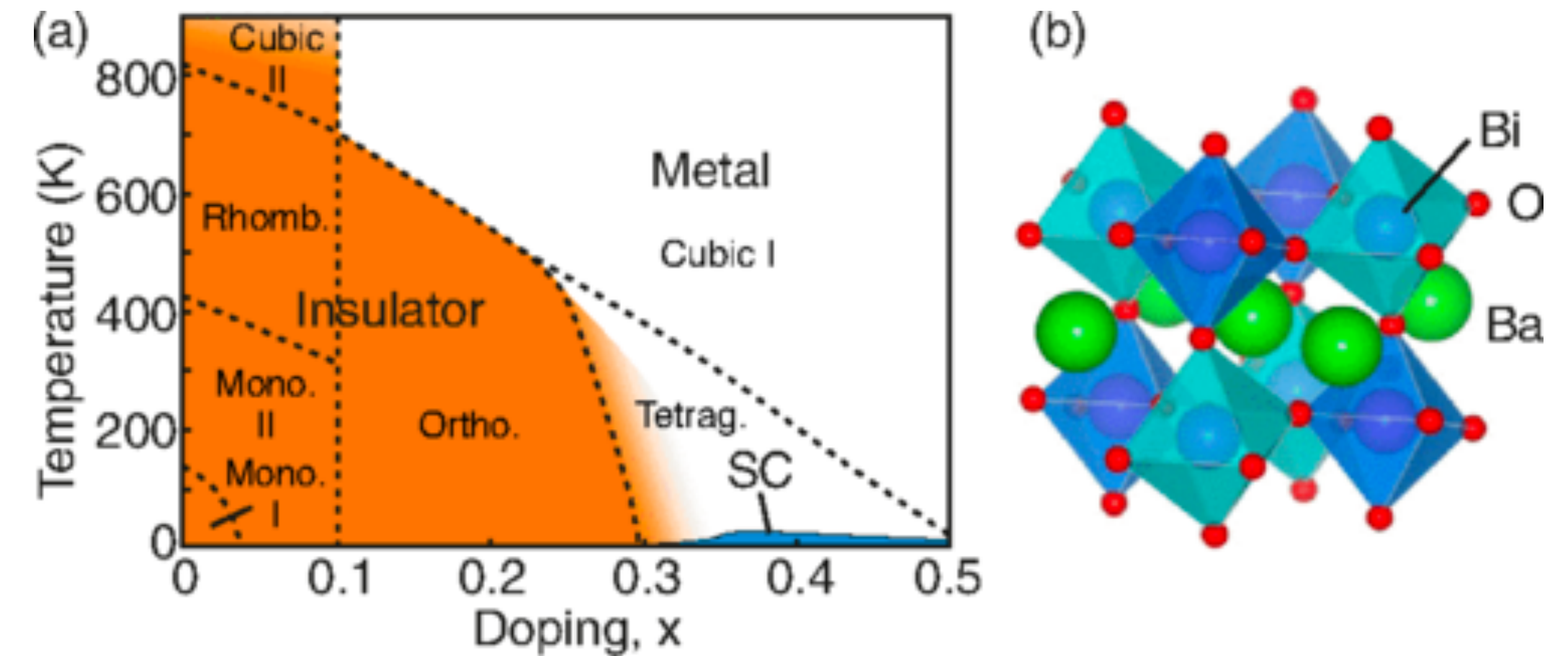
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# DQMC study of bond phonons in $\text{Ba}_{1-x}\text{K}_x\text{BiO}_3$

Li et al., arXiv:1901.07612

## $\text{Ba}_{1-x}\text{K}_x\text{BiO}_3$

- Negative charge transfer regime
- Holes self-dope from cation to ligand oxygen atoms
- Hybridization between cation and oxygen atom leads to sizable electron-phonon coupling
- **Charge-ordered insulator** for  $x=0$
- **Superconductor** ( $T_c \sim 30$  K) at finite  $x$



From Plumb et al., PRL '16

## 3-orbital Su-Schrieffer-Heger (SSH) model

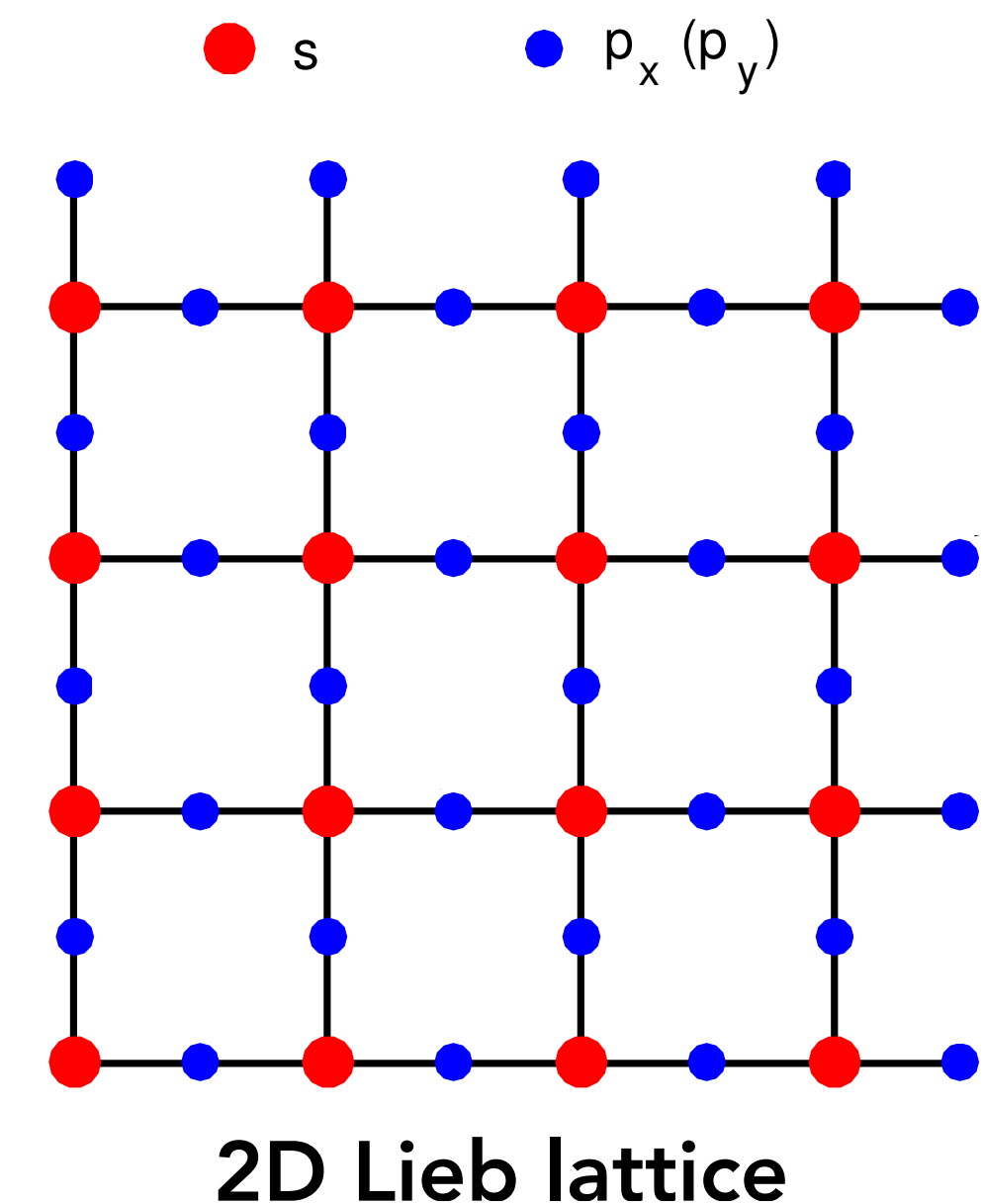
$$H_0 = -t_{sp} \sum_{\langle \mathbf{r}, \delta \rangle, \sigma} (P_{\delta} s_{\mathbf{r}, \sigma}^{\dagger} p_{\mathbf{r}, \delta, \sigma} + \text{h.c.}) + t_{pp} \sum_{\langle \mathbf{r}, \delta, \delta' \rangle, \sigma} P_{\delta, \delta'} p_{\mathbf{r}, \delta, \sigma}^{\dagger} p_{\mathbf{r}, \delta', \sigma} + \sum_{\mathbf{r}, \sigma} \left[ (\epsilon_s - \mu) \hat{n}_{\mathbf{r}, \sigma}^s + (\epsilon_p - \mu) (\hat{n}_{\mathbf{r}, \sigma}^{p_x} + \hat{n}_{\mathbf{r}, \sigma}^{p_y}) \right]$$

