

Superconductivity in the doped Hubbard and t-J models on the square lattice

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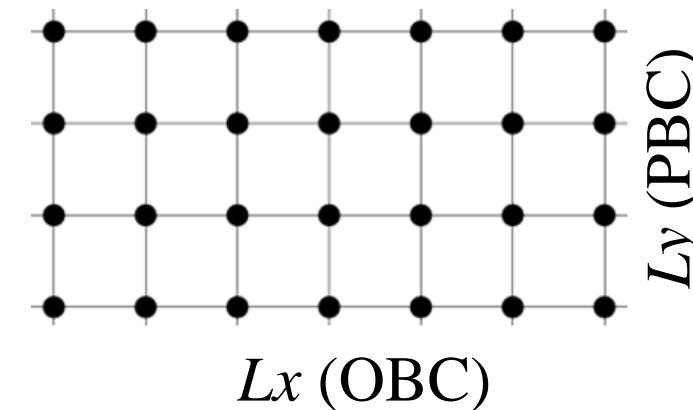
❖ Introduction and Motivation

- The Hubbard and t-J models are widely regarded as the starting point to understand high-T_c superconductivity, such as cuprates.[1-2]
- Enormous effort has been devoted to studying the properties of the models at intermediate couplings, however, no general theoretically controlled methods or consensus.[1-2]
- Previous studies suggest striped ground state with unidirectional charge-density-wave order, many low-lying states close in energy. However, no direct evidence for the presence of superconductivity.[3]
- **Question:** Can we have long-range superconductivity in doped Hubbard and t-J models on square lattice wider than 2-leg ladder?

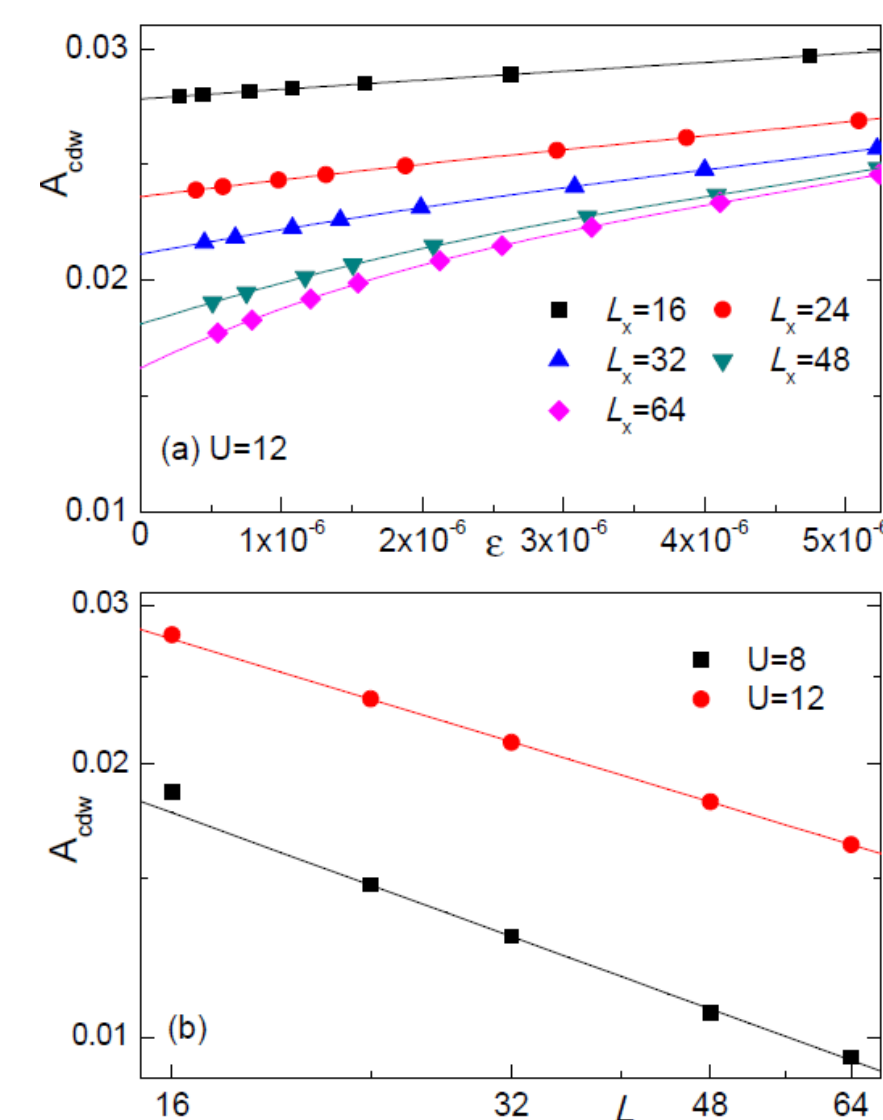
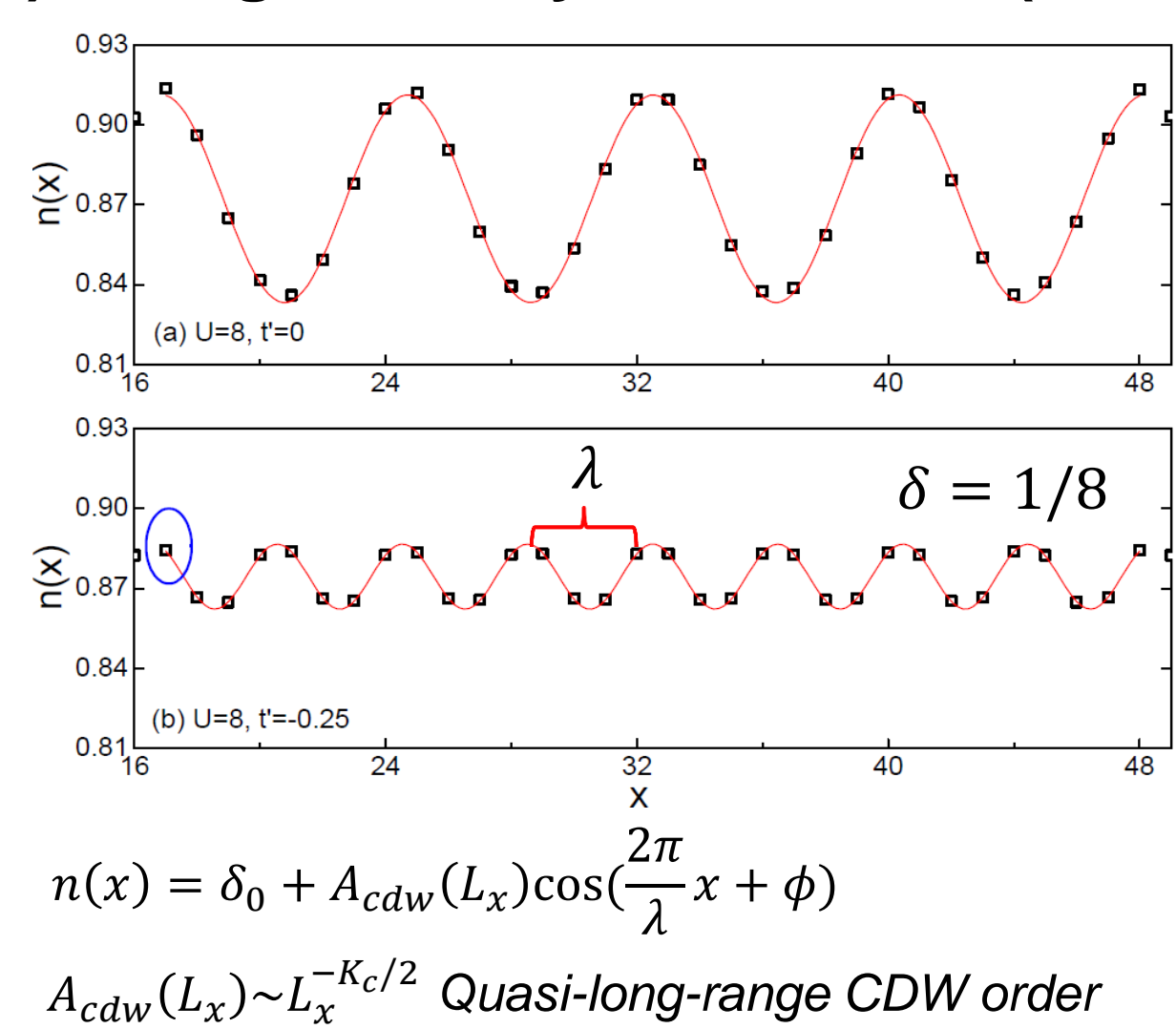
❖ Hubbard Model and DMRG study [4-5]

