Stability analysis of numerical interface conditions in coupled ocean-atmosphere models

CANGA

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Overview

Motivation

Current coupling algorithms for climate models are equivalent to performing a single step of an iterative procedure [Lemarie et al. 2015]. This may not be enough to ensure numerical stability.

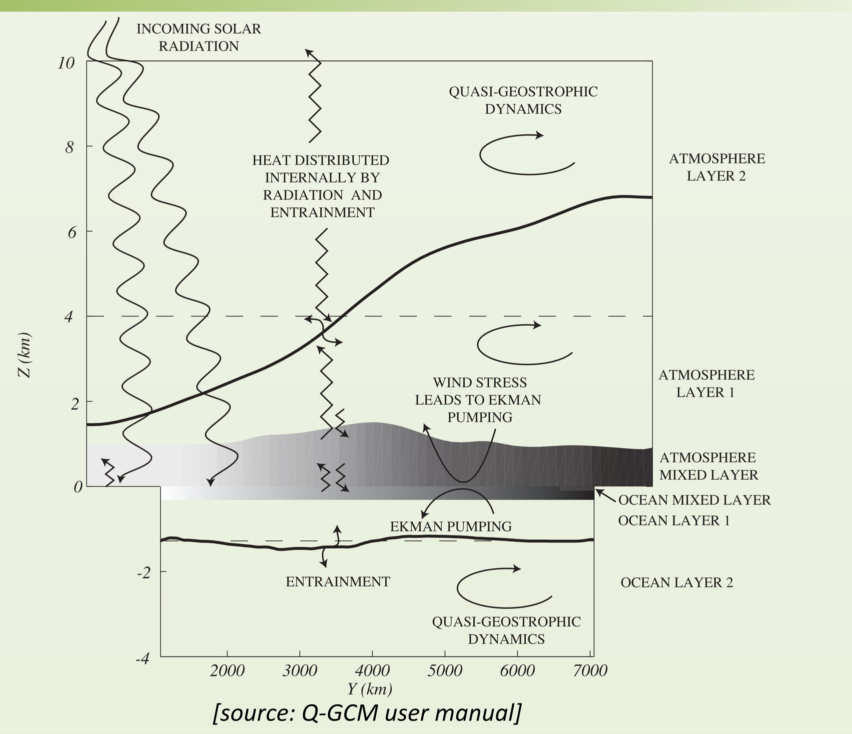
Objectives

- Perform rigorous stability analysis of interface conditions for a variety of coupling algorithms
- Gain insight on how to improve the existing methods or propose new coupling schemes

Model Exemplars: Q-GCM

The Quasi-Geostrophic Coupled Model

- Intermediate-complexity mid-latitude coupled climate model
- Candidate testbed for comparing different strategies and analyzing instability



[source: Q-GCM user manual]

Coupling mechanism

- Includes mixed layers to allow the exchange of heat flux and momentum
- At the interface, Ekman pumping drives momentum from atmosphere to ocean; latitudinal variations in solar radiation drives surface heat flux in both direction

$$ {}^a{q_k}_t + ({}^a u_k\, {}^a q_k)_x + ({}^a v_k\, {}^a q_k)_y = \frac{f_0\,({}^a e_k - {}^a e_{k-1})}{{}^a H_k} - \frac{{}^a A_4}{f_0}\nabla_H^6\, {}^a p_k $$

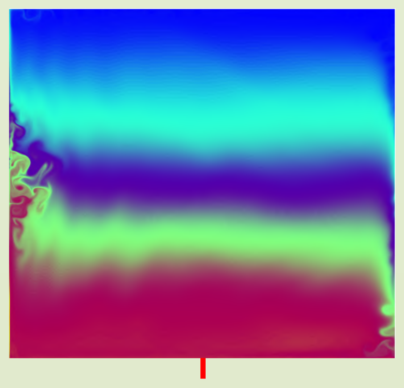
$$ {}^o{q_k}_t + ({}^o u_k\, {}^o q_k)_x + ({}^o v_k\, {}^o q_k)_y = \frac{f_0\,({}^o e_k - 1 - {}^a e_k)}{{}^a H_k} + \frac{{}^o A_2}{f_0}\nabla_H^4\, {}^o p_k - \frac{{}^a A_4}{f_0}\nabla_H^6\, {}^a p_k $$

notation: k – layer number, q – vorticity, a – atmosphere, p – pressure, o – ocean, e – entrainment