$$H_{\text{lat}} = \sum_{\mathbf{r}} \left( \frac{\hat{p}_{\mathbf{r}, x}^2}{2M} + K \hat{X}_{\mathbf{r}, x}^2 + \frac{\hat{p}_{\mathbf{r}, y}^2}{2M} + K \hat{X}_{\mathbf{r}, y}^2 \right)$$

$$\text{Phonon frequency } \Omega = \sqrt{2} t_{sp}$$

$$H_{\text{e-ph}} = \alpha t_{sp} \sum_{\langle \mathbf{r}, \delta \rangle, \sigma} \left( \hat{u}_{\mathbf{r}, \delta} s_{\mathbf{r}, \sigma}^{\dagger} p_{\mathbf{r}, \delta, \sigma} + \text{h.c.} \right)$$

$$\text{e-ph coupling strength } \alpha = 4a^{-1}$$



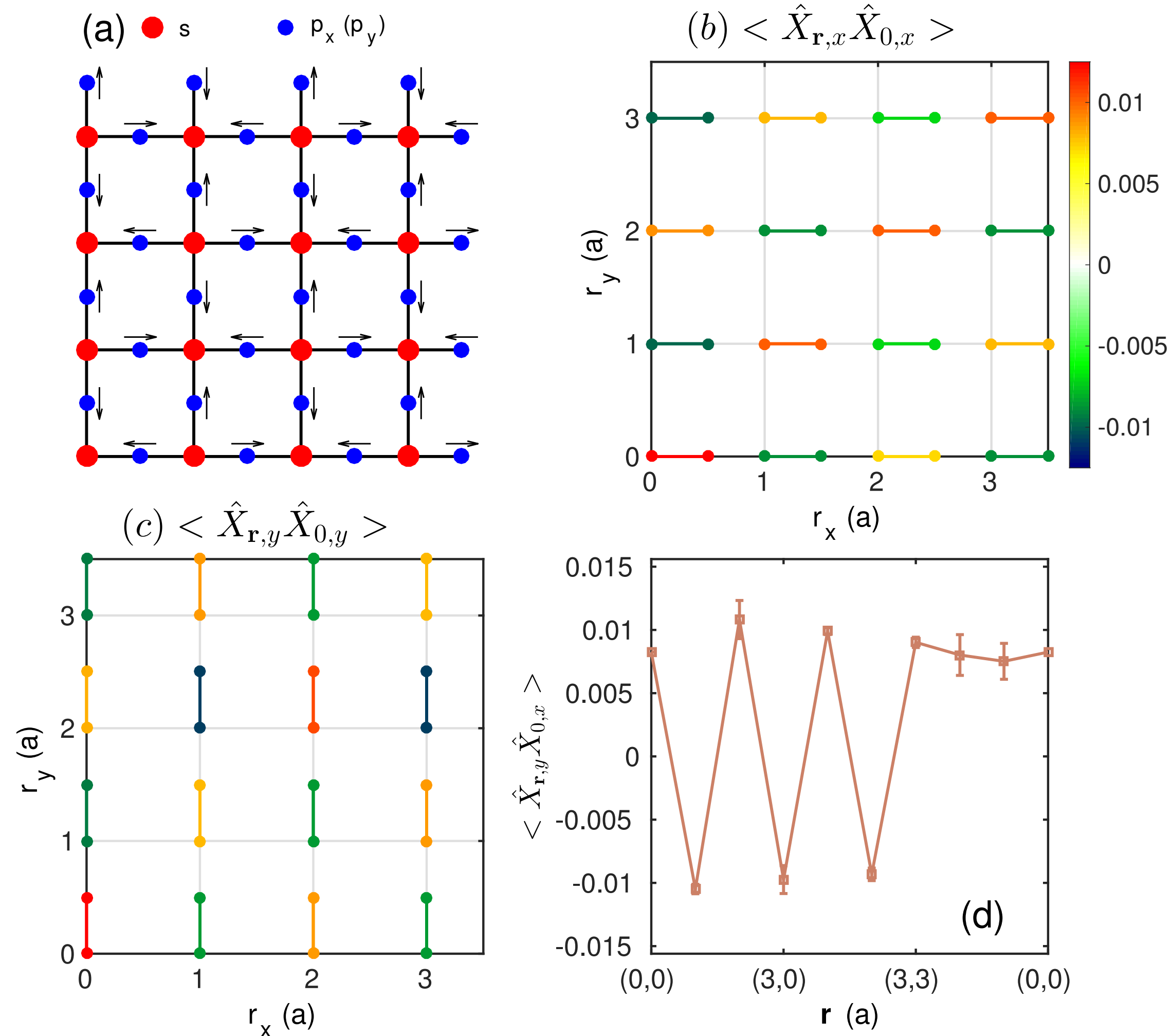
Sign-problem free DQMC algorithm!

First non-perturbative treatment in > 1D!