$$H = - \sum_{ij,\sigma} t_{ij} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

✓ $N=L_x^*L_y$, $L_y=4$, $L_x=16\sim 64$, hole doping $\delta=1/8$
 ✓ Isotropic hopping $t=1$, $t'=0.25$, and $U=8\sim 12$
 ✓ Keep $m=4096\sim 20000$ states with ~ 100 sweeps



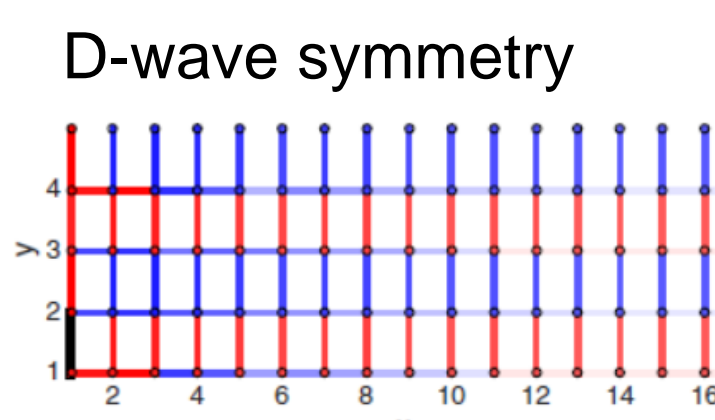
1) Charge-density-wave order (CDW)



2) Superconductivity (SC) $\alpha = \hat{x}, \hat{y}$

$$\Phi_{\alpha\beta}(x) = \frac{1}{L_y} \sum_{y=1}^{L_y} \langle \Delta_\alpha^\dagger(x_0, y) \Delta_\beta(x_0 + x, y) \rangle$$

