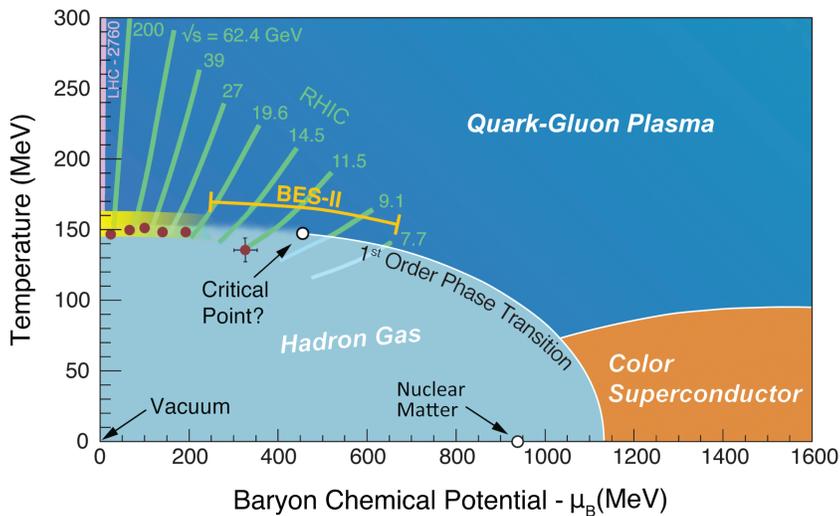
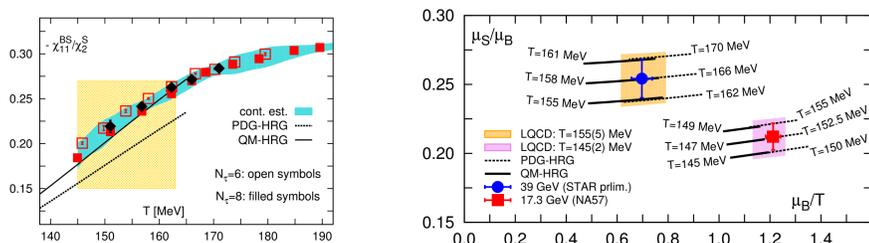


Introduction



In a heavy-ion collision (HIC), a dense medium of quarks and gluons is created which subsequently expands and cools down. After equilibration, this cooling process is characterized by a trajectory in the space of temperature T and baryon chemical potential μ_B . The system eventually crosses the “phase boundary” between the partonic and hadronic phase of strong-interaction matter. Shortly after this, hadrons are expected to form at the so-called freeze-out point (T_f, μ_B^f) . This point can be varied in HIC experiments by changing the beam energy. In the beam energy scan (BES) at RHIC, a freeze-out line is mapped out. The expectation is that this line is close to the QCD phase boundary. Information on the phase structure, i.e. possibly the existence of a critical point as well as a line of first order phase transitions at higher energy, can be obtained by analyzing physical observables on the freeze-out line. By examining fluctuations of conserved charges and their correlations, one tries to: (i) determine the freeze-out parameters [1], (ii) establish their relation with the QCD transition line and (iii) hopefully find evidence for the existence of a critical point.

2. Additional strange hadrons



We have calculated higher order cumulants of strangeness (χ_2^S) fluctuations and their correlations with baryon number (χ_{11}^{BS}) and electric charge (χ_{11}^{QS}) at $\mu = 0$ and evaluated them in a next-to-leading-order Taylor expansion in μ_B . For our simulations, we used the Highly Improved Staggered Quark (HISQ) action with physical strange quark mass m_s and degenerate up and down quarks with $m_l = m_s/20$ corresponding to a pion mass of about 160 MeV.

$$\left(\frac{\mu_S}{\mu_B}\right)_{LO} = -\frac{\chi_{11}^{BS}}{\chi_2^S} - \frac{\mu_Q}{\mu_B} \frac{\chi_{11}^{QS}}{\chi_2^S} \quad (1)$$

We compare our results [2] to predictions of the hadron resonance gas model having different strange hadron content. The first formulation includes only observed hadrons (PDG-HRG) and is very successful in describing the hadronic phase. However, near the cross-over region it fails to match lattice QCD results. This is why we applied a second formulation which takes additional strange hadrons from quark models into account (QM-HRG). The shaded regions in the plots indicate the chiral crossover region $T_c = 154(9)$ MeV. On the r.h.s., we compare experimentally extracted values of (μ_S^f/μ_B^f) , (μ_B^f/T_f) with lattice QCD results for μ_S/μ_B . The results are shown for $\mu_B/T = \mu_B^f/T_f$. The temperature range where lattice QCD results match with μ_S^f/μ_B^f provides values for T_f .

We conclude that these unobserved strange hadrons become thermodynamically relevant close to the QCD crossover region and lead to a significant reduction in the chemical freeze-out temperature. These calculations currently get extended to 6th order [3].