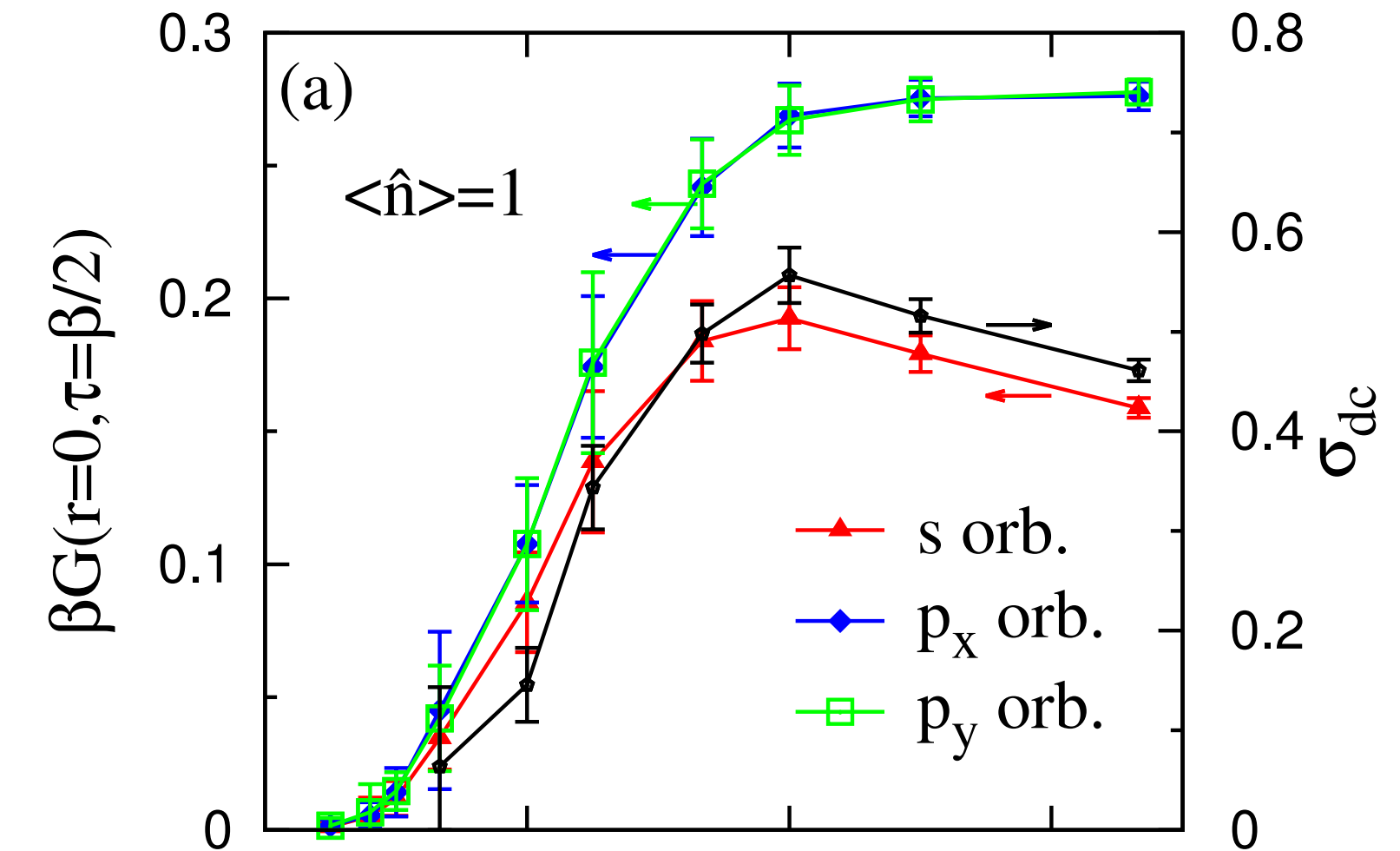


# Bond disproportionated, charge ordered state at half-filling

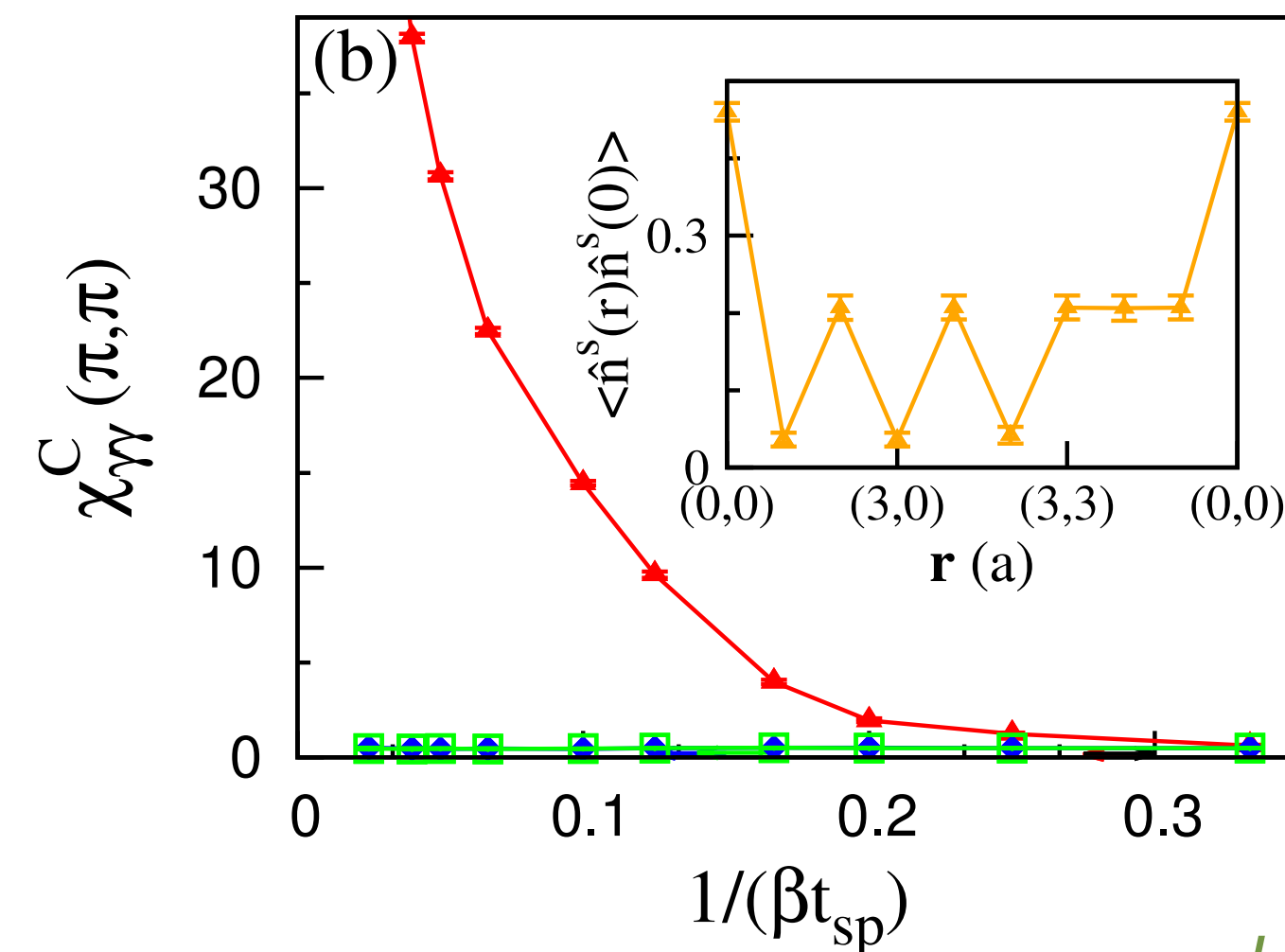
## Displacement correlation functions



## dc conductivity and spectral weight at $E_F$



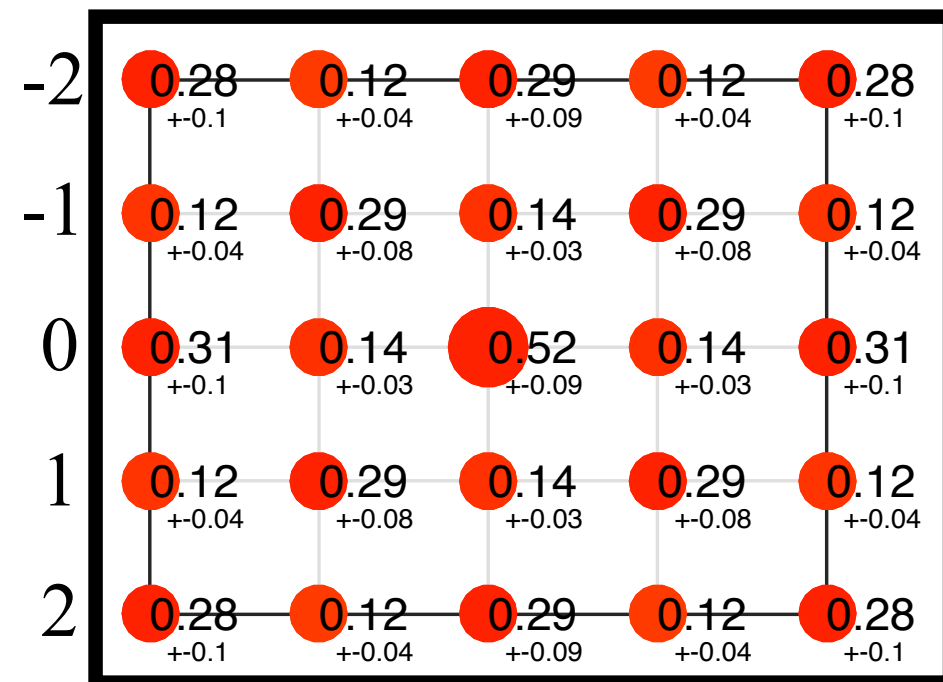
## CDW susceptibility



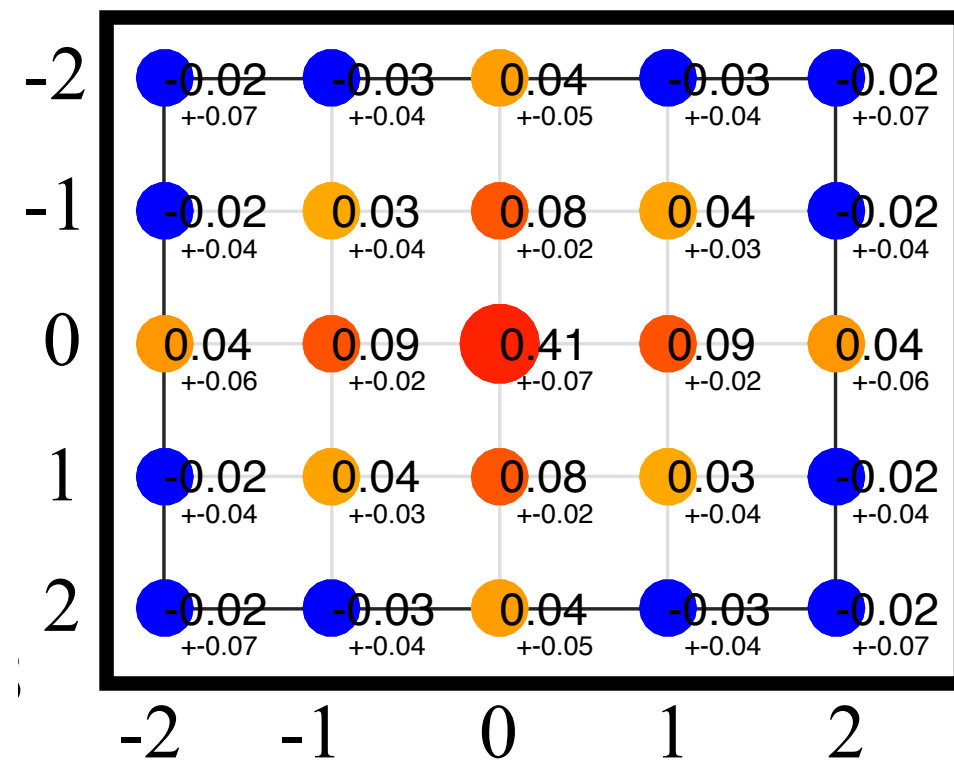
# Polaronic liquid and superconductivity at finite doping