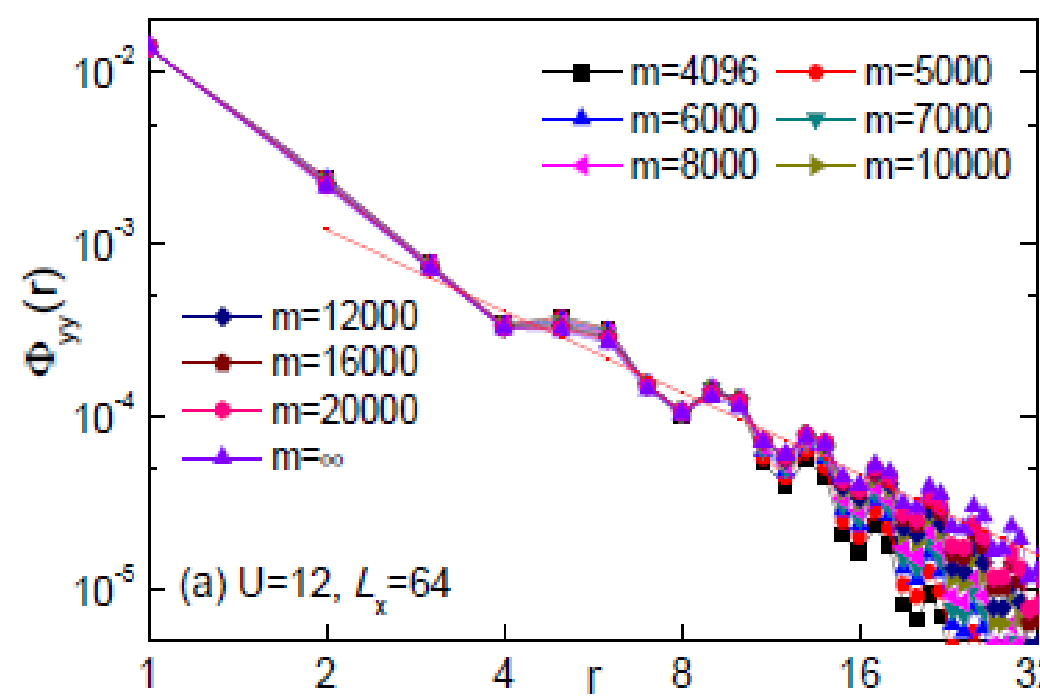
$$\Delta_\alpha^\dagger(x, y) = \frac{1}{\sqrt{2}} [c_{(x,y),\uparrow}^\dagger c_{(x,y)+\alpha,\downarrow}^\dagger - c_{(x,y),\downarrow}^\dagger c_{(x,y)+\alpha,\uparrow}^\dagger]$$



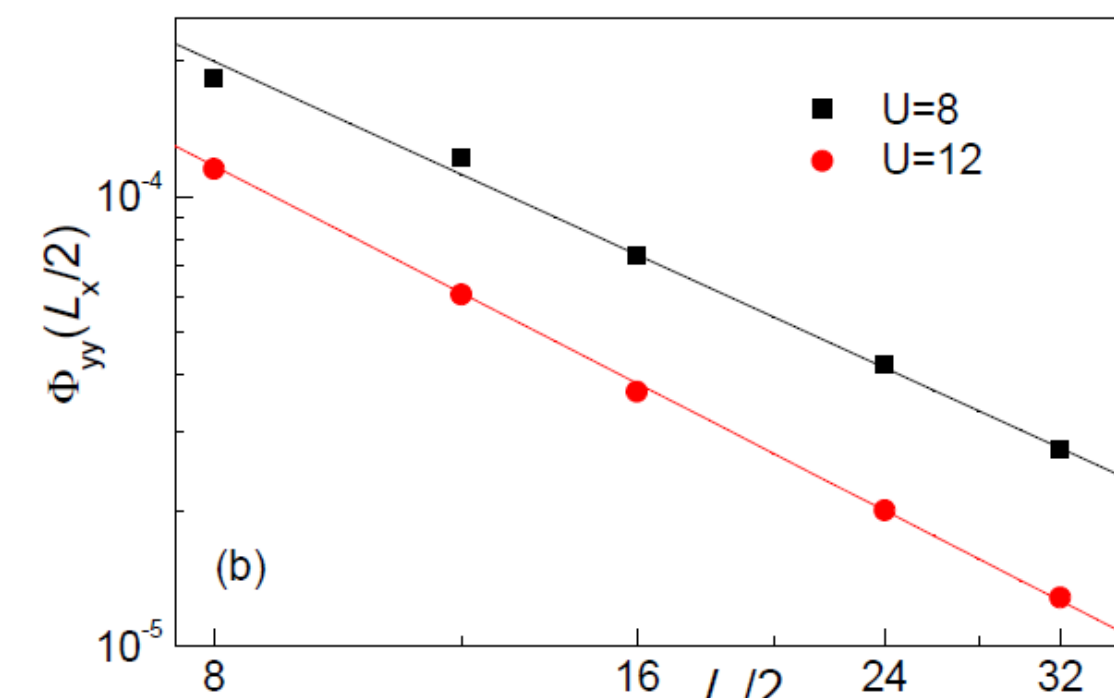
References

- [1] E. Dagotto, Rev. Mod. Phys. 66, 763 (1994);
- [2] P. A. Lee, N. Nagaosa, X. G. Wen, Rev. Mod. Phys. 78, 17 (2006);
- [3] B. X. Zheng et al., Science 358, 1155 (2017);
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- [7] H. C. Jiang, Z. Y. Weng, S. Kivelson, 1805.11163.

