

Nuclear Physics from First Principles

Symmetries in Nuclei from Lattice QCD and Effective Theories

Thorsten Kurth and Evan Berkowitz for the Collaboration



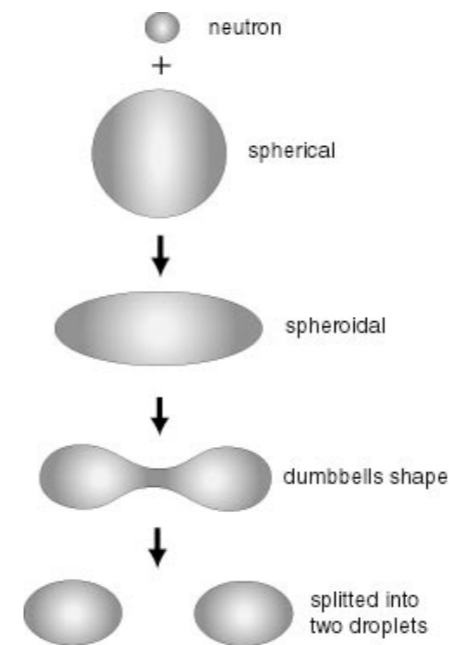
Team

- LLNL: Robert Falgout, Ron Soltz, Pavlos Vranas, Chris Schroeder, Evan Berkowitz, Enrico Rinaldi, Joe Wasem
- LBL/UCB: Wick Haxton, Thorsten Kurth, Ken McElvain, Mark Strother
- SDSU: Calvin Johnson
- nVidia: Michael Clark
- BNL: Sergey Syritsyn
- JLAB/W&M: André Walker-Loud
- JSC/Bonn University: Tom Luu

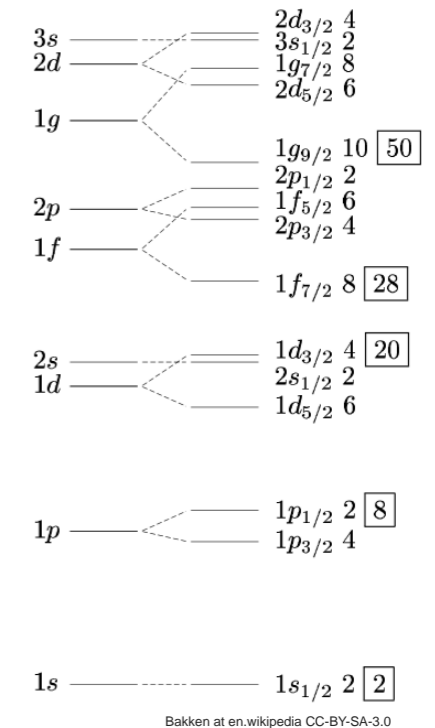
Motivation

- Historically, nuclear physics has relied on models.
- Nuclei are made of protons and neutrons, which in turn are made of quarks and gluons.
- We can use QCD to extract nucleon properties and interactions,
 - connecting model parameters to the Standard Model.
 - giving first-principles understanding.