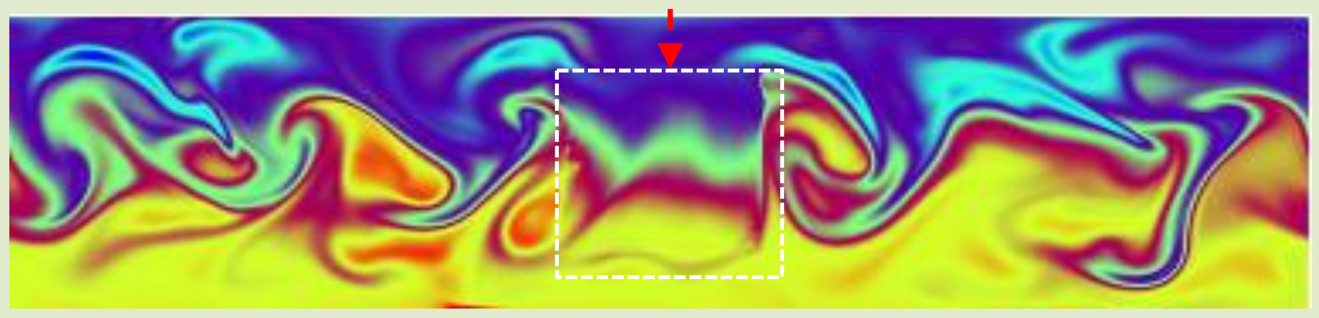
$$ {}^a T_{mt} + ({}^a u_m\, {}^a T_m)_x + ({}^a v_m\, {}^a T_m)_y + \frac{{}^a w_{ek}\, {}^a T_m}{{}^a H_m} = {}^a K_2 \nabla_H^2\, {}^a T_m - {}^a K_4 \nabla_H^4\, {}^a T_m + \frac{-{}^a F_m + {}^a F_0}{{}^a\rho\, {}^a C_p\, {}^a h_m} $$

$$ {}^o T_{mt} + ({}^o u_m\, {}^o T_m)_x + ({}^o v_m\, {}^o T_m)_y + \frac{{}^o w_{ek}\, {}^o T_m}{{}^o H_m} = {}^o K_2 \nabla_H^2\, {}^o T_m - {}^o K_4 \nabla_H^4\, {}^o T_m + \frac{-{}^o F_m + {}^o F_0}{{}^o\rho\, {}^o C_p\, {}^o H_m} $$

Ocean
4790 km X 4790 km



Atmosphere
30640 km X 7600 km



Surface Temperature for the double gyre example (6 months)

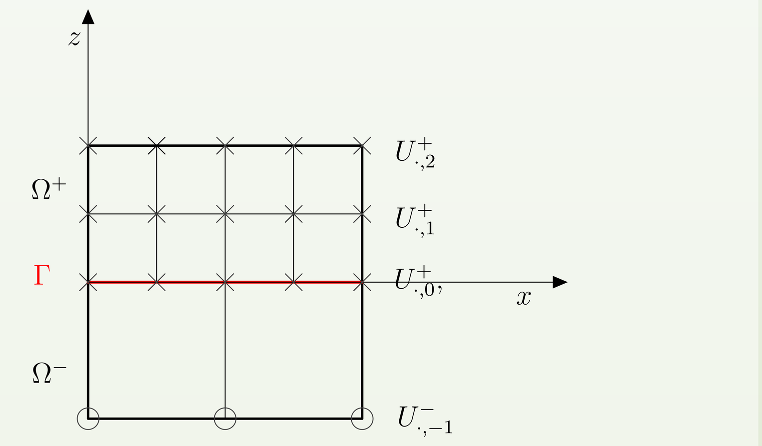
Observations

- Running in the coupled mode, the ocean model stays stable with a timestep size of up to 40 minutes; running in the ocean-only mode, the timestep size can be larger (>72 minutes)