Staggered polaron correlations

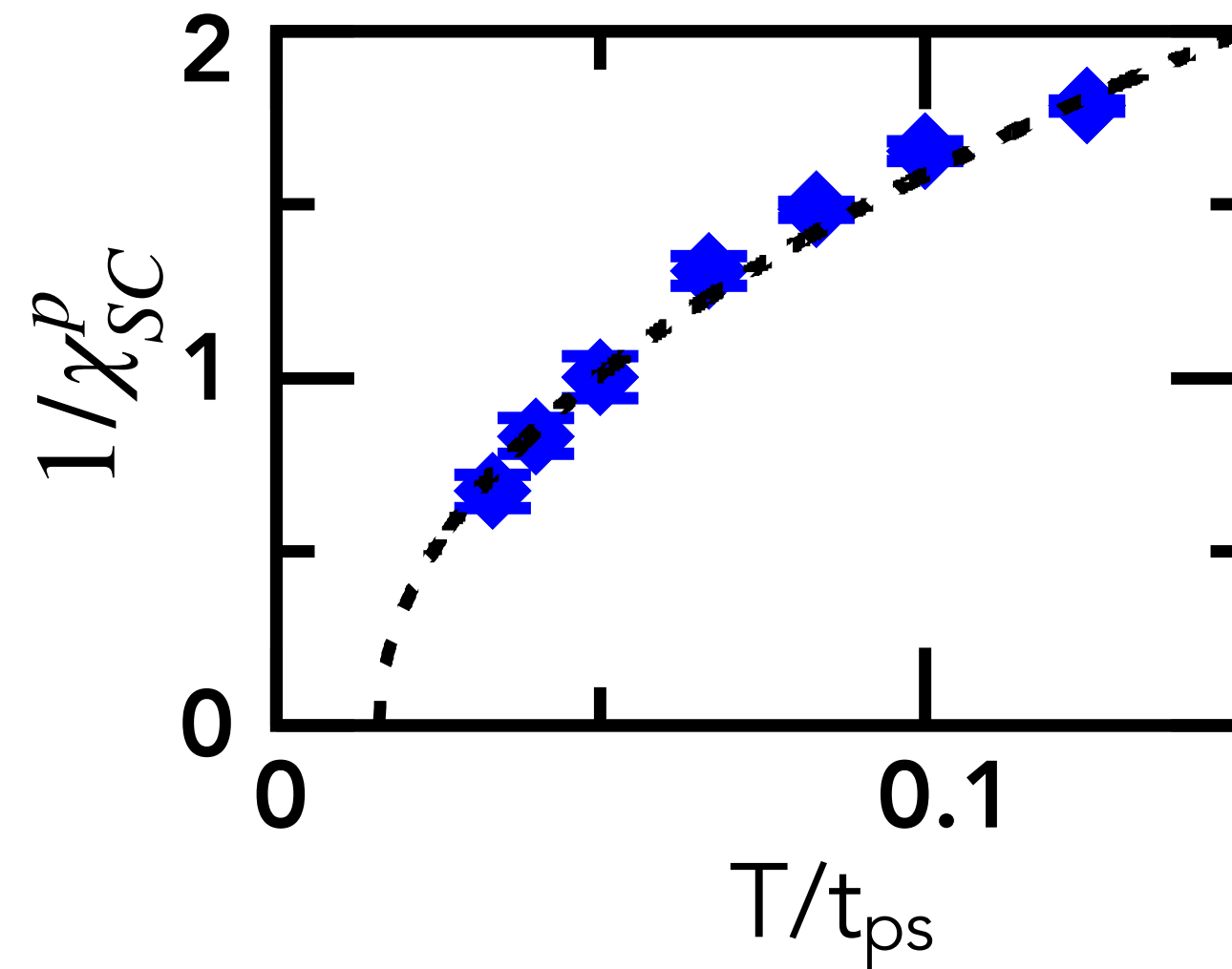
(c)  $\langle \hat{n} \rangle = 1$



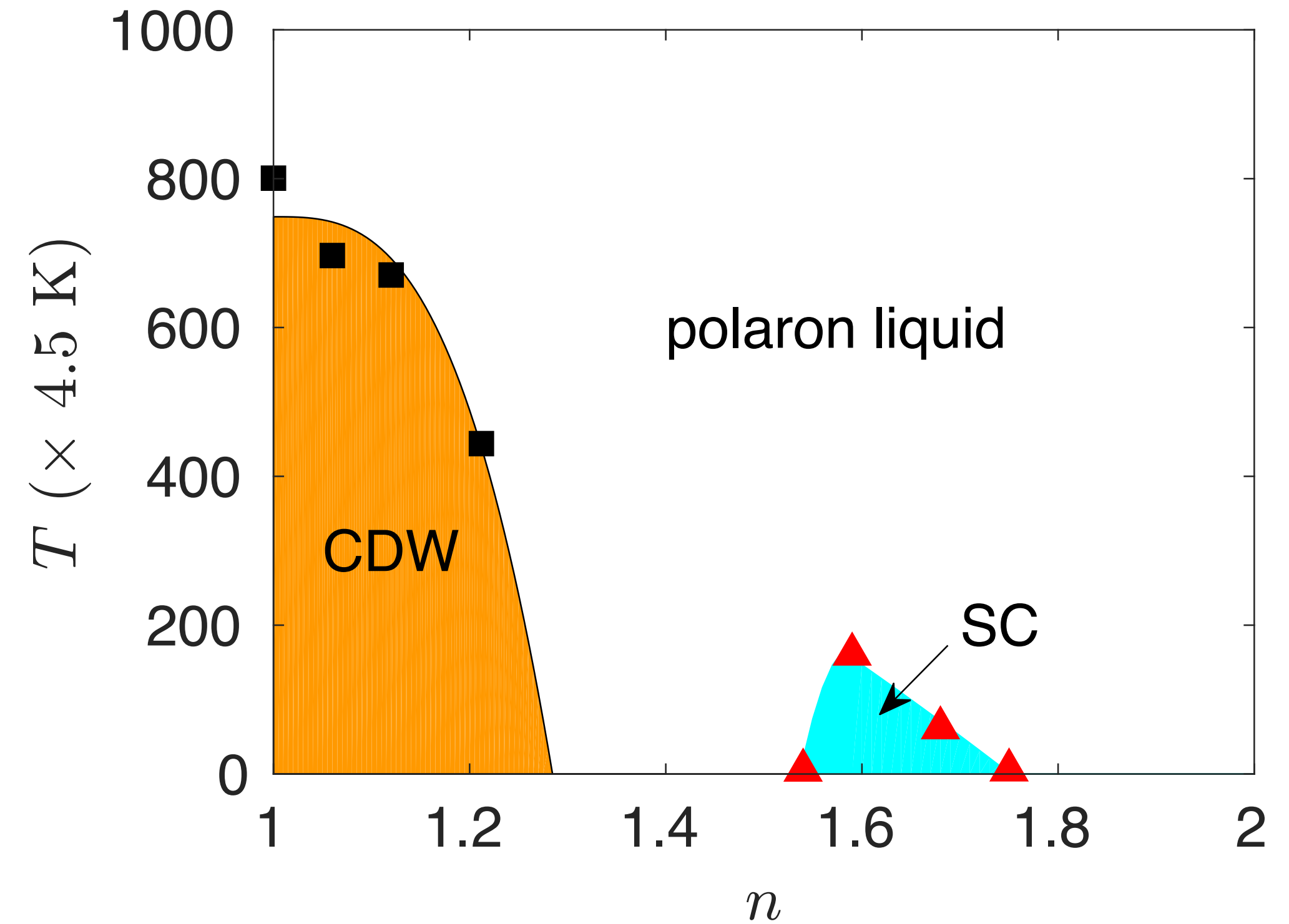
(e)  $\langle \hat{n} \rangle = 1.44$



S-wave pairfield susceptibility



Phase diagram



- Bipolaronic state at half-filling that melts into a polaron-liquid-like metallic state with a superconducting ground state upon doping.