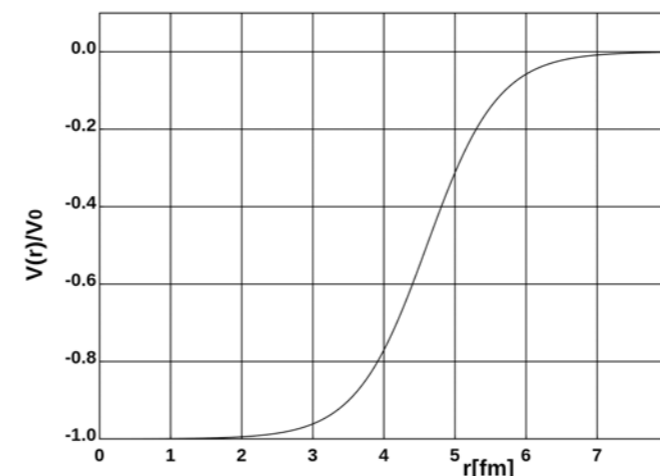
Liquid Drop



Shell



Mean Field Potentials

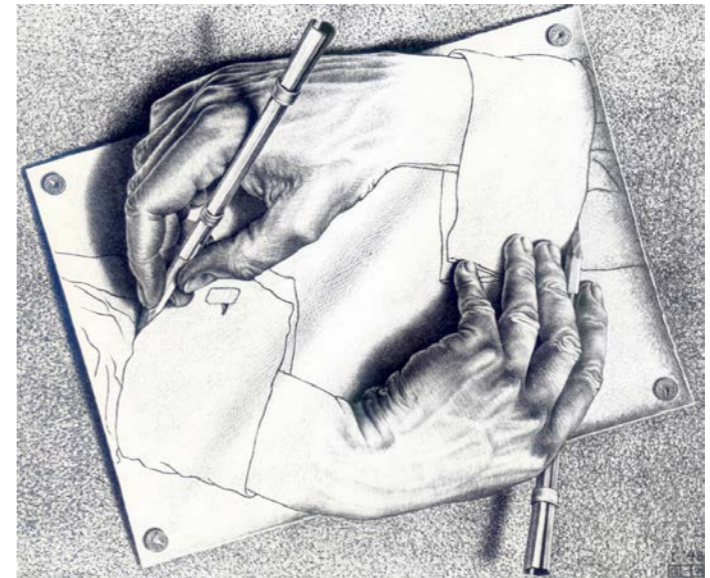


QCD

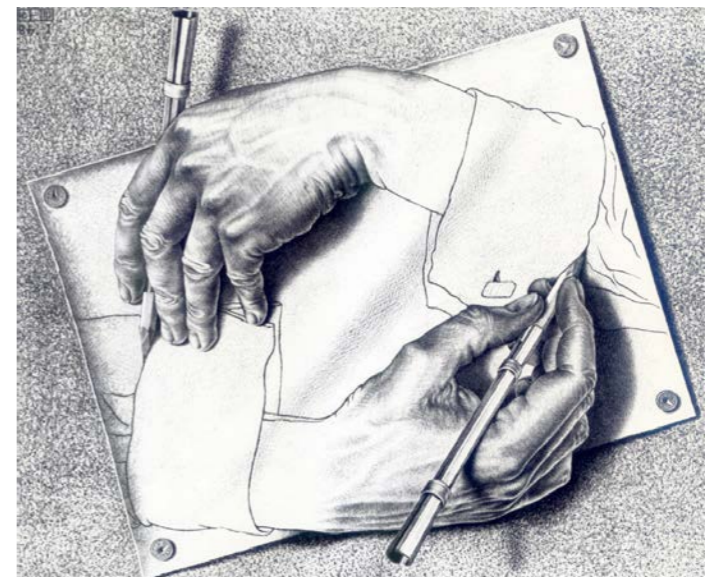
$$\mathcal{L} = \bar{\psi}(i\not{D} + m)\psi - \frac{1}{4}\text{Tr}(F^2)$$

Nuclei as Laboratories

- There are observables that are *hard to measure experimentally*.
- Nuclei provide a low-energy environment where high-precision “experiments” happen all the time.
- Can test many-body physics.
- Hadronic electroweak neutral current responsible for parity violation (PV) is the least constrained observable in the SM.
 - Weak interaction is $\sim G_F F_\pi^2 = \mathcal{O}(10^{-7})$ compared to strong interaction $\sim \mathcal{O}(1)$.
 - Nuclear data show *enhanced* isoscalar and *suppressed* isovector PV.

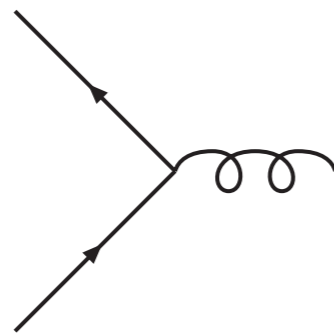


PV \neq VP



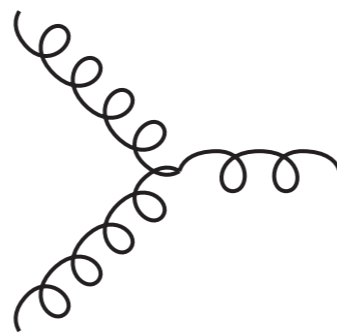
What is QCD?

- Quantum Chromodynamics is the fundamental theory of strong/nuclear interactions (fusion, fission, α -decay, ...)
- QCD describes interactions between Quarks and Gluons



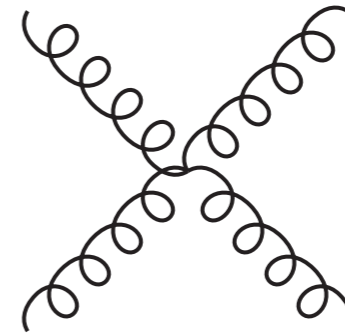
g

Quark-Gluon coupling
(QED-like)