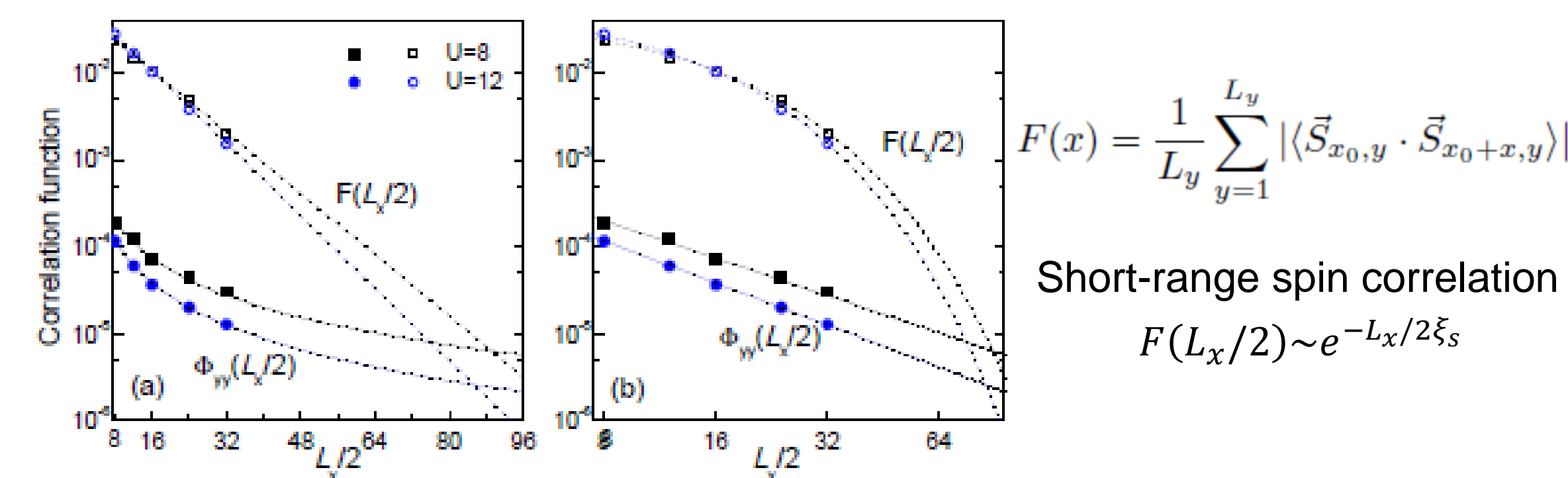
Quasi-long-range SC: $\Phi_{yy}(r) \sim r^{-K_{sc}}$



$\Phi_{yy}(L_x/2) \sim (L_x/2)^{-K_{sc}}$



3) Spin-spin correlation



4) Luther-Emery liquid [6]

$$A_{cdw}(L_x) \sim L_x^{-K_c/2}$$

$$\Phi\left(\frac{L_x}{2}\right) \sim \left(\frac{L_x}{2}\right)^{-1/K_{sc}}$$

$$K_c \times K_{sc} = 1$$

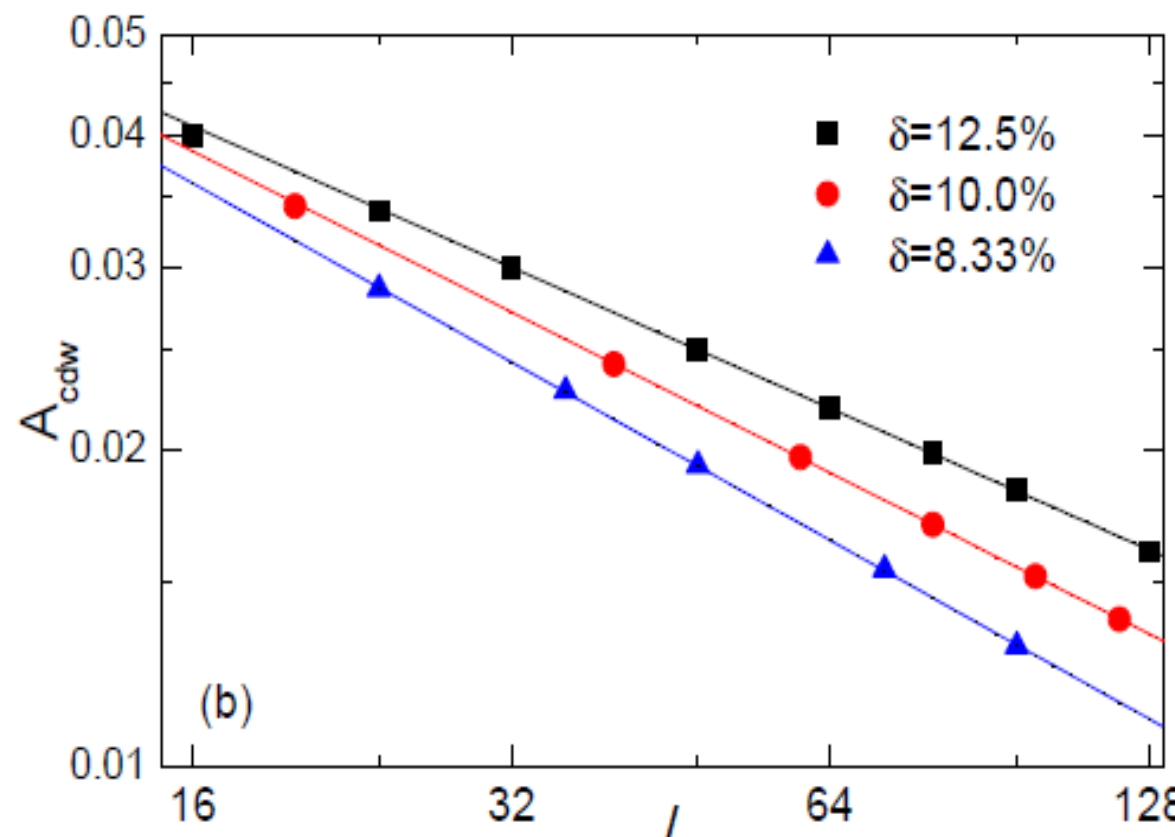
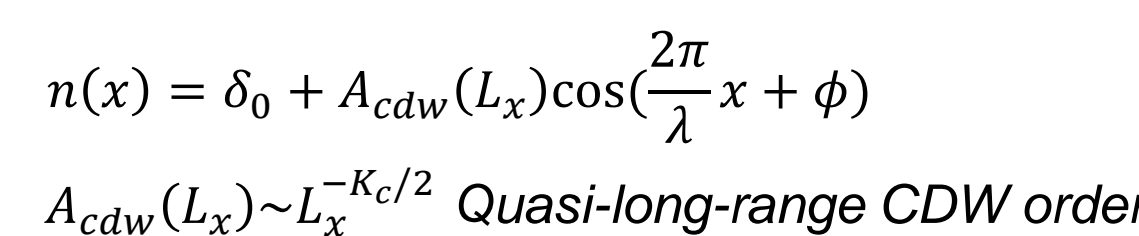
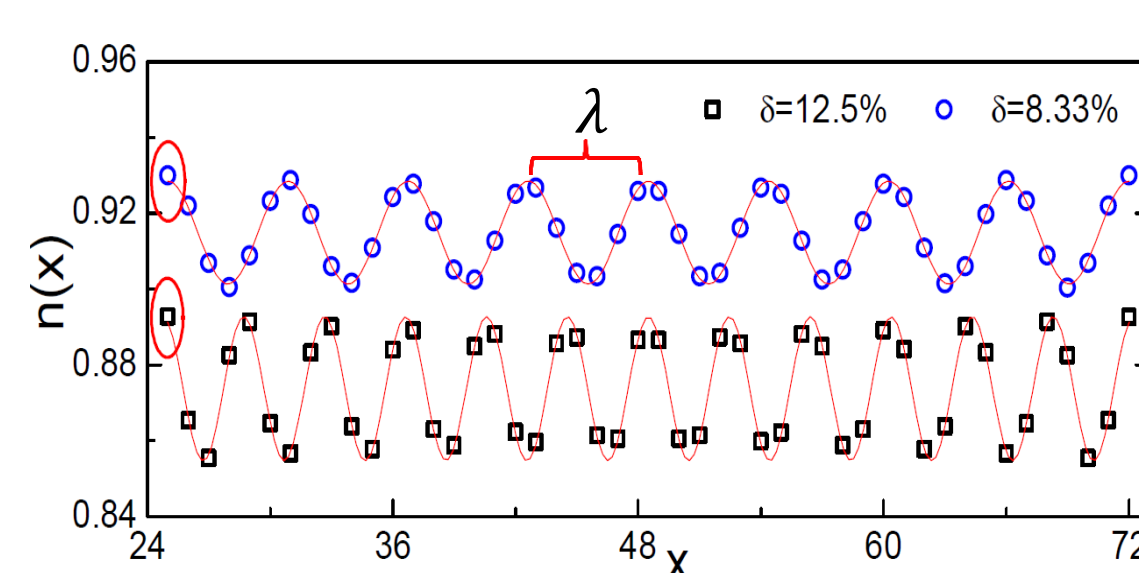
U	K_c	K_{sc}	$K_c K_{sc}$	ξ_s
8	0.90(6)	1.43(8)	1.3 (2)	9.8(6)
12	0.75(6)	1.60(7)	1.2(2)	8.3(4)

❖ t-J Model and DMRG study [7]

$$H = -t \sum_{\langle ij \rangle} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + J \sum_{\langle ij \rangle} (S_i \cdot S_j - \frac{n_i n_j}{4})$$

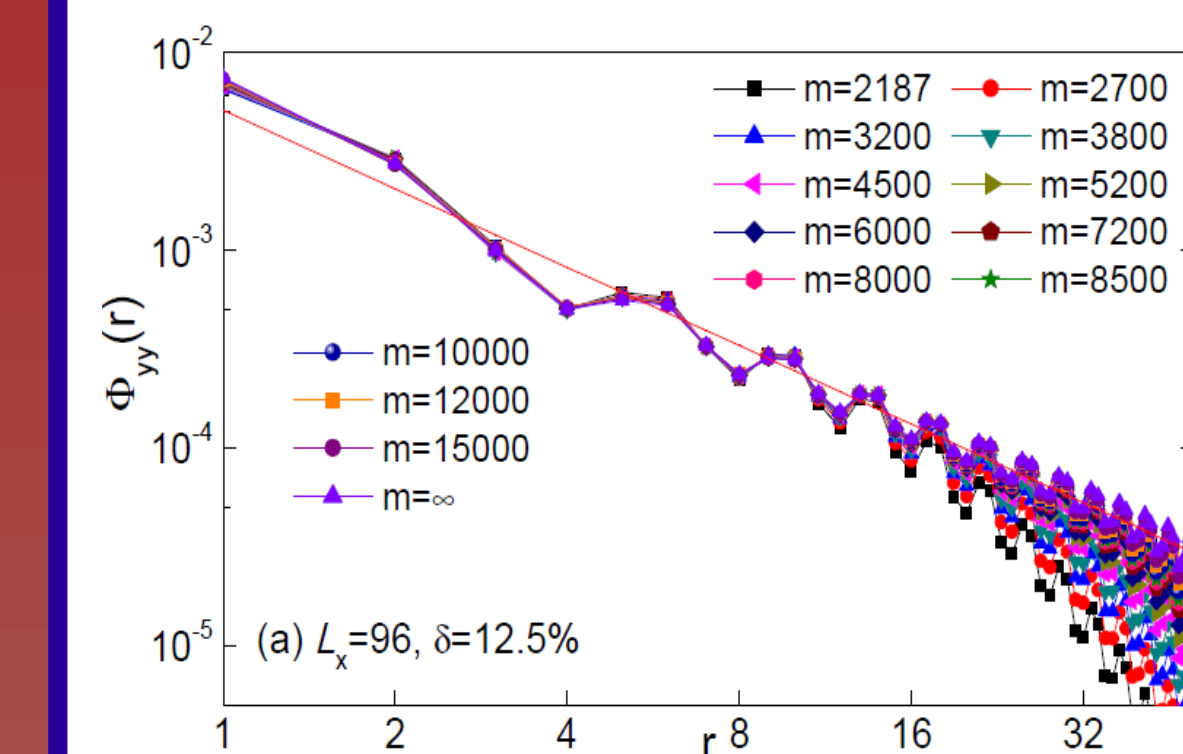
- ✓ $N=L_x^*L_y$, $L_y=4$, $L_x=16\sim 128$, hole doping $\delta=5\%\sim 12.5\%$
- ✓ Isotropic hopping and spin interaction $t=3, J=1$
- ✓ Keep $m=2187\sim 15000$ states with ~ 100 sweeps

1) Charge-density-wave order (CDW)

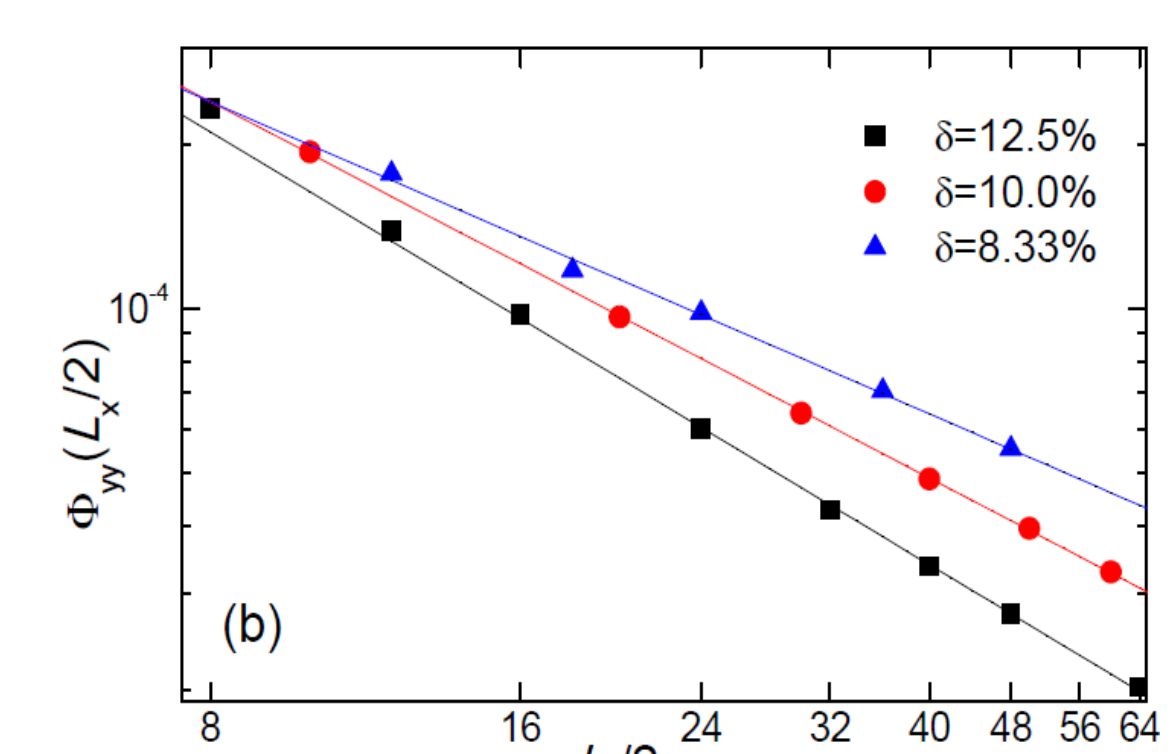


2) Superconductivity (SC)

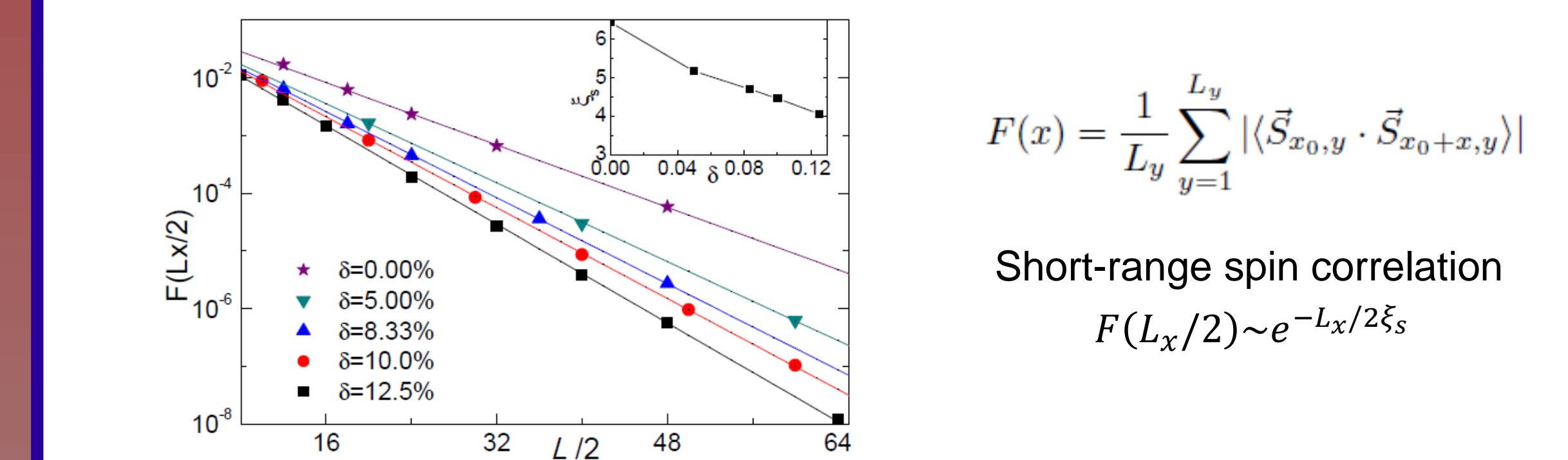
Quasi-long-range SC: $\Phi_{yy}(r) \sim r^{-K_{sc}}$



$\Phi_{yy}(L_x/2) \sim (L_x/2)^{-K_{sc}}$



3) Spin-spin correlation

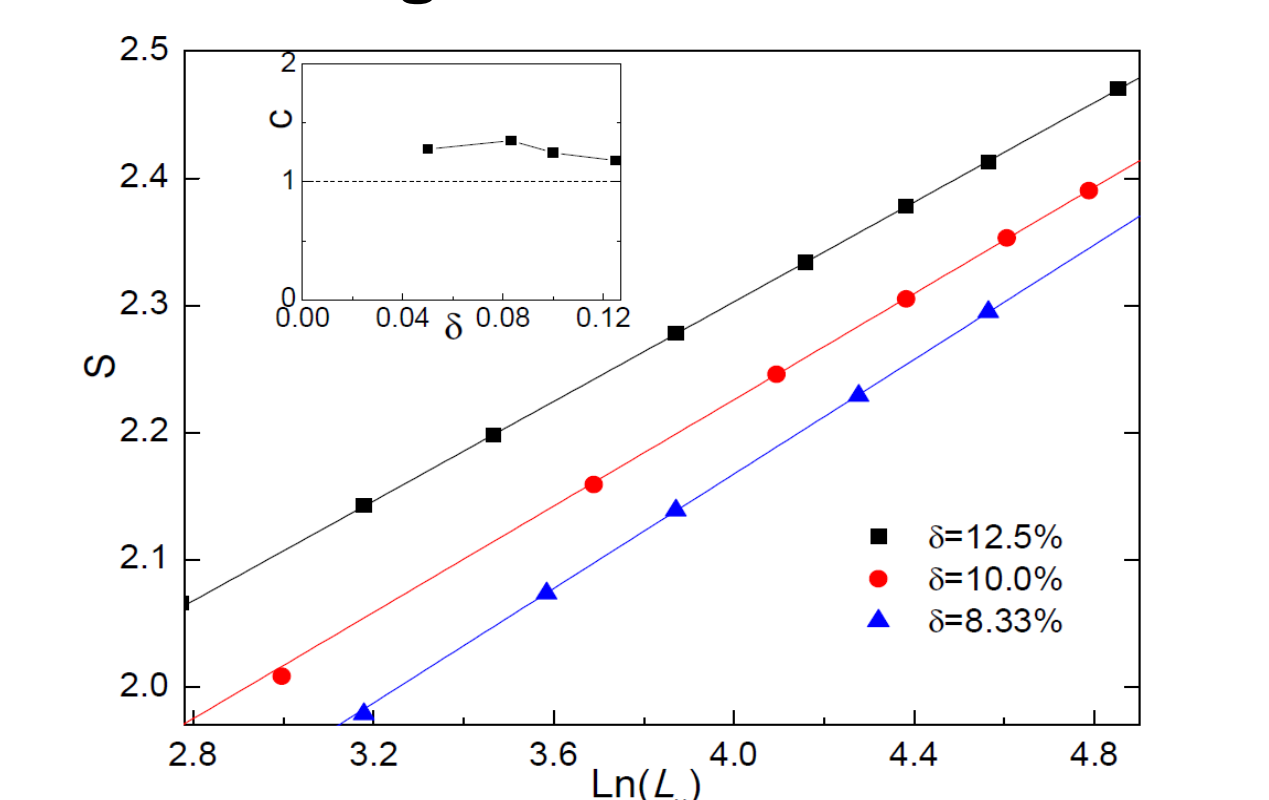


4) Entanglement entropy and central charge

$$S = -\text{Tr} \rho \ln \rho$$

$$S\left(\frac{L_x}{2}\right) = \frac{c}{6} \ln(L_x) + \tilde{c}$$

c =central charge

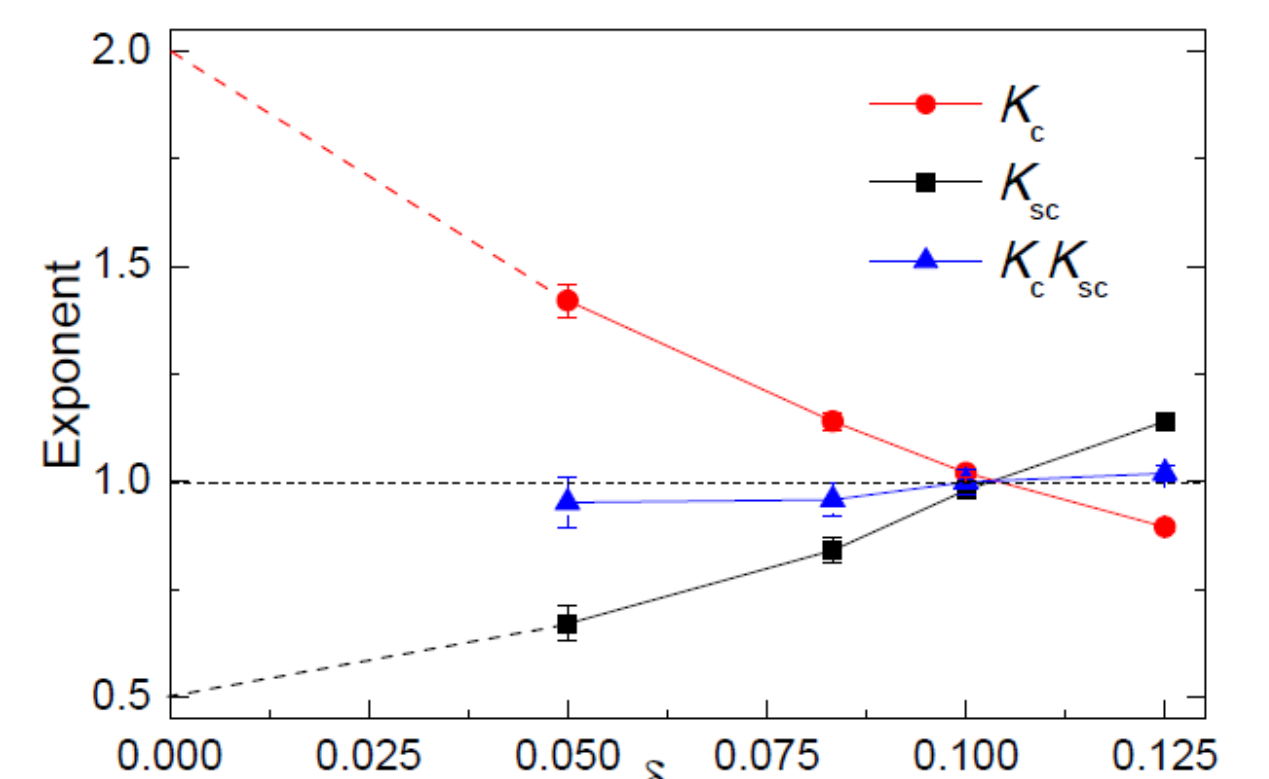


5) Luther-Emery liquid

$$A_{cdw}(L_x) \sim L_x^{-K_c/2}$$

$$\Phi\left(\frac{L_x}{2}\right) \sim \left(\frac{L_x}{2}\right)^{-1/K_{sc}}$$

$$K_c \times K_{sc} = 1$$



❖ Conclusion

- ✓ The ground state of the lightly hole-doped Hubbard and t-J models is a Luther-Emery liquid with a gap in the spin sector.
- ✓ Both superconducting order and charge-density-wave order are quasi-long-ranged.

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