Interface Conditions

Practical assumptions

- Both components can be run simultaneously to promote task parallelism
- Ocean step size is a multiple of atmosphere step



Dirichlet-Neumann condition

$$ \begin{cases} \mathcal{L}^+ U^+ = f^+ & \text{in } \Omega^+ \times [0,\mathcal{T}] \\ U_0^+ = U_0^- & \text{on } \Gamma \times [0,\mathcal{T}] \end{cases} $$

$$ \begin{cases} \mathcal{L}^- U^- = f^- & \text{in } \Omega^- \times [0,\mathcal{T}] \\ \mathcal{F}(U_0^-) = \mathcal{F}(U_0^+) & \text{on } \Gamma \times [0,\mathcal{T}] \end{cases} $$

Bulk interface condition

$$ \begin{cases} \mathcal{L}^+ U^+ = f^+ & \text{in } \Omega^+ \times [0,\mathcal{T}] \\ \rho^+ \mathcal{F}(U^+) = \tau & \text{on } \Gamma \times [0,\mathcal{T}] \end{cases} $$

$$ \begin{cases} \mathcal{L}^- U^- = f^- & \text{in } \Omega^\times [0,\mathcal{T}] \\ \rho^- \mathcal{F}(U^-) = \tau & \text{on } \Gamma \times [0,\mathcal{T}] \end{cases} $$

Discretize for 1D Diffusion equation

$$ U_{\cdot,j}^{-,n+1} - U_{\cdot,j}^{-,n} = \left(\frac{\nu_- \Delta t}{(\Delta z_-)^2}\right)(U_{\cdot,j+1}^{-,n} - 2U_{\cdot,j}^{-,n} + U_{\cdot,j-1}^{-,n}) -, \quad j<0 $$

$$ U_{\cdot,0}^{-,n+1} - U_{\cdot,0}^{-,n} = \Delta t \frac{\left(\mathcal{F}^{+,n}_{\cdot,\frac12} - \mathcal{F}^{-,n}_{\cdot,-\frac12}\right)}{(\Delta z_+ + \Delta z_-)/2}, \quad \text{where } \mathcal{F} \text{ is the flux} $$

$$ U_{\cdot,j}^{+,n+1} - U_{\cdot,j}^{+,n} = \left(\frac{\nu_+ \Delta t}{(\Delta z_+)^2}\right)(U_{\cdot,j+1}^{+,n} - 2U_{\cdot,j}^{+,n} + U_{\cdot,j-1}^{+,n}) -, \quad j>0 $$

$$ U^{+,n+1}_{\cdot,j+\frac12} = U^{+,n}_{\cdot,j+\frac12} + \frac{\nu_+\Delta t}{(\Delta z_+)^2}\left(U^{+,n}_{\cdot,j+\frac32} - 2U^{+,n}_{\cdot,j+\frac12} + U^{+,n}_{\cdot,j-\frac12}\right), \quad j \ge 1 $$

$$ U^{+,n+1}_{\cdot,\frac12} = U^{+,n}_{\cdot,\frac12} + \frac{\Delta t}{\Delta z_+}\left(\frac{\nu_+}{\Delta z_+}(U^{+,n}_{\cdot,\frac32} - U^{+,n}_{\cdot,\frac12}) - \alpha(U^{+,n}_{\cdot,\frac12} - U^{-,n}_{\cdot,-\frac12})\right), \quad j = 0 $$

$$ U^{-,n+1}_{\cdot,j+\frac12} = U^{-,n}_{\cdot,j+\frac12} + \frac{\nu_-\Delta t}{(\Delta z_-)^2}\left(U^{-,n}_{\cdot,j+\frac32} - 2U^{-,n}_{\cdot,j+\frac12} + U^{-,n}_{\cdot,j-\frac12}\right), \quad j \le -2 $$

$$ U^{-,n+1}_{\cdot,-\frac12} = U^{-,n}_{\cdot,-\frac12} + \frac{\Delta t}{\Delta z_-}\left(\frac{\rho_2}{\rho_1}\alpha(U^{+,n}_{\cdot,\frac12} - U^{-,n}_{\cdot,-\frac12}) - \frac{\nu_-}{\Delta z_-}(U^{-,n}_{\cdot,-\frac12} - U^{-,n}_{\cdot,-\frac32})\right), \quad j = -1 $$