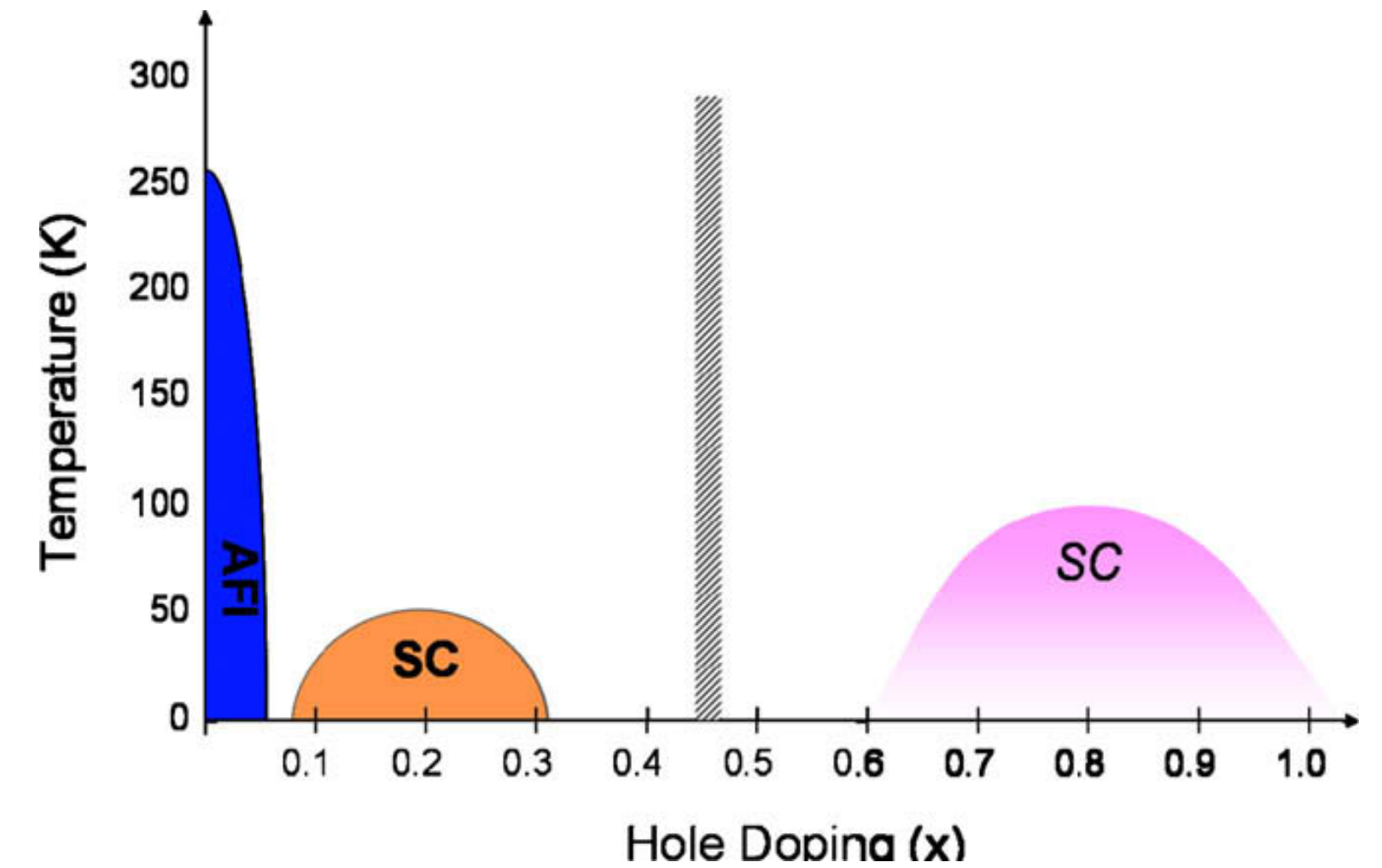
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# Two pairing domes as $\text{Cu}^{2+}$ varies to $\text{Cu}^{3+}$

## Highly (hole) overdoped curates

- $\text{Sr}_2\text{CuO}_{4-\delta}$  ( $T_c \sim 95$  K; isostructural to 214  $\text{La}_2\text{CuO}_4$ )
- $\text{Cu}_{0.75}\text{Mo}_{0.25}\text{Sr}_2\text{YCu}_2\text{O}_{7.54}$  ( $T_c \sim 84$  K)
- $\text{Ba}_2\text{CuO}_{4-\delta}$  ( $T_c \sim 70$  K)
- Monolayer  $\text{CuO}_2$  films ( $T_c \sim 100$  K)
- High pressure oxidized synthesis
- Reduced Cu - apical O spacing
- $d_{3z^2-r^2}$  orbital important, in addition to  $d_{x^2-y^2}$  orbital

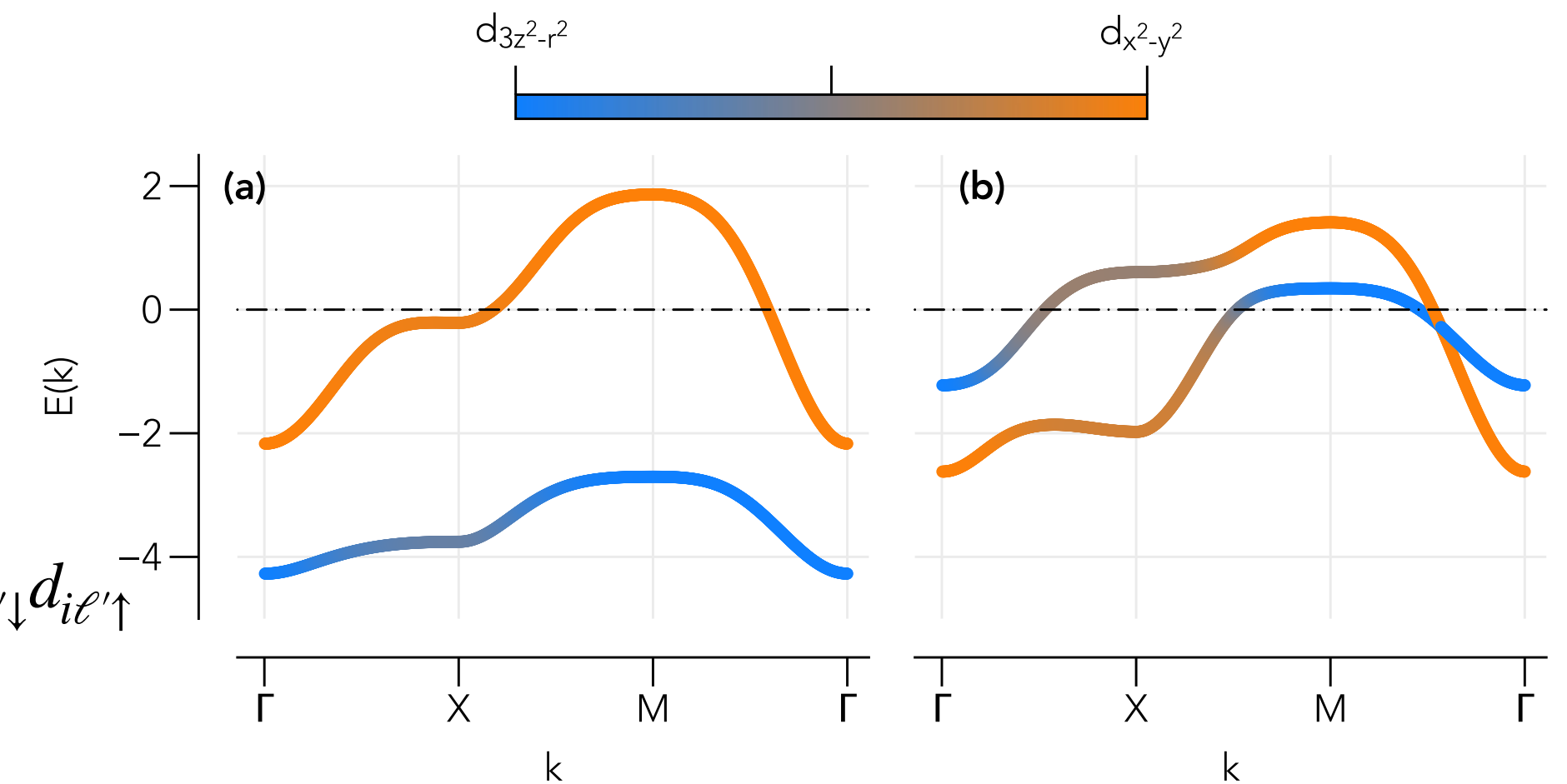


From Geballe & Marezi, *Physica C* '09

## Two-orbital tight-binding Hubbard-Hund model

$$H_0(x) = \sum_{k\sigma} \sum_{\ell\ell'} \left( \xi_{\ell\ell'}(k) + (\varepsilon_{\ell}(x) - \mu) \delta_{\ell\ell'} \right) d_{\ell\sigma}^\dagger(k) d_{\ell'\sigma}(k)$$

$$H_1 = U \sum_{i,\ell} n_{i\ell\uparrow} n_{i\ell\downarrow} + U' \sum_{i,\ell' < \ell} n_{i\ell} n_{i\ell'} + J \sum_{i,\ell' < \ell, \sigma, \sigma'} \sum_{\sigma} d_{i\ell\sigma}^\dagger d_{i\ell'\sigma}^\dagger d_{i\ell\sigma} d_{i\ell'\sigma} + J' \sum_{i,\ell' \neq \ell} d_{i\ell}^\dagger d_{i\ell\downarrow}^\dagger d_{i\ell'\downarrow} d_{i\ell'\uparrow}$$



