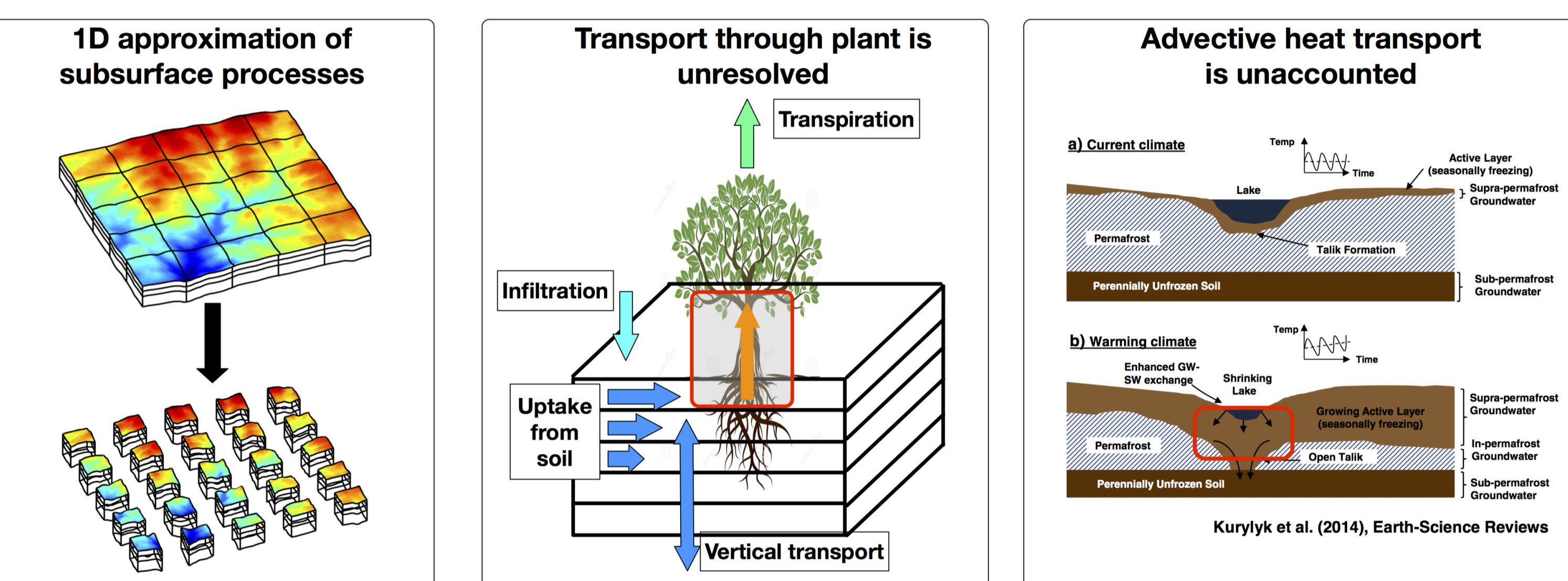


Motivation

- Developing a **predictive understanding of the terrestrial water cycle** at local to global scale is essential for accurate assessment of water resources, agricultural production, and energy generation given current climate variability.
- Terrestrial component of the U.S. Department of Energy's (DOE's) E3SM **excludes many critical physical processes**



- Due to missing process representations, E3SM is **unable to answer following key research questions**:
 - How topography may mitigate drought effects on vegetation along a hillslope gradient?
 - How hydraulic functional traits of root, stem, and leaf will determine the response of tree to future drought in presence of topographic gradients?
 - Will inclusion of advective energy transport significantly alter prediction of permafrost thaw?
- Computational challenges associated with E3SM's 10-year vision of resolving terrestrial processes at sub-kilometer resolution include:
 - Solving nonlinear parabolic PDEs with approximately **10 billion unknowns**
 - Using spatial discretization of PDEs that account for **non-orthogonal grids**
 - Flexibility to solve **tightly coupled multi-component, multi-physics problems**