Normal Mode Analysis

For explicit Euler consider the solution

$$ U_j^n = \begin{cases} \mathcal{A}^n k_-^j, & j = 0, -1, -2, \ldots \\ \mathcal{A}^n k_+^j, & j = 1, 2, \ldots \end{cases} $$

Define $r_\pm = \frac{\Delta z_\pm}{\Delta z_+ + \Delta z_-}$ $d_\pm = \frac{\nu_\pm \Delta t}{(\Delta z_\pm)^2}$

$$ A = 1 + d_+(k_+ - 2 + k_+^{-1}) $$
$$ A = 1 + 2d_+ r_+(k_+ - 1) - 2d_- r_-(1 - k_-^{-1}) $$
$$ A = 1 + d_-(k_- - 2 + k_-^{-1}) $$

Solve for k and choose negative root $1 = r_+\left(1 - \sqrt{1 + \frac{4d_+}{A-1}}\right) + r_-\left(1 - \sqrt{1 + \frac{4d_-}{A-1}}\right)$

Asymptotic solution of the requirement $|\mathcal{A}| < 1$ yields

- If $\Delta z_- \ll \Delta z_+$ then $d_+ < \frac12$
- If $\Delta z_+ \ll \Delta z_-$ then $d_- < \frac12$
- If $d_\pm \gg 1$ then $\frac{\nu_-}{\nu_+} \approx 1$

For backward Euler we have to use $U^{+,n+1}_{\cdot,0^+} = U^{-,n}_{\cdot,0^-}$

Thus consider the solution

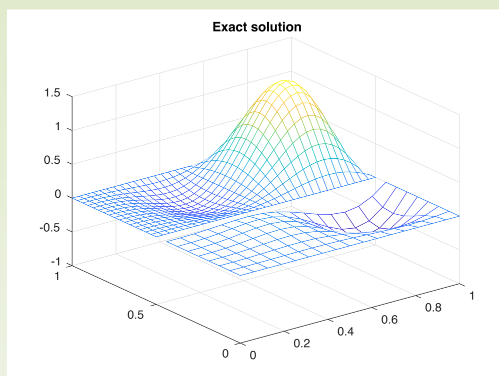
$$ U_j^n = \begin{cases} \mathcal{A}^n k_-^j, & j = 0^-, -1, -2, \ldots \\ \mathcal{A}^{n-1} k_+^j, & j = 0^+, 1, 2, \ldots \end{cases} $$

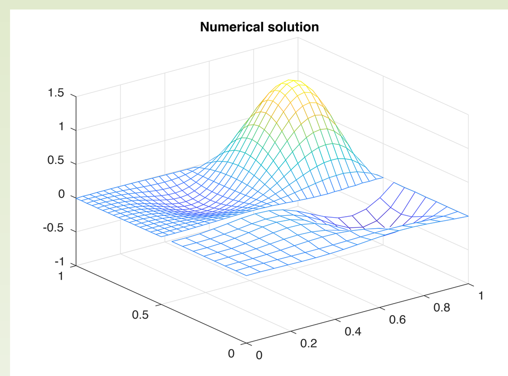
- If $\Delta z_- \ll \Delta z_+$ the scheme is stable
- Otherwise the scheme can be unstable

Analysis on bulk interface conditions also implies additional constraints on step size

Conclusions and Future Work

- Instability originates from interface condition and the implementation
- Validated on an advection-diffusion model
- Extending to a passive tracer model
- Adding the iterative Schwarz method in Q-GCM to improve the step size





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