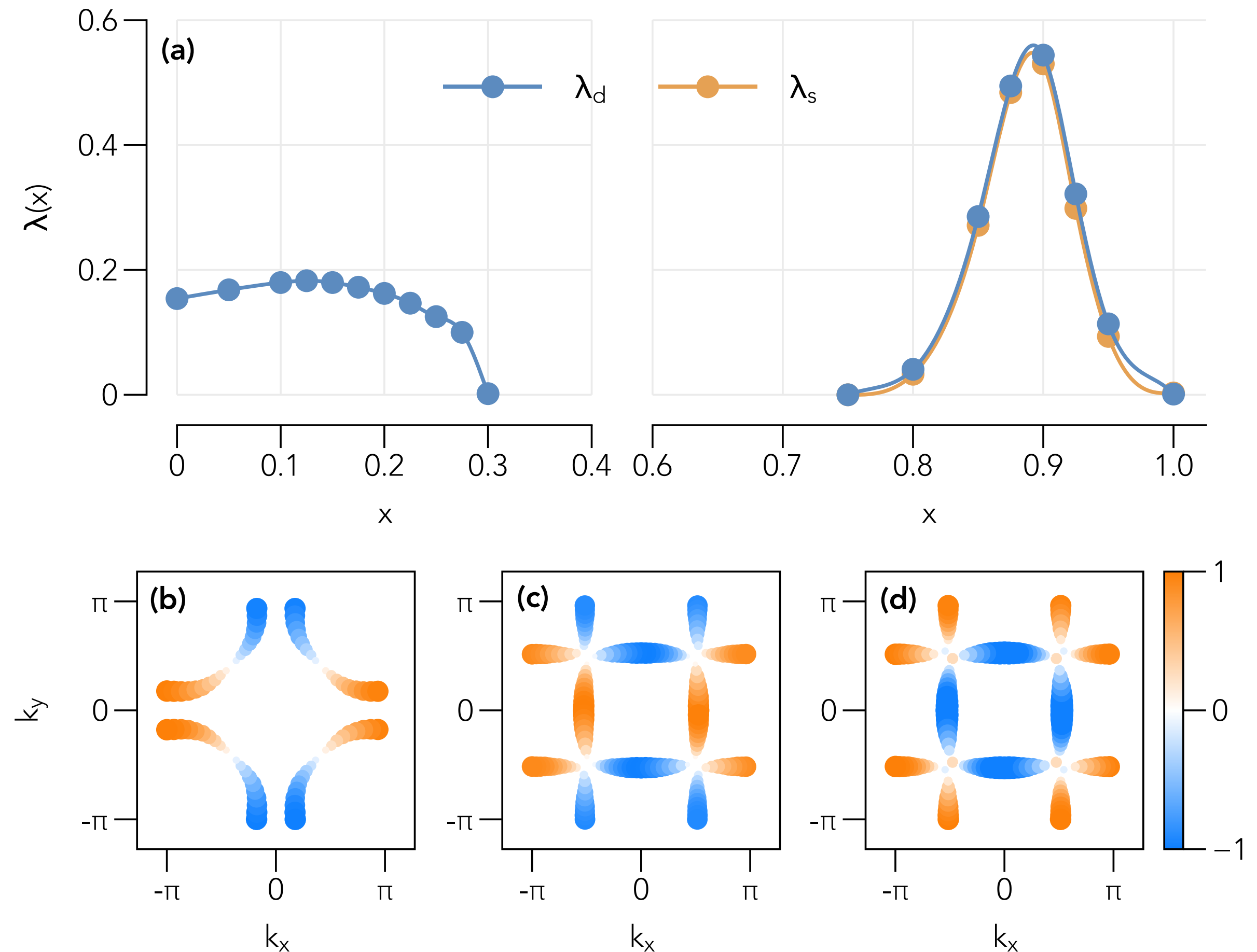
Maier, Berlijn & Scalapino, *PRB* '19



# Random Phase Approximation (Weak Coupling) Prediction

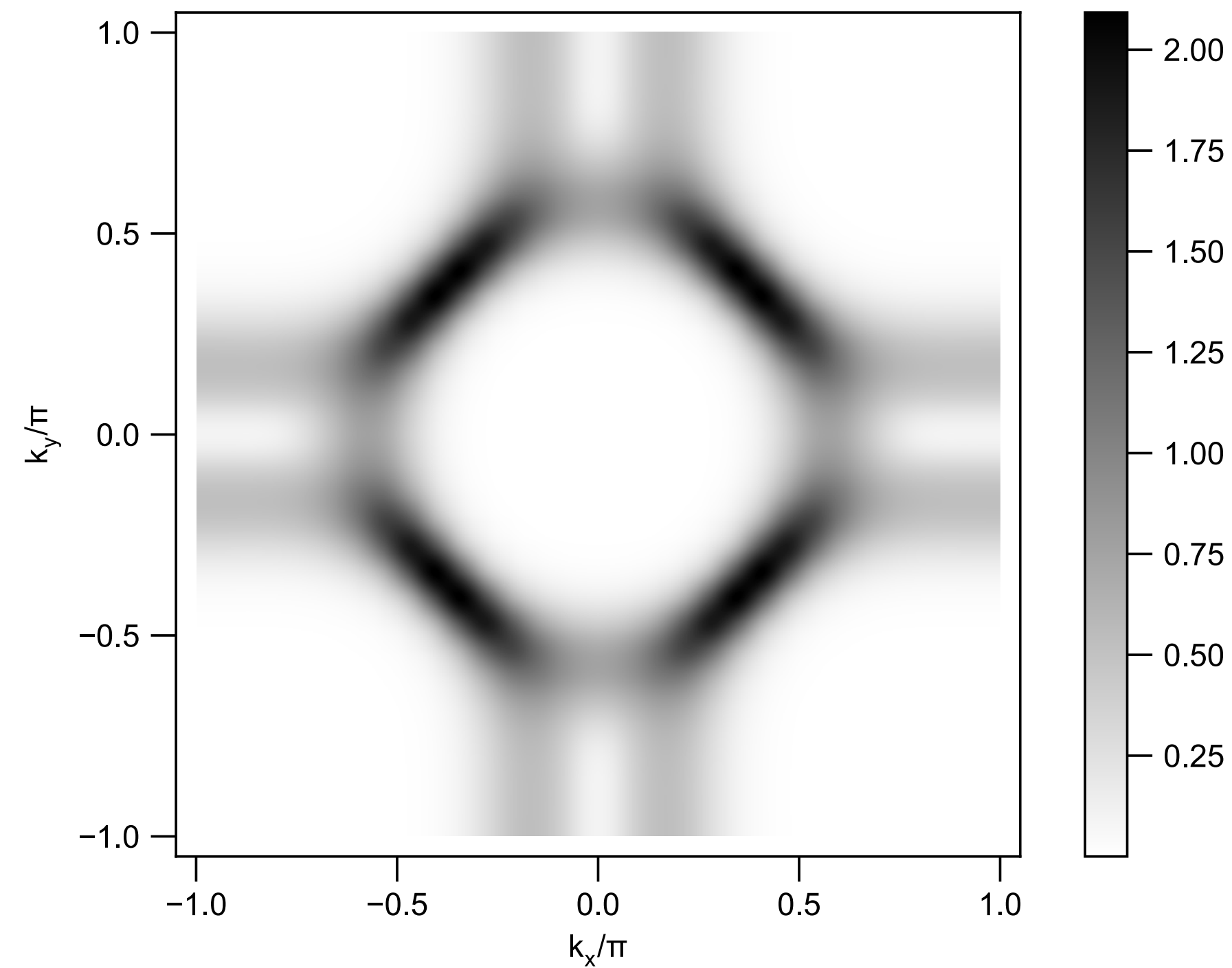
## Two pairing domes

- **First dome at small doping  $x$**  describes superconductivity in usual, lightly doped cuprates
- **Second dome at much larger doping** describes recently observed superconductivity in highly overdoped cuprates
- **Second dome has much larger pairing strength**
- For large doping, mixing of different  $d$ -orbitals allows for strong nesting between electron and hole Fermi surfaces
- Possibility of Bardassis-Schrieffer mode in Raman scattering due to near degeneracy of  $d$ - and  $s^\pm$  pairing channels