Objectives

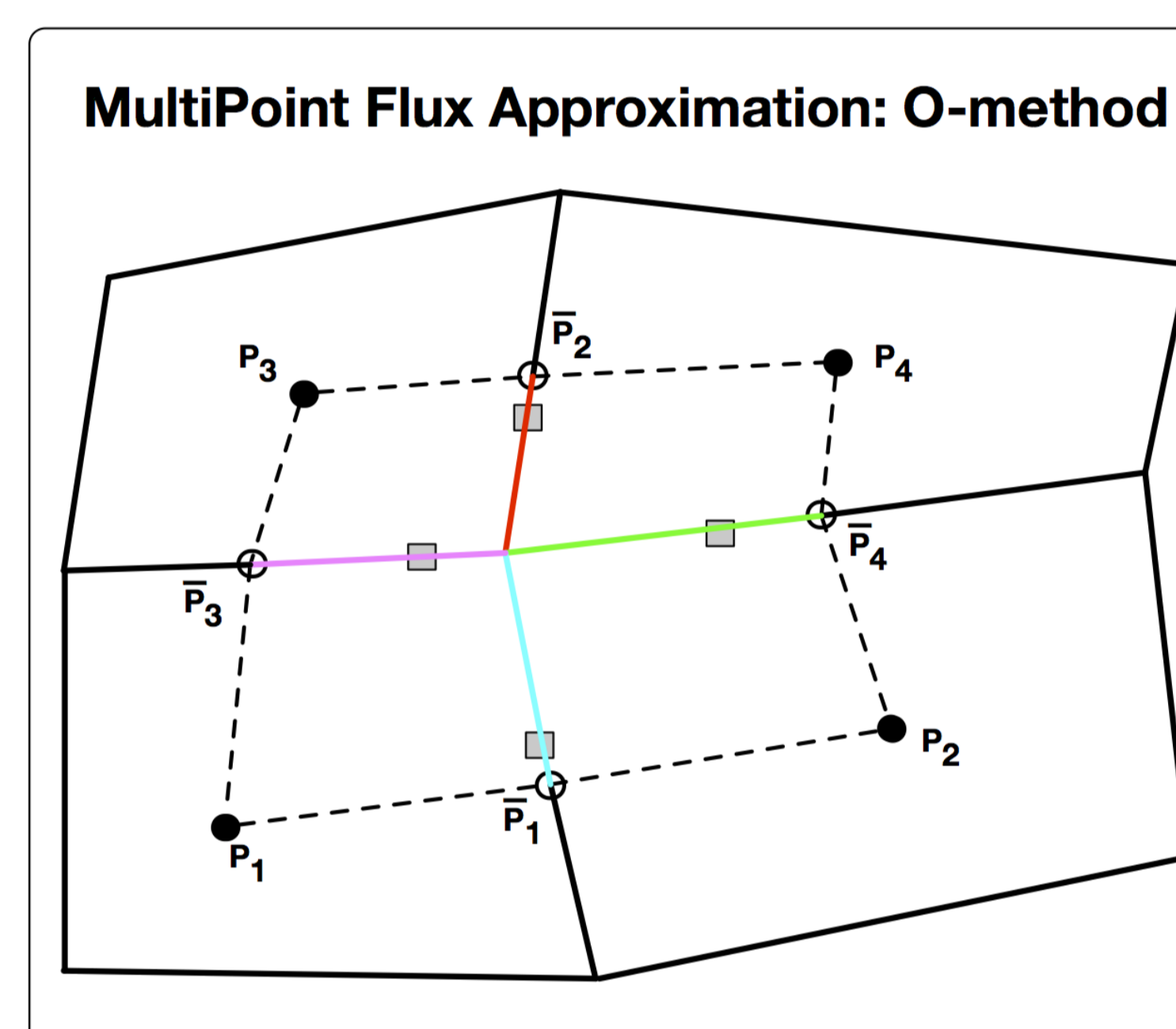
- In Phase-I of this project, we will develop a **rigorously verified, spatially adaptive, scalable, multi-physics dynamical core** (dycore) for global-scale modeling of three-dimensional subsurface processes
- The dycore will use the **Portable, Extensible Toolkit for Scientific Computation** (PETSc) library to provide numerical solution of discretized equations