1. Charge fluctuations and correlations

In order to study the QCD phase diagram, it is mandatory to consider physical observables which reflect the thermal conditions of the system, i.e. are sensitive to the inner structure of the fireball as created in heavy-ion collision experiments. Fluctuations of globally conserved charges (B, Q, S) vary strongly between the confined and deconfined phase, thus, can be calculated in lattice simulations to distinguish between the phases of QCD. In experiment fluctuations can be studied at the time of freeze-out, i.e. are obtained from measured ensembles of conserved charges in several heavy-ion collision events. The idea of these fluctuation probes of the QCD phases is based on a simple picture: In the Hadron Resonance Gas (HRG) model all hadrons have electric charge of ± 1 or ± 2 , while in the quark-gluon plasma the unit of charge is $\pm 1/3$ or $\pm 2/3$. Thus, fluctuations of charged particles in or out of a sub-volume in the fireball produces a larger mean square fluctuation of the net electric charge while the system is in the hadronic phase. The coefficients in the Taylor series of the pressure

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V, T, \mu_B, \mu_Q, \mu_S) = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk,0}^{BQS} \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k, \quad \hat{\mu}_X = \mu_X/T \quad (2)$$

are higher orders of charge fluctuations and correlations given by

$$\chi_{ijk,\mu}^{BQS} = \frac{1}{VT^3} \left. \frac{\partial^{i+j+k} \ln Z(V, T, \mu_B, \mu_Q, \mu_S)}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\mu=\{\mu_B, \mu_Q, \mu_S\}}, \quad (3)$$

which can be directly related to physical observables as measured in heavy-ion collisions. In particular, these generalized susceptibilities are related to higher order moments: mean M_X , variance σ_X^2 , skewness S_X and kurtosis κ_X which are obtained by net charge fluctuations $\delta N_X = N_X - \langle N_X \rangle$ for each conserved charge $X = B, Q, S$ in the thermodynamic ensemble.

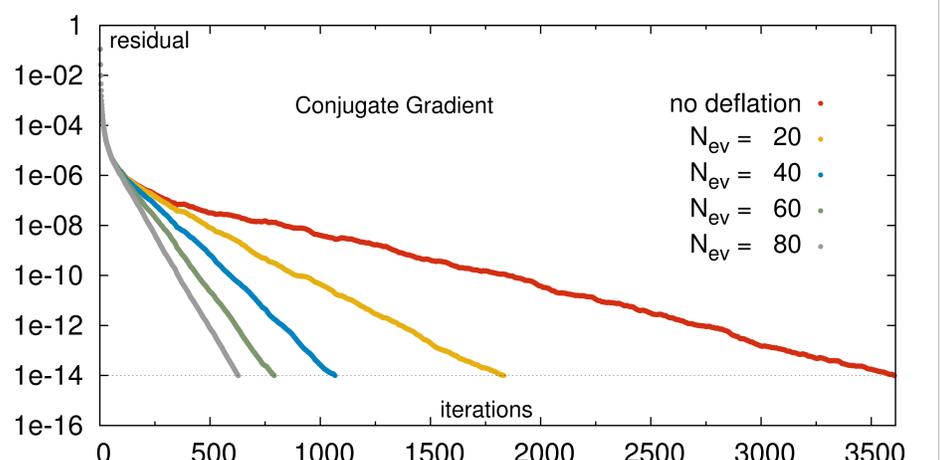
3. Deflated solvers

Higher order charge fluctuations require to calculate traces containing fermion matrix inversions and derivatives. We estimate these traces using the so-called random noise method. In this method, the trace is replaced by a sum over N random noise vectors η_k each with properties of white noise

$$\text{Tr} \left(Q^{-1} \frac{\partial^n Q}{\partial \hat{\mu}^n} \right) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N \eta_k^\dagger Q^{-1} \frac{\partial^n Q}{\partial \hat{\mu}^n} \eta_k. \quad (4)$$

For inverting the fermion matrix Q , we use a Conjugate Gradient (CG) solver ($Q^{-1} \mathbf{b} = \mathbf{x}$). The convergence speed is dominated by small eigenvalues λ_i . We can reduce the iteration count by computing an initial guess \mathbf{x}_0 for the CG using the lowest N_{ev} eigenvalues and eigenvectors \mathbf{q}_i . It is given by

$$\mathbf{x}_0 = \sum_{i=1}^{N_{ev}} \frac{\langle \mathbf{q}_i, \mathbf{b} \rangle}{\lambda_i} \mathbf{q}_i. \quad (5)$$



[1] A. Bazavov et al., Phys. Rev. Lett. 109, 192302 (2012).

[2] A. Bazavov et al., Phys. Rev. Lett. 113, 072001 (2014).

[3] in preparation Bielefeld, BNL and CCNU.