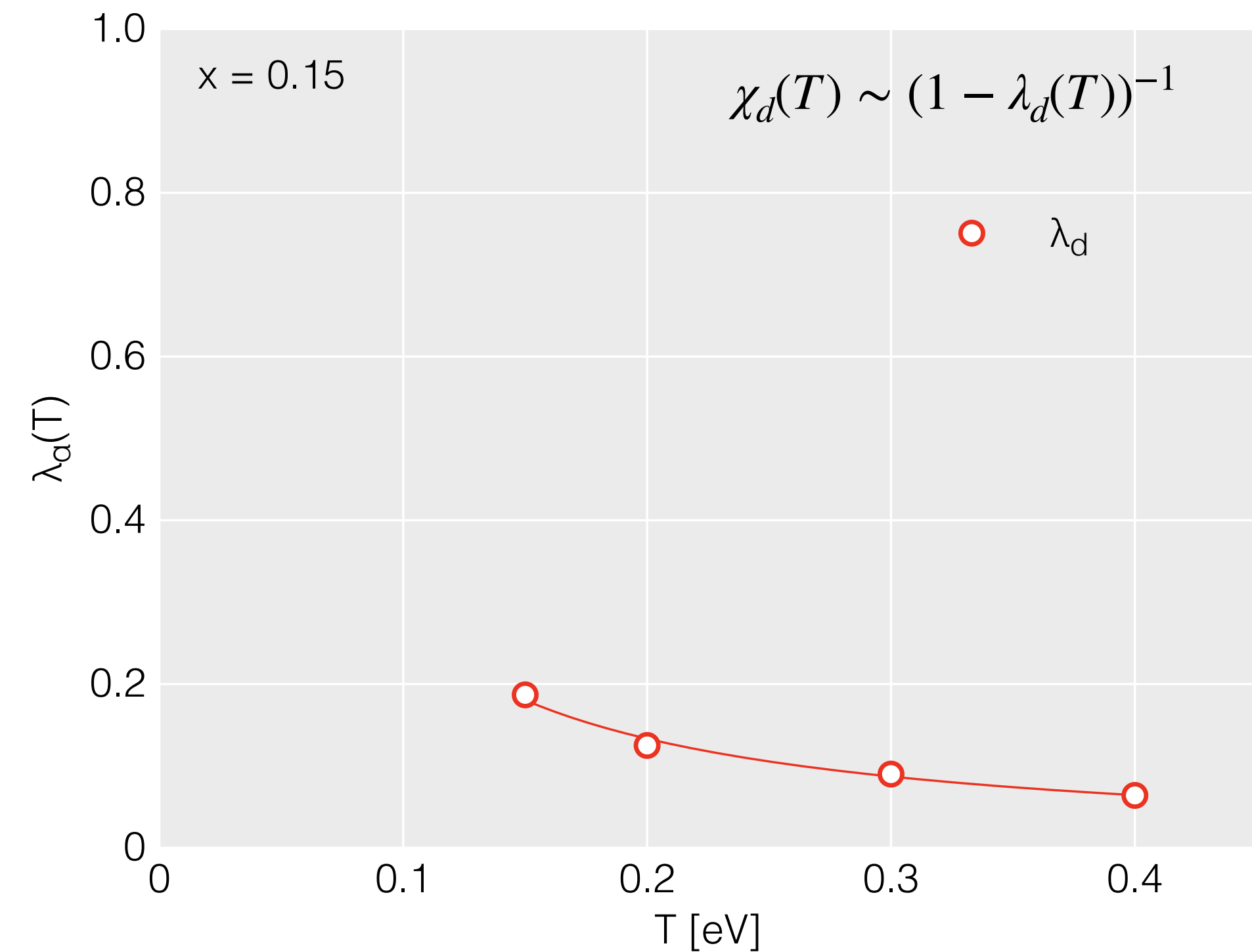


# Dynamic Cluster Quantum Monte Carlo results: $\langle n \rangle = 2.85$ ( $x = 0.15$ )

Fermi surface (  $|\nabla n(k)|$  )



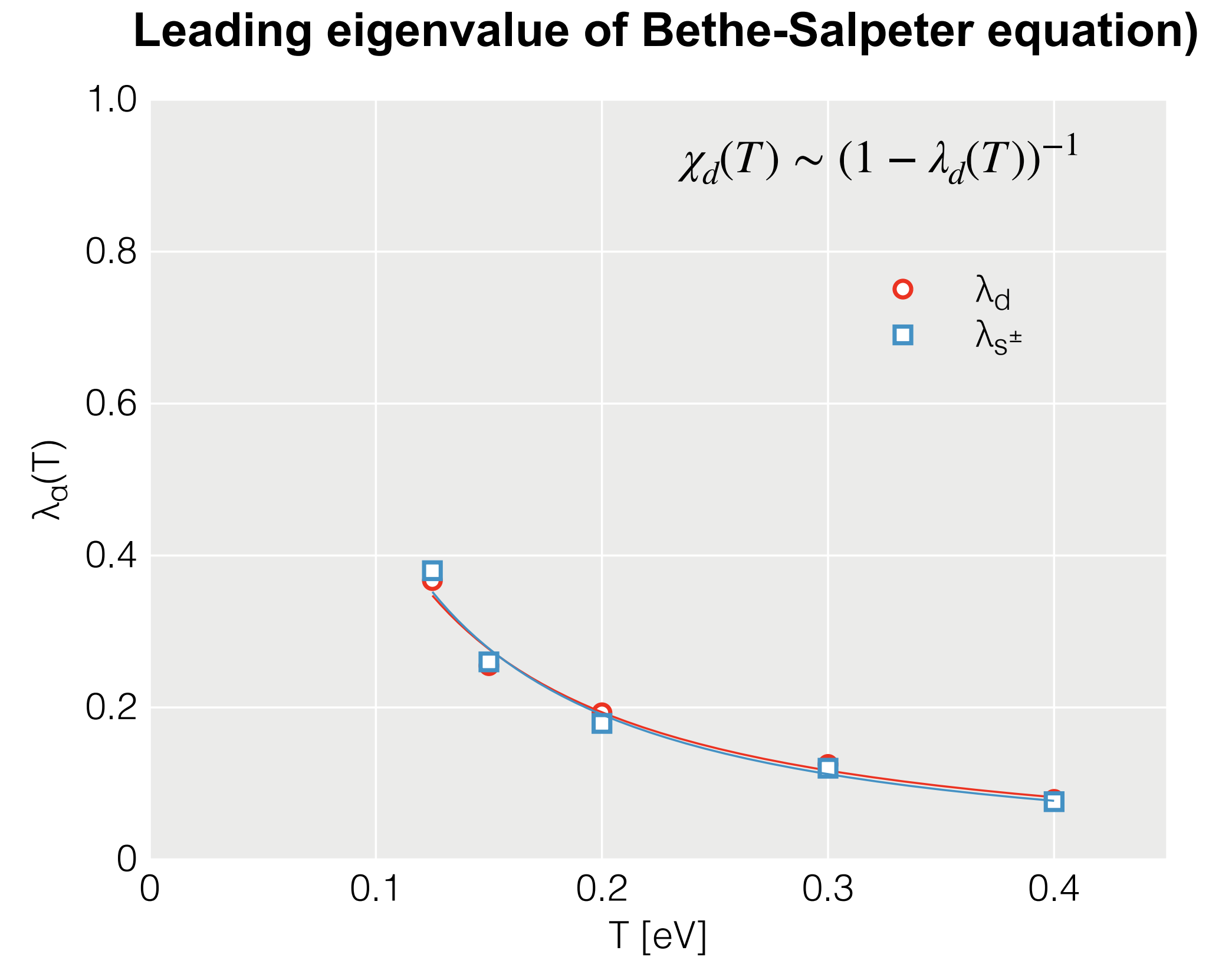
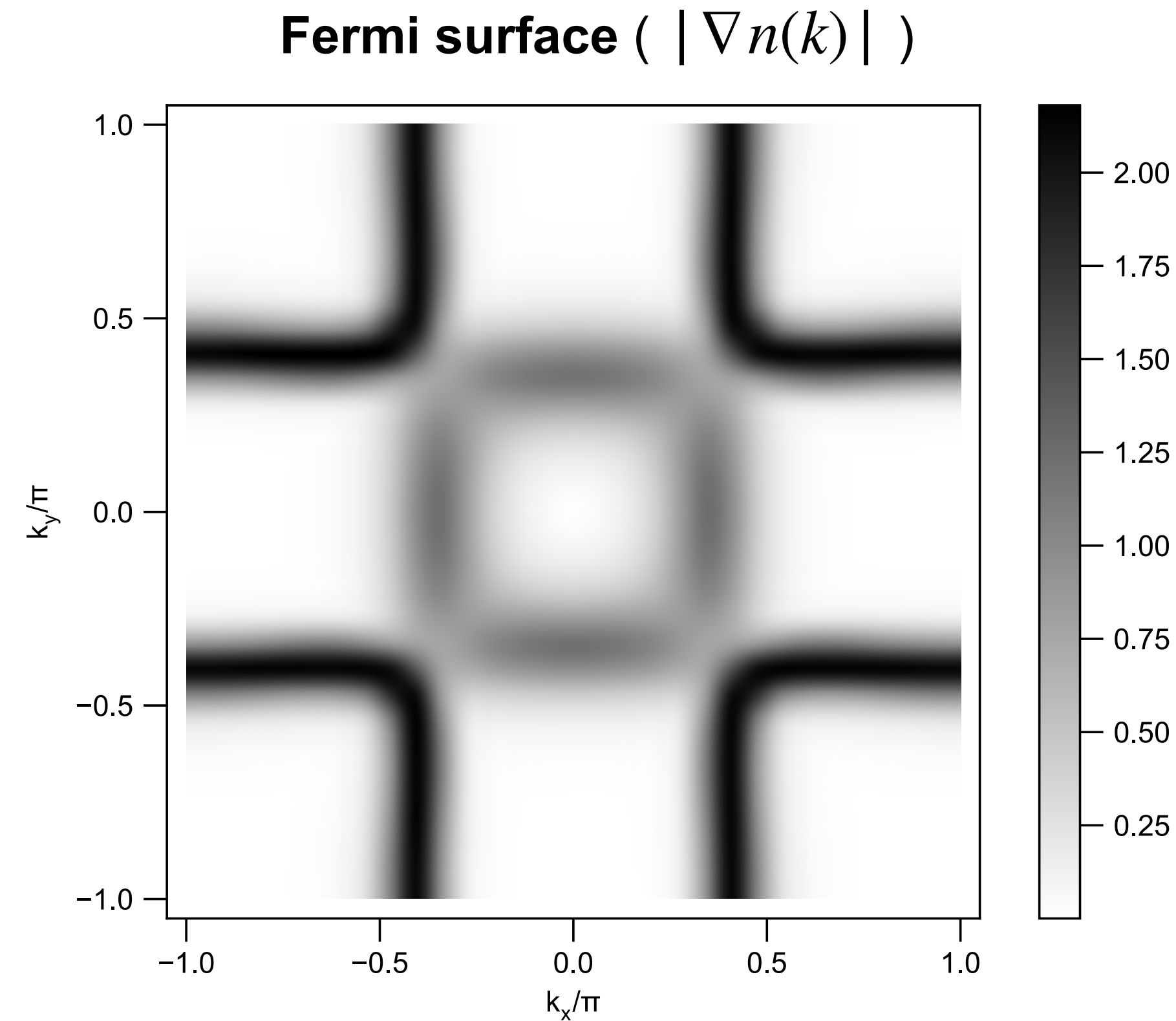
Leading eigenvalue of Bethe-Salpeter equation)