Spatial Discretization: Methods

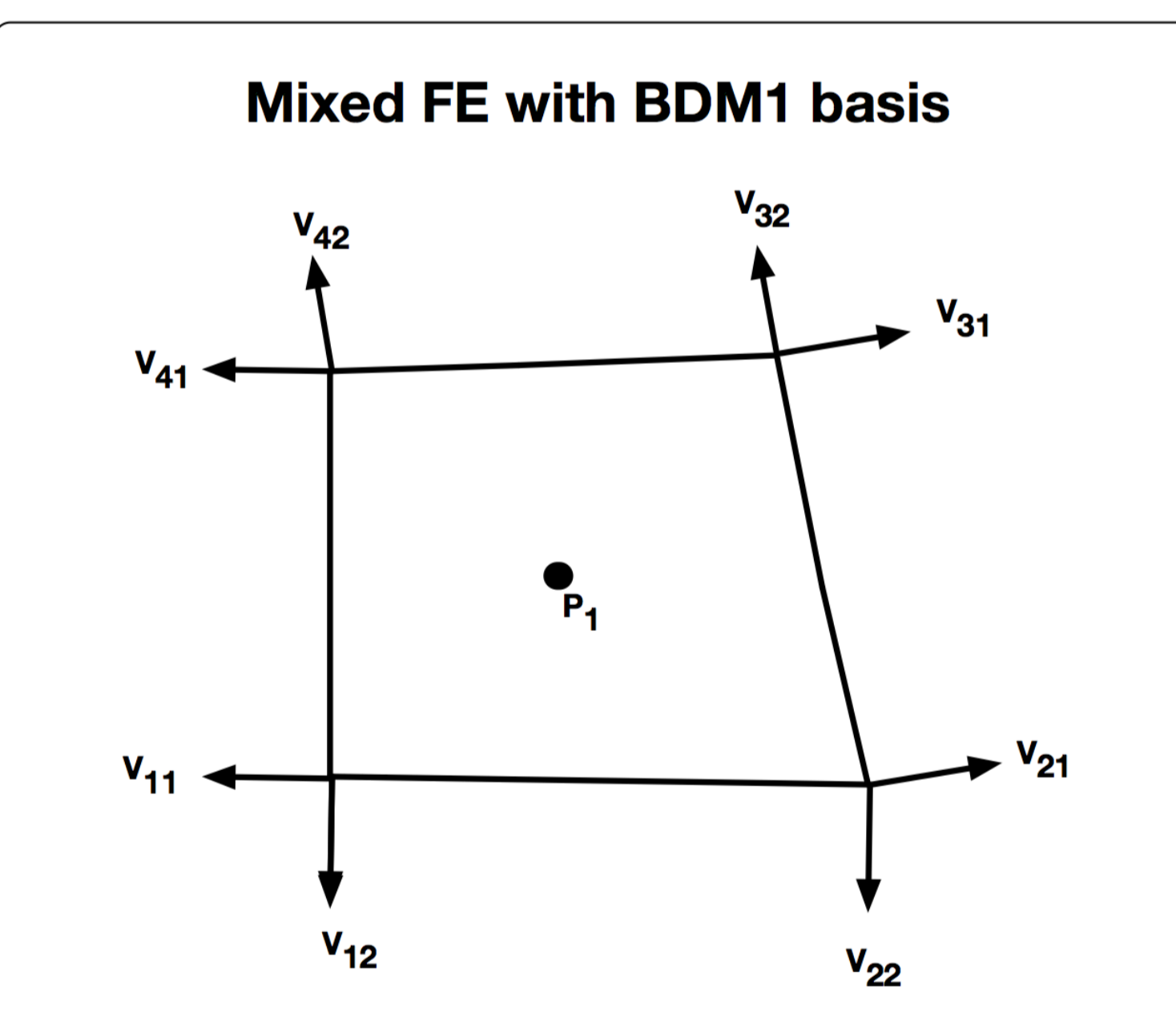
- High spatial resolution in complex terrain leads to **non-orthogonal three dimensional grids**
- Two spatial discretization methods** have been identified that account for non-orthogonal grids and have been previously applied to solve for subsurface flow and transport processes

1. Multi Point Flux Approximation (MPFA) - O method

- Based on finite volume in which control volumes are subdivided into interaction volumes (IVs)
- Pressure varies linearly and flux continuity is enforced across IVs
- Discretization can be performed in physical or reference space
- Number of unknowns are cell-centered pressure values



2. Mixed Finite Element (MFE) method



- BDM1 basis assumes that normal velocity may vary linearly along an edge
- Discretization is performed in a reference space
- Choice of numerical quadrature reduces the number of unknowns to cell centered pressure values

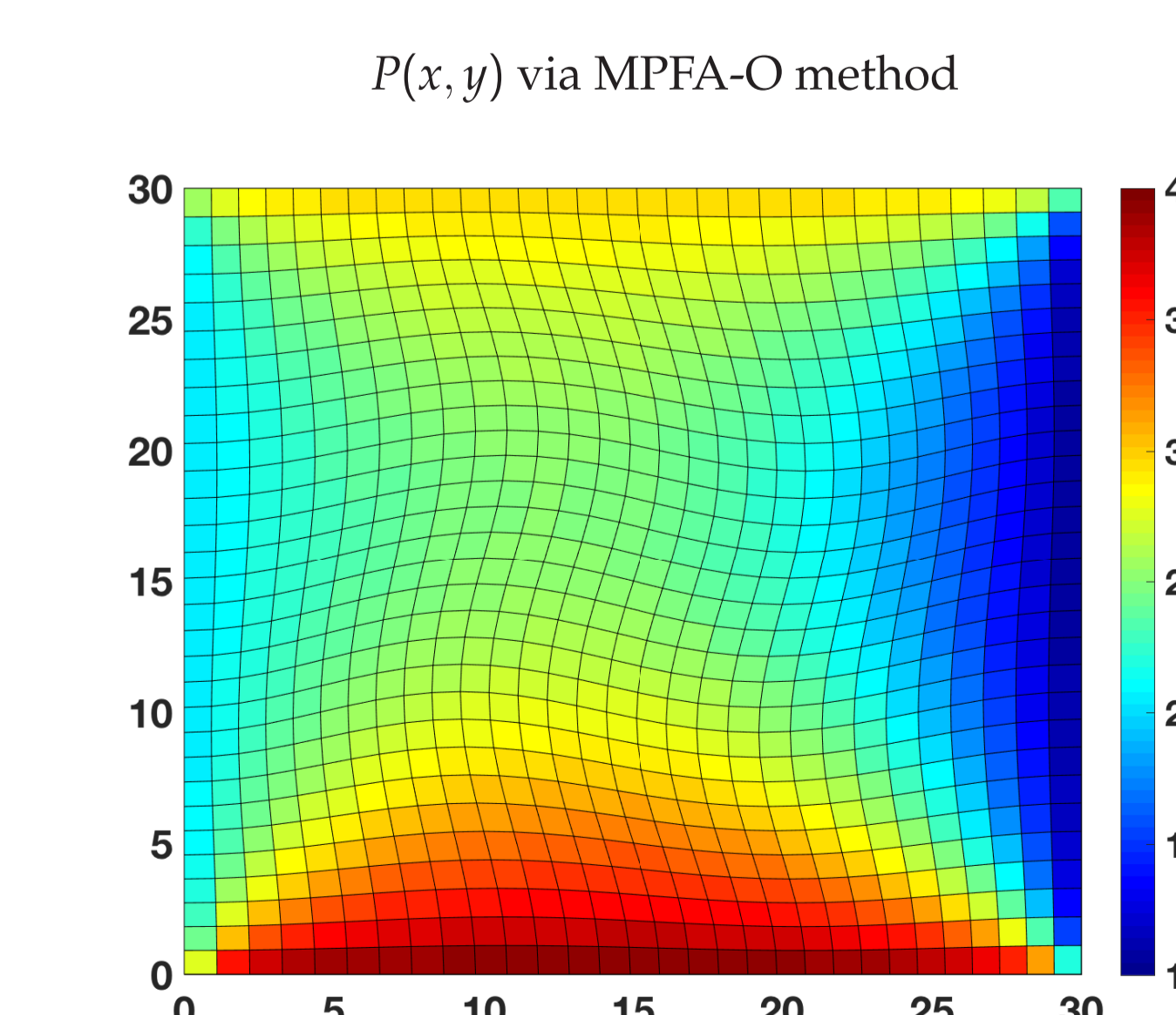
- MPFA-O and MFE methods have **different error convergence properties** for rough grids

Spatial Discretization: Results

- Developed a prototype code** for solving 2D steady-state diffusion equation using MPFA-O method

- Solves $\nabla \cdot (K \nabla P) = 0$ on a 2D non-orthogonal grid with $K = 1$, $P_{south} = 4$, $P_{north} = 3$, $P_{right} = 1$, and $P_{left} = 2$

- Preliminary comparison of our results show good agreement with the MATLAB Reservoir Simulation Toolbox