g

Gluon self-interaction

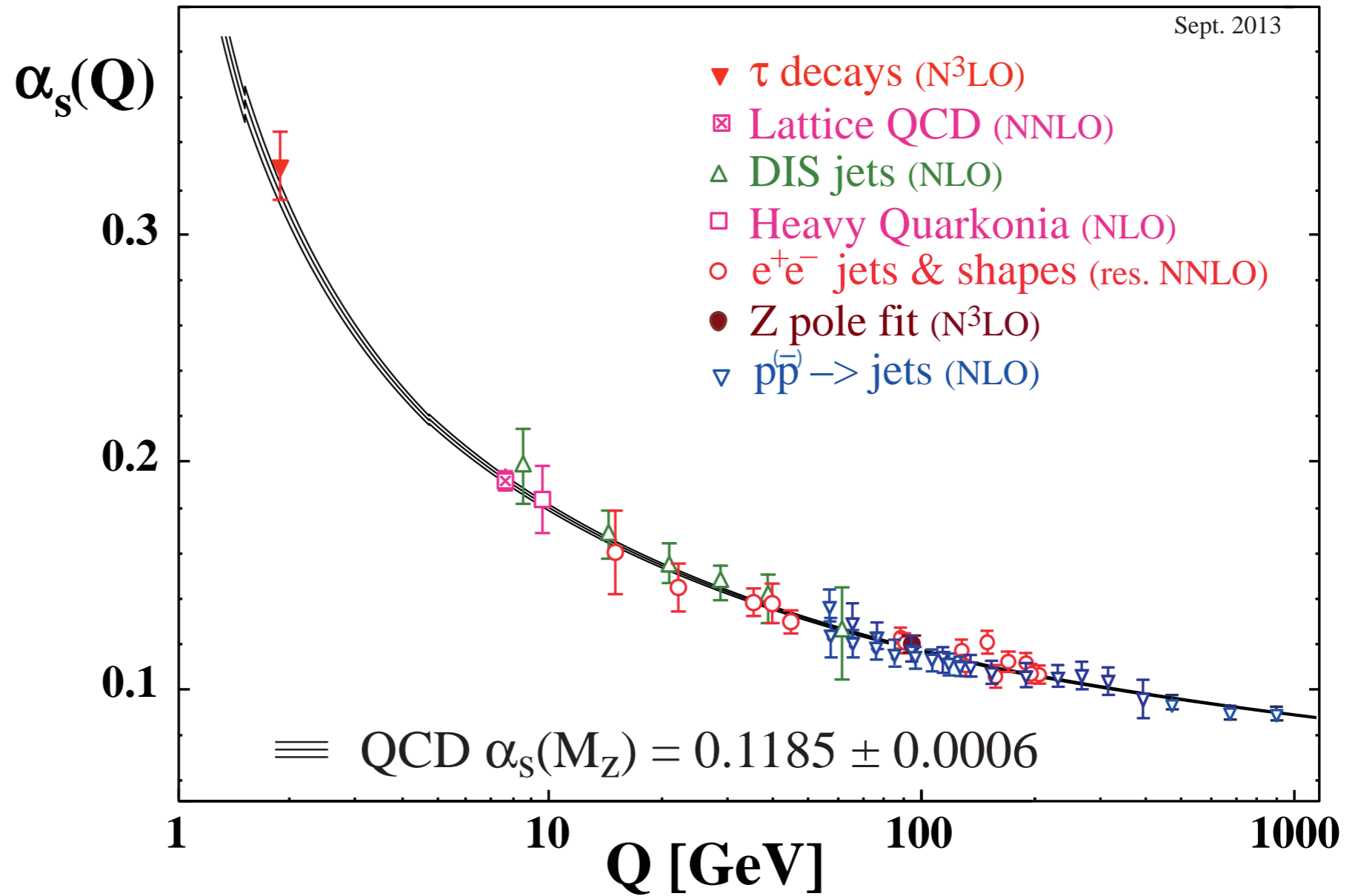
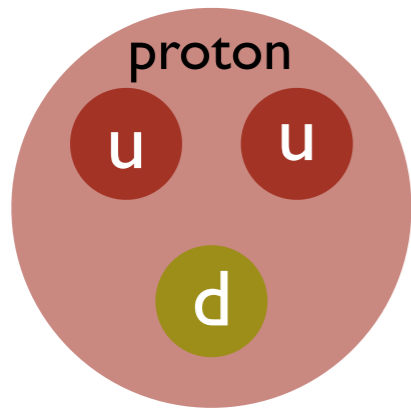
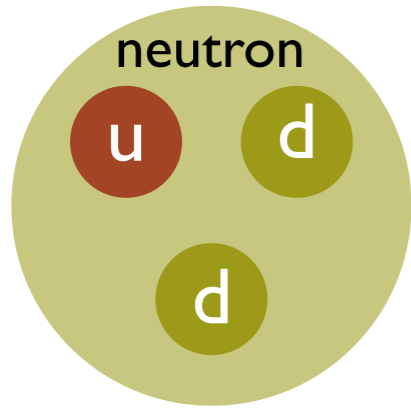


g^2

- QCD features confinement (small energies) and asymptotic freedom (large energies)

long distance

short distance



How to solve QCD?

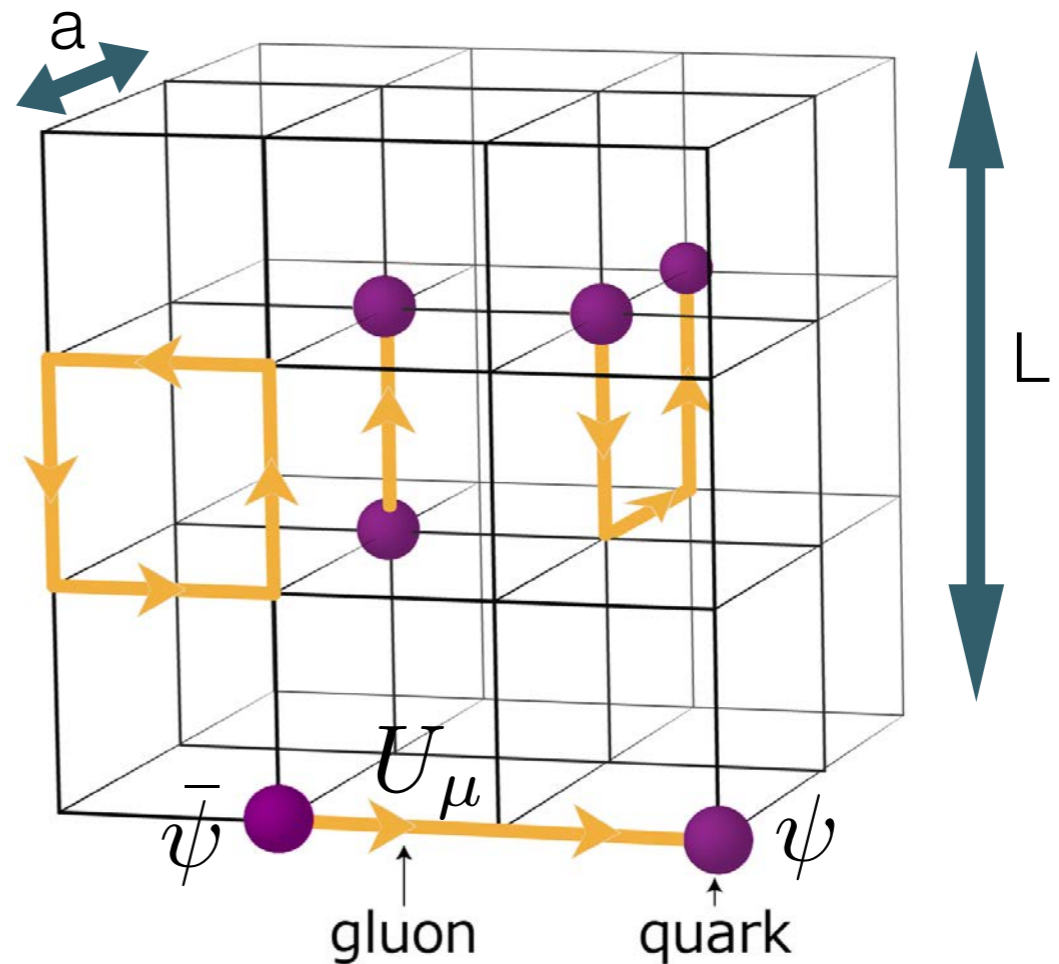
- QCD is described by infinite dimensional integrals