Parameters: 4 x 4 cluster;  $U = 4, U' = 2, J = J' = 0$ ; Doping  $x=0.15$

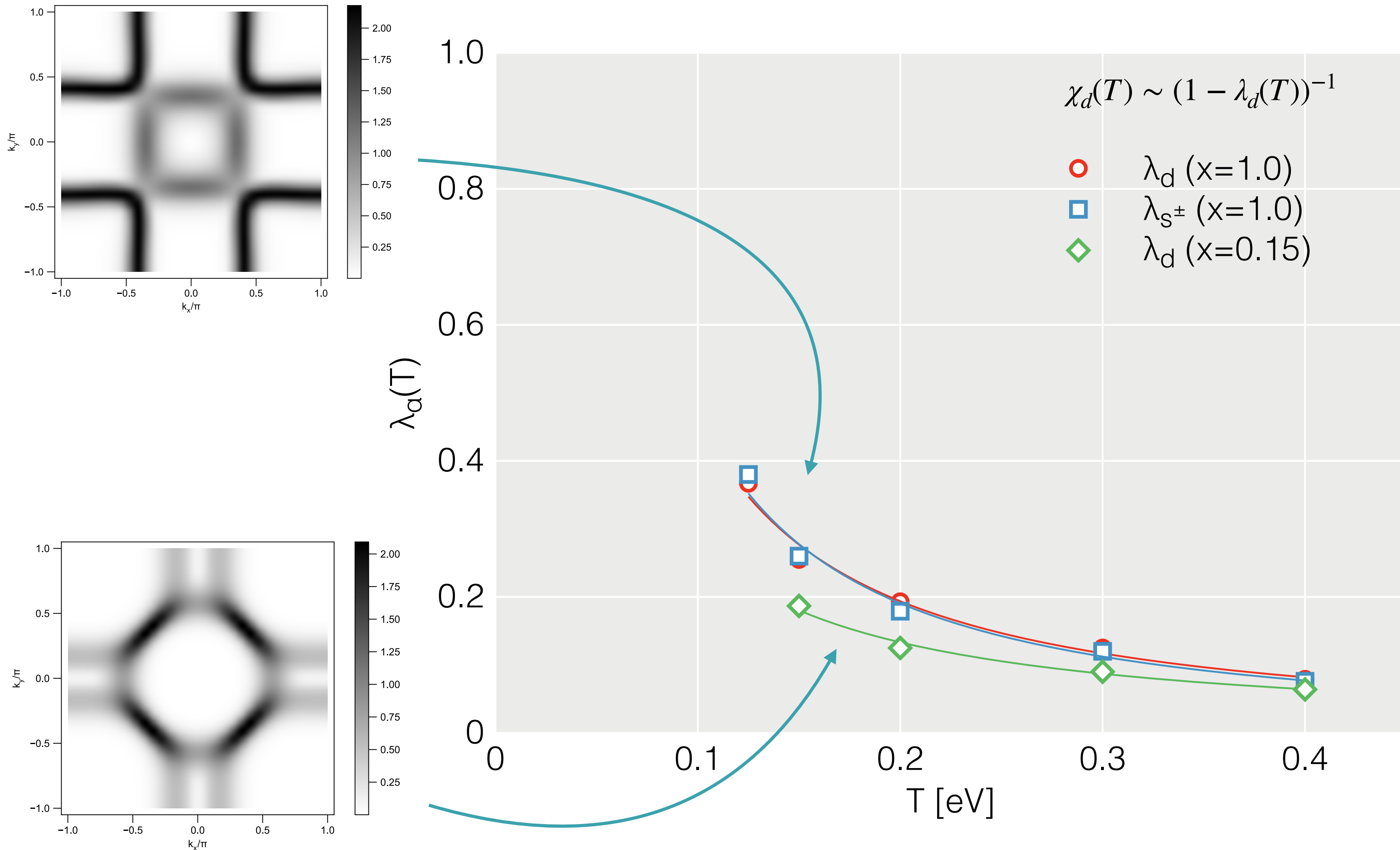


# Dynamic Cluster Quantum Monte Carlo results: $\langle n \rangle = 2.0$ ( $x = 1$ )



Parameters: 4 x 4 cluster;  $U = 4, U' = 2, J = J' = 0$ ; Doping  $x=1$

# Pairing in 2-orbital model for cuprates



- ▶ Pairing is stronger in heavily overdoped region with hole and electron Fermi surface pockets
- ▶  $d$ -wave and  $s^\pm$  - pairing almost degenerate in heavily overdoped region



# Acknowledgments

## CompFUSE SciDAC Team (ORNL / LANL / UTK / UCSB / UFL):

- Gonzalo Alvarez
- Matthew Bachstein
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- Philip Dee
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- Wael Elwasif
- Oscar Hernandez
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- Aaron Kirby
- Pontus Laurell
- Ying Wai Li
- Peizhi Mai
- Satoshi Okamoto
- Doug Scalapino
- Clayton Webster
- Xuping Xie

## ETH Zürich

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- Urs Hähner
- Thomas Schulthess

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