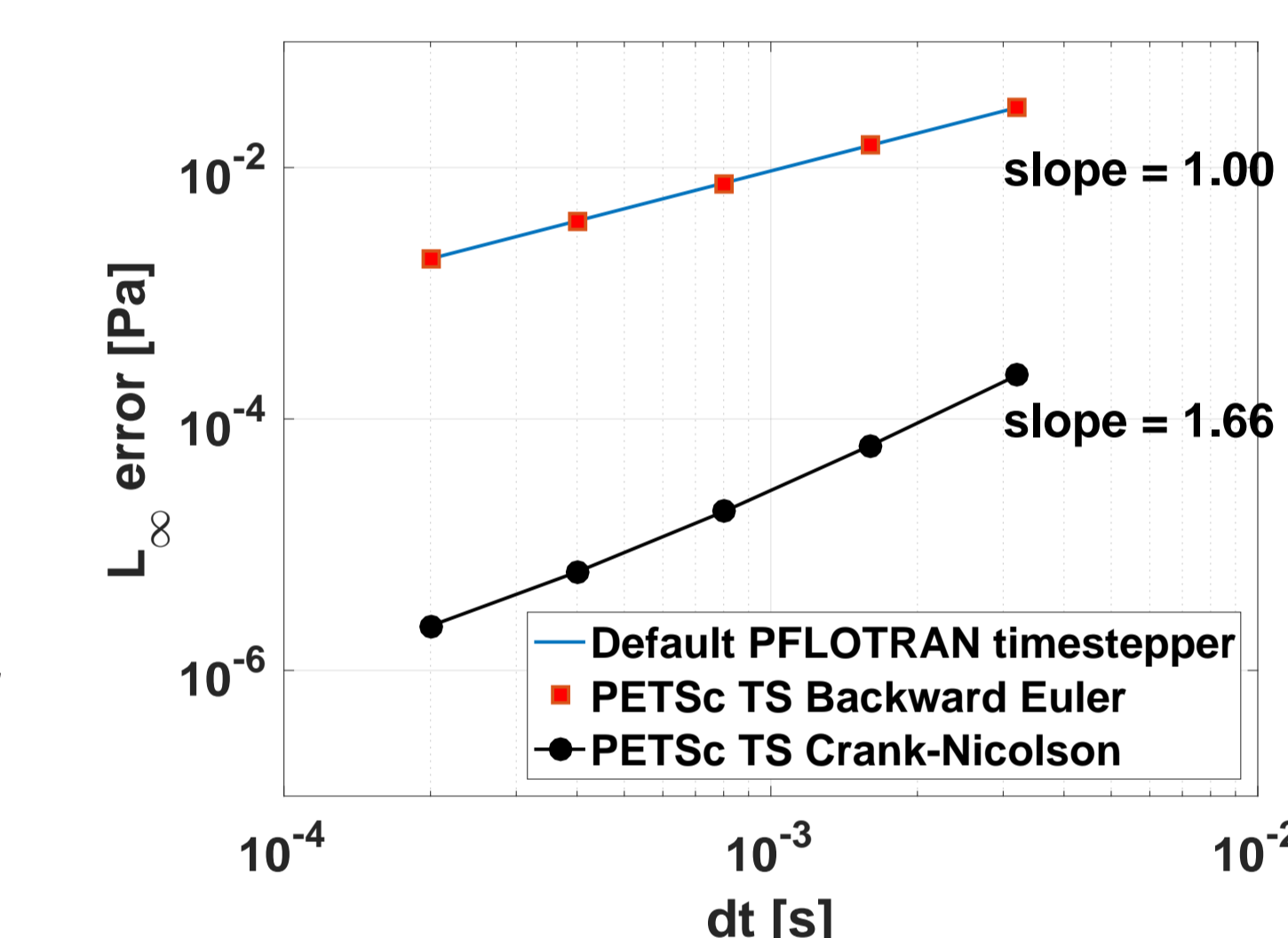
Temporal Discretization: Methods

- The dycore will use **PETSc time-stepper (TS)** for temporal discretization
- Use of TS will enable easy **experimentation with multiple time integration algorithms** and allow the development of **an adjoint model in the future** for data assimilation

Temporal Discretization: Results

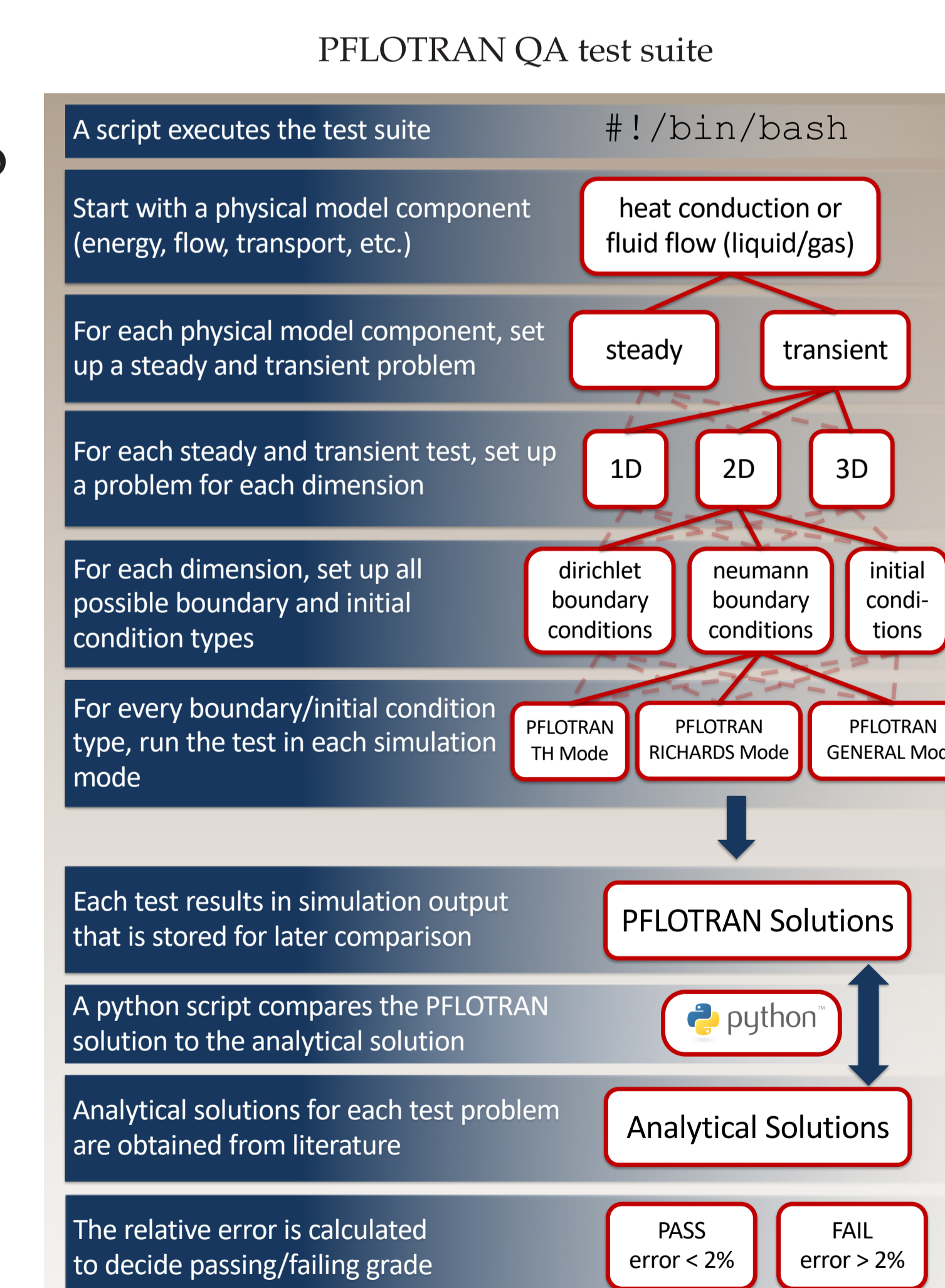
- Implemented PETSc TS based solver** in PFLOTRAN, a subsurface flow and transport model that uses first-order spatial discretization
- Error convergence study** is performed for a 1D soil column problem in which liquid pressure evolves towards a hydrostatic equilibrium starting with a spatially homogenous initial condition

- Error for the default PFLOTRAN timestepper and TS backward Euler converges at the same expected rate
- Error for TS Crank-Nicolson converges at a higher rate than TS backward Euler



Next steps

- Extend prototype codes for MPFA-O and MFE methods to **support 3D problems**
- Study error convergence** for MPFA-O and MFE for non-orthogonal grids
- Develop a **verification and validation test suite** for the dycore
- Apply the dycore** for a range of terrestrial multi-physics problems at global scales



Acknowledgements

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