Improving convergence for stochastic physics parameterizations

Motivation

Stochastic parameterizations are attractive since they can help improve statistical properties of simulations and account for uncertainties, but naive application of traditional numerical methods can lead to large errors

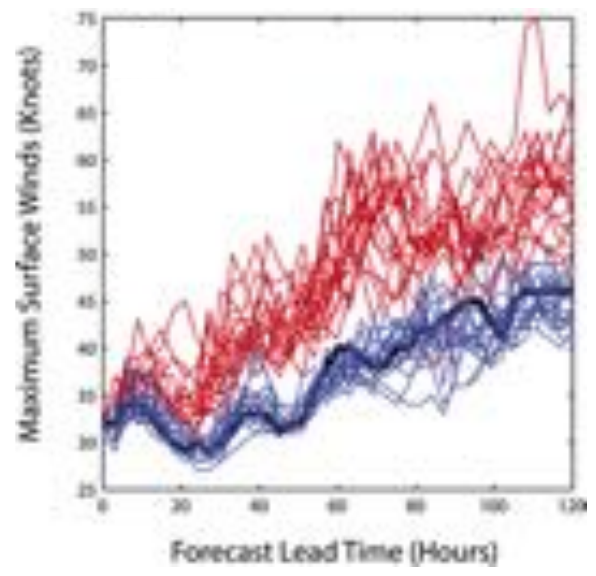

Figure 1: Intensity of hurricane Isaac (2012) in ensemble forecasts made by the NAVGEM model. Red lines correspond to a traditional time integration method which failed to capture the correct ensemble mean shown as thick black line. Correct results (blue lines) were obtained by adding a Ito correction to the time integration. Figure reproduced from Hodyss et al. (2014, Mon. Wea. Rev.) with permission from the American Meteorological Society.

Take-home Messages

Our first results from a simple 1D model show that the generalized Ito correction can help improve solution accuracy and convergence for a broad range of stochastic processes. This is a promising approach worth further exploration.

Toward Application to E3SM

We are working on applying the approach to a more complex and realistic problem with stochastic turbulence parameterization, and potentially applying to the parameterizations in E3SM.

Our Approach & First Results

- Add a **mean effect** (Ito correction) to complement the stochastic term
- Use Ito correction to improve accuracy for **white** noise; Generalize the Ito correction for **colored** noise
- Develop *appropriate* numerical implementations

- For the *white* noise case, the use of the Ito correction restores convergence to the physically relevant solution.
- For the *colored* noise case, the use of the Ito correction improves the numerical convergence rate.

P. Stinis, H. Lei, J. Li and H. Wan, Improving solution accuracy and convergence for stochastic physics parameterizations with colored noise, 2019, Mon. Wea. Rev., in review. Available from arXiv:1904.08550

Example: advection-diffusion equation with space-dependent advection and fast forcing

$$\frac{\partial u}{\partial t} = -\left[c + \frac{\epsilon}{2}\cos(x)\right]\frac{\partial u}{\partial x} + \mu\frac{\partial^2 u}{\partial x^2} + g(u)n(t)$$

$g(u)n(t)$ — fast forcing (Figure 2)

1) If forcing spectrum is *white*, solve the following equation:

$$\frac{\partial u}{\partial t} = -\left[c + \frac{\epsilon}{2}\cos(x)\right]\frac{\partial u}{\partial x} + \mu\frac{\partial^2 u}{\partial x^2} + \underbrace{\frac{1}{2}\frac{dg}{du}g(u)}_{\text{mean effect}} + \underbrace{g(u)\dot{W}(t)}_{\text{noise effect}}$$

$\dot{W}_t$ — white noise

2) If forcing spectrum is *colored*, the mean effect in integral form is:

$$\lim_{\Delta t \to 0}\sum_j \left(\frac{1}{2}\frac{dg(u)}{du}g[u]\right)\Bigg|_{t_j}\mathbb{E}\left[(\Delta n_j)^2\right]$$

$\Delta n_j = n(t_{j+1}) - n(t_j)$ — noise increment

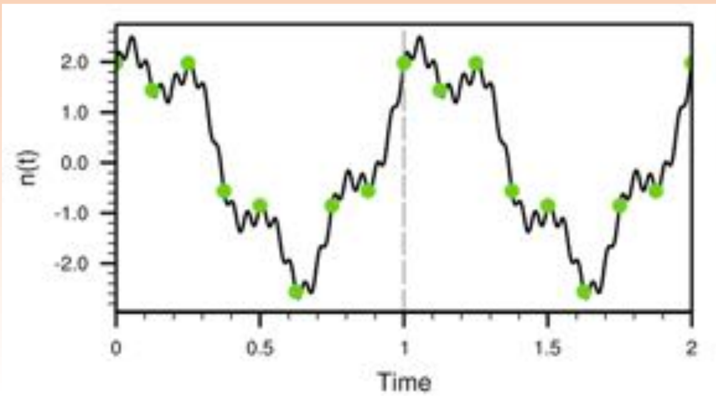

Figure 2: Black curve shows *n(t)* in the advection-diffusion equation. Green dots are discrete time steps.

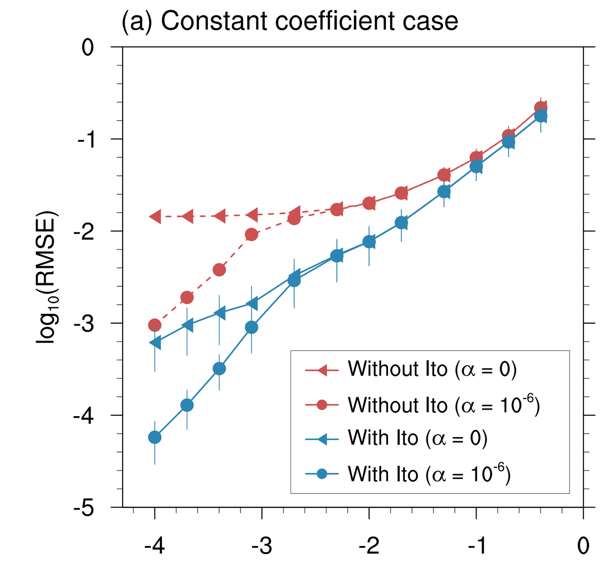

Figure 3: Convergence rate for the *constant* coefficient case.

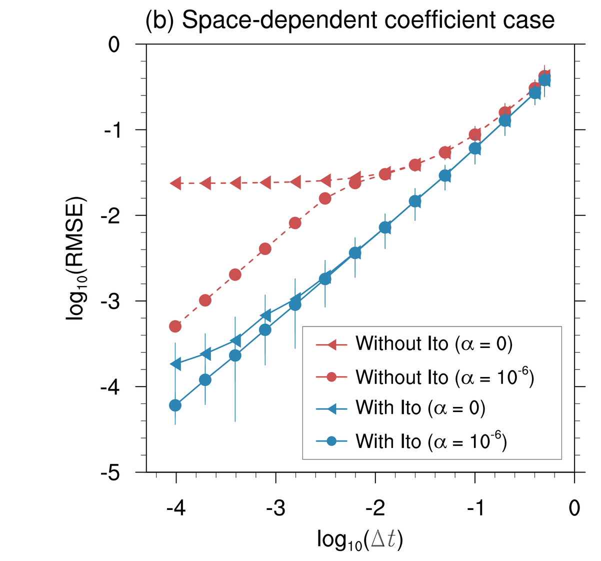

Figure 4: Convergence rate for the *space-dependent* coefficient case.

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This work is part of the SCiDAC project *"Assessing and Improving the Numerical Solution of Atmospheric Physics in E3SM"*