$$Z_E = \int \mathcal{D}U_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp \left(-S_g[U] - \int_{\mathbb{R}^4} d^4x \bar{\psi}(x) D[U] \psi(x) \right)$$

How to solve QCD?

- QCD is described by infinite dimensional integrals
- use (euclidian) 4D lattice as regulator

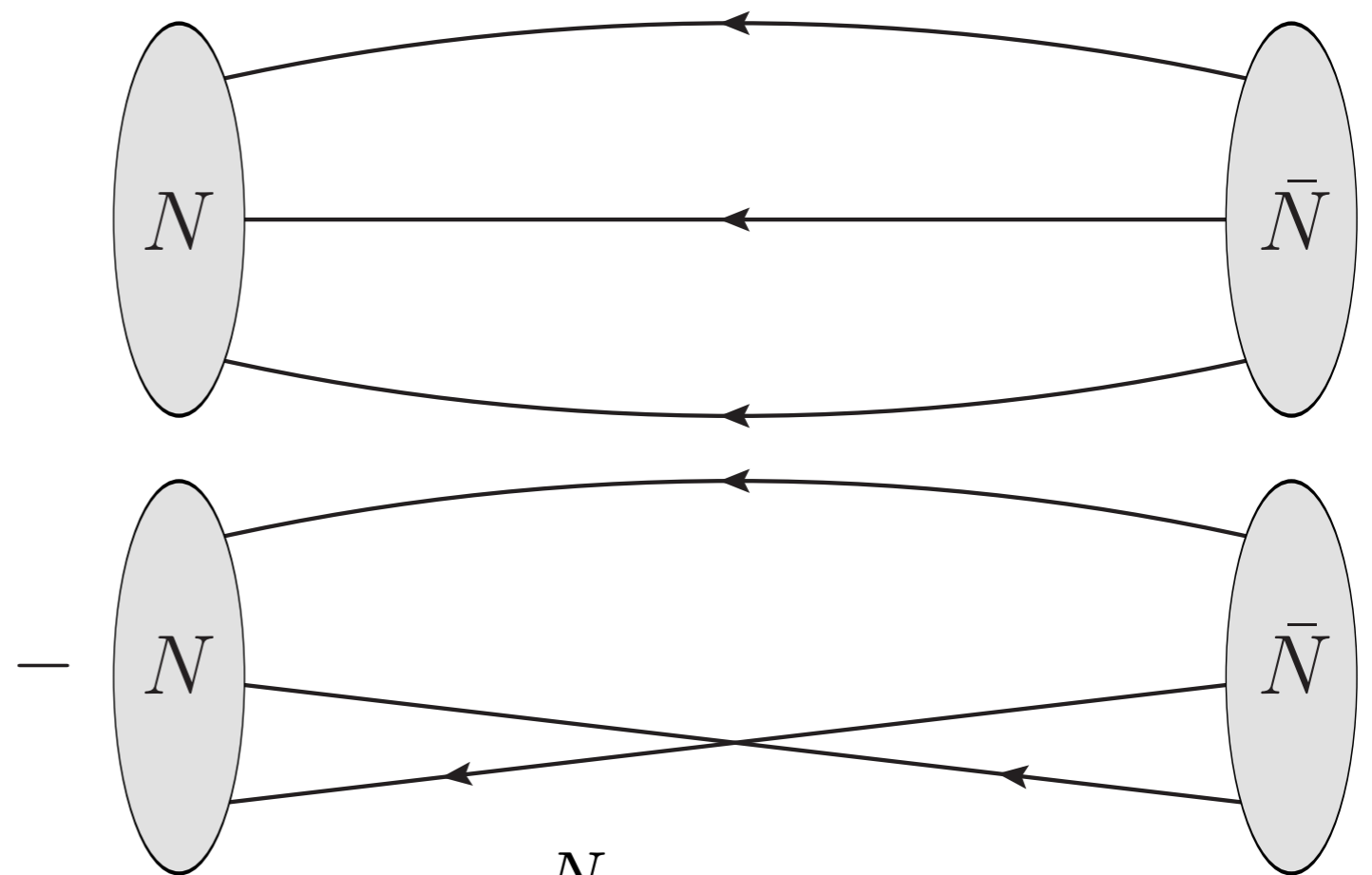
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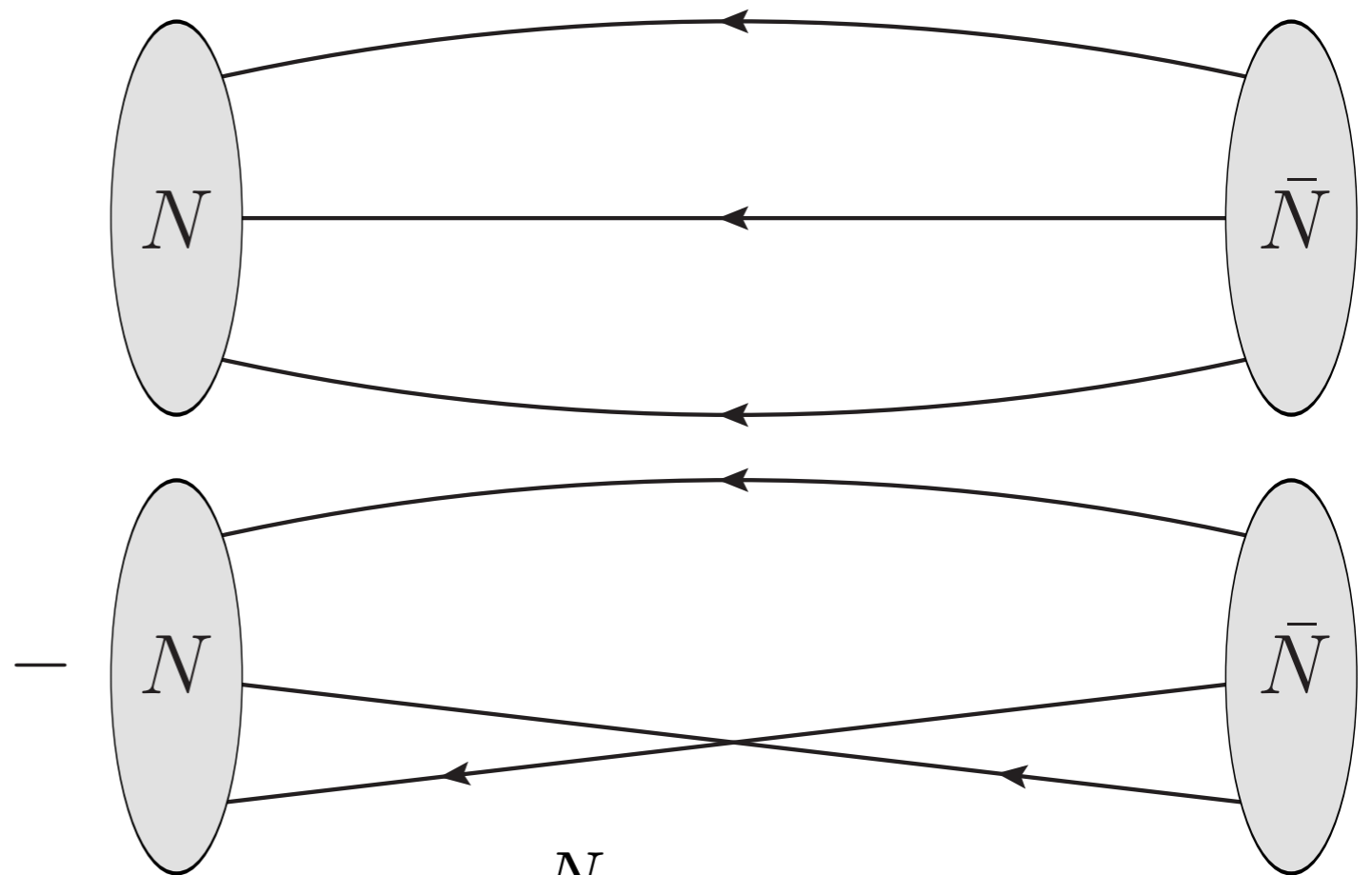


$$\langle \mathcal{O} \rangle \approx \frac{1}{N} \sum_{n=1}^N \mathcal{O}[U_n, D[U_n]^{-1}]$$

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- perform limit $a \rightarrow 0, L \rightarrow \infty$ to recover continuum QCD

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- but: $t_{\text{CPU}} \sim a^{-5}$

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$$= \int \mathcal{D}U_\mu \mathcal{D}\phi \mathcal{D}\phi^\dagger \exp \left(-S_g[U] - \int_{\mathbb{R}^4} d^4x \phi^\dagger(x) D[U]^{-\frac{1}{2}} \phi(x) \right)$$

expensive matrix inversions

$$S(x, y) = \overbrace{\psi(x) \bar{\psi}(y)} = D[U]^{-1}(x, y)$$

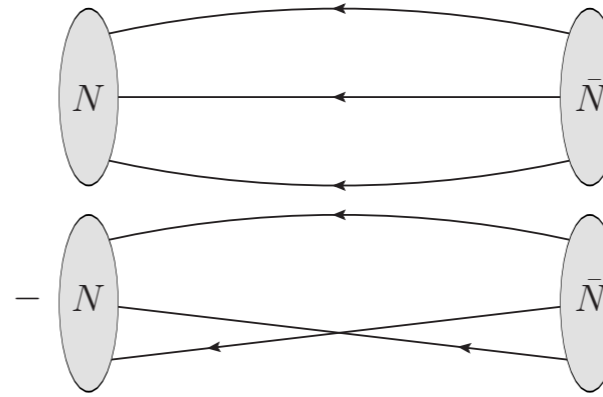
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- but: $t_{\text{CPU}} \sim a^{-5}$
- big computers needed

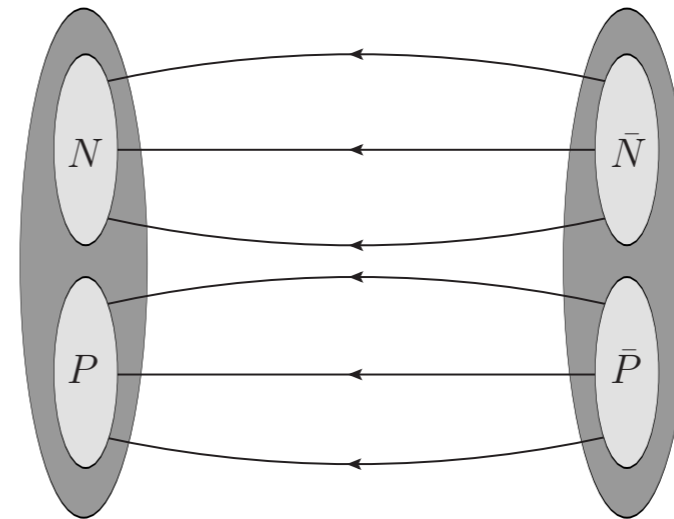


Nuclear Physics from Bottom Up

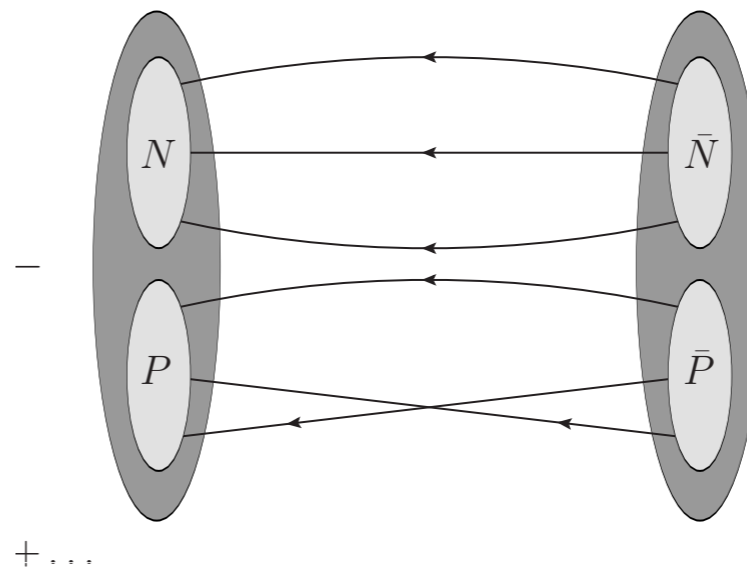
- compute multi-nucleon correlation functions



single Nucleon
 $2!1!=2$



deuteron
 $3!3!=36$

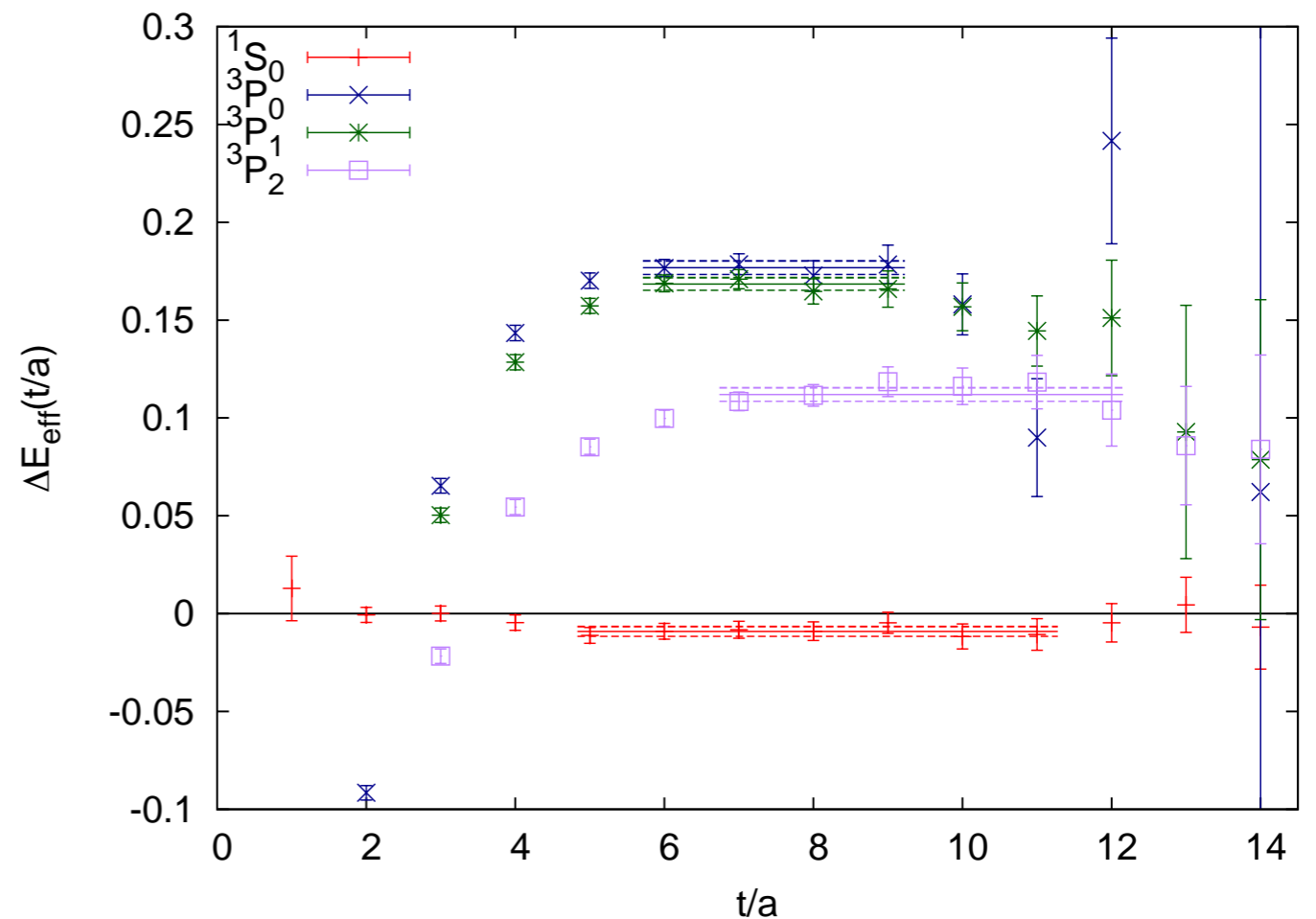


He₃-scattering

$5!4!=2880$

Nuclear Physics from Bottom Up

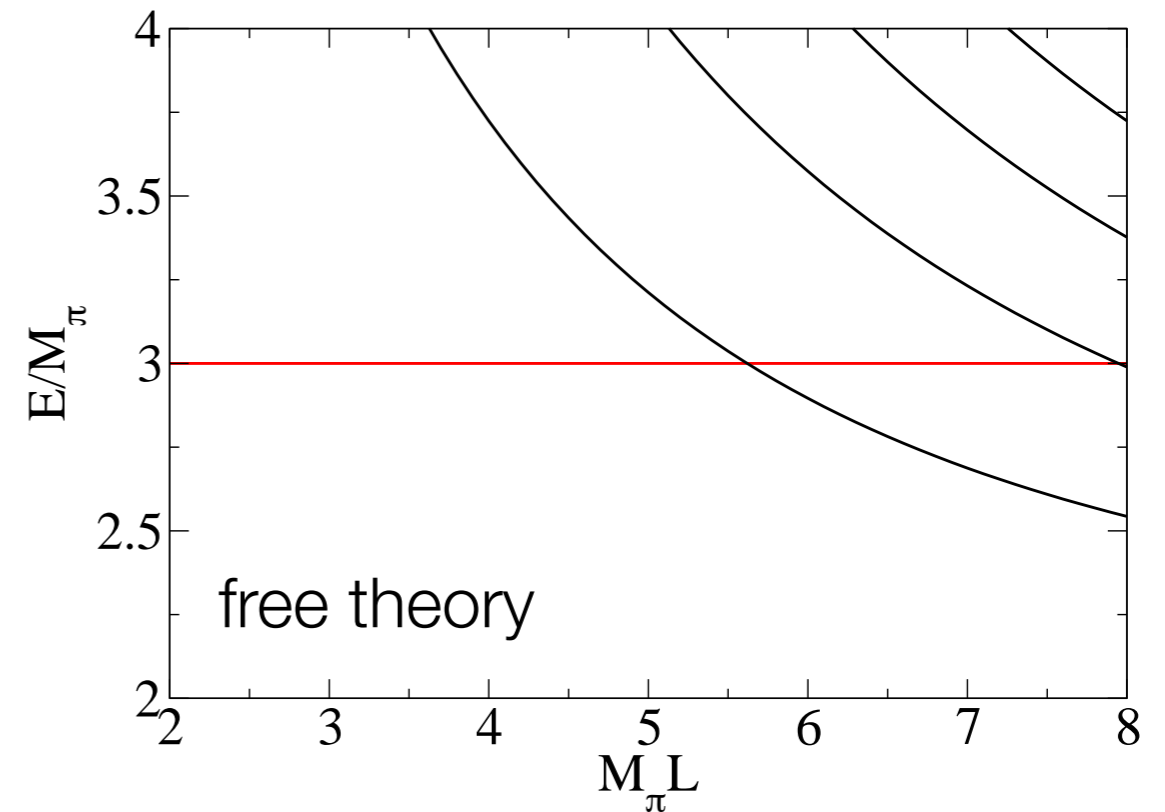
- compute multi-nucleon correlation functions
- measure energy difference between interacting and non-interacting system



$$\Delta E_{\text{eff}}^{\Gamma}(t) = \frac{1}{2} \log \left(\frac{C_{NN}^{\Gamma}(t-1) C_N^2(t+1)}{C_{NN}^{\Gamma}(t+1) C_N^2(t-1)} \right)$$

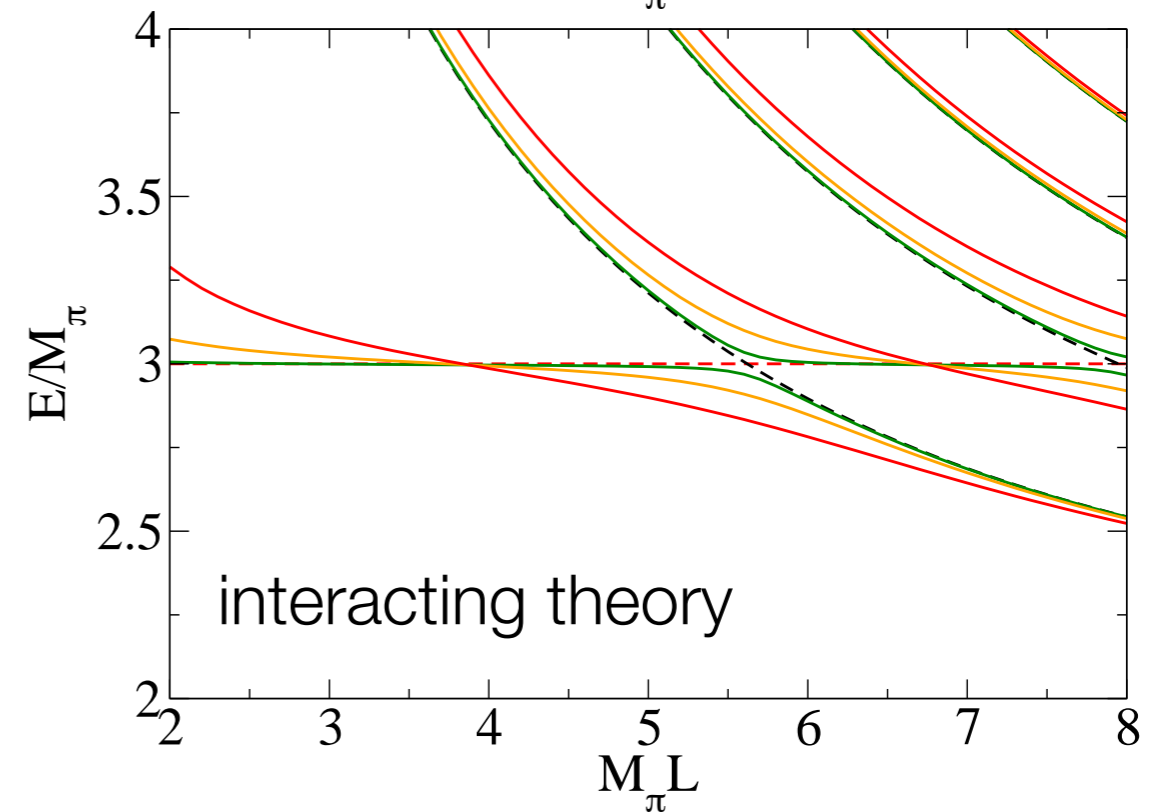
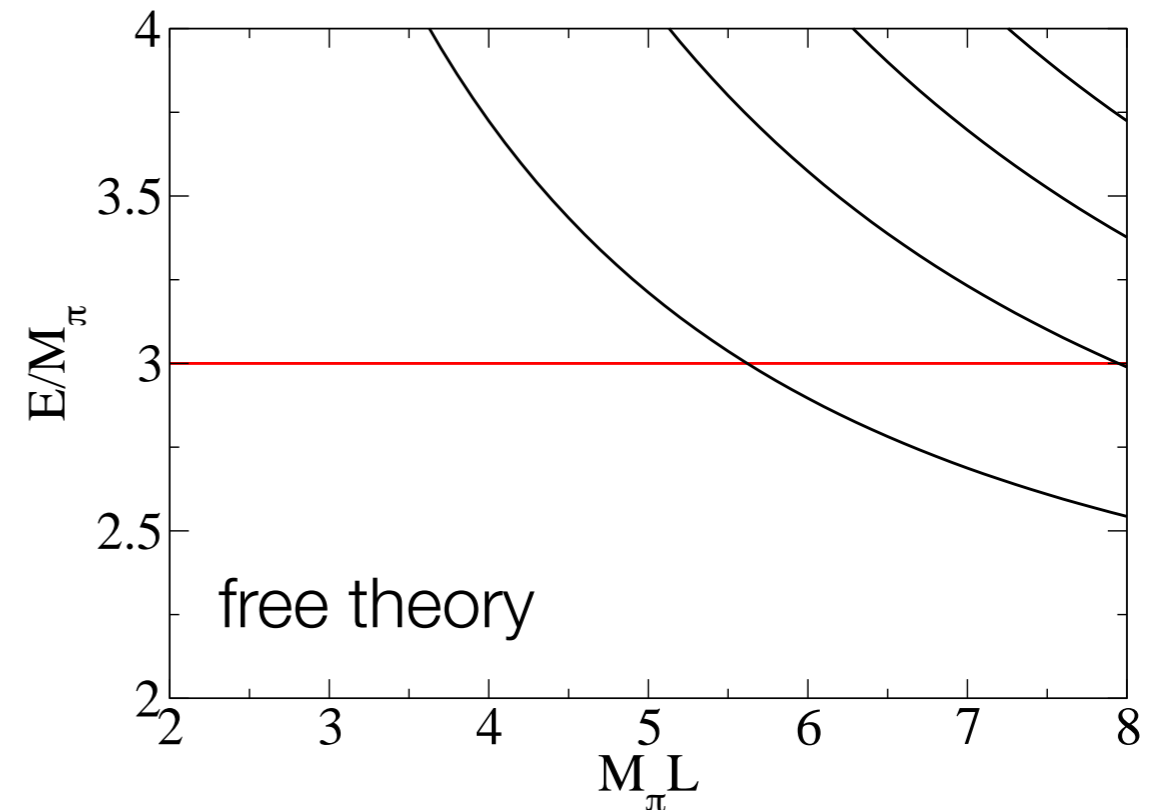
Nuclear Physics from Bottom Up

- compute multi-nucleon correlation functions
- measure energy difference between interacting and non-interacting system
- use Lüscher's finite volume formula to relate measured energies to phase shifts $\delta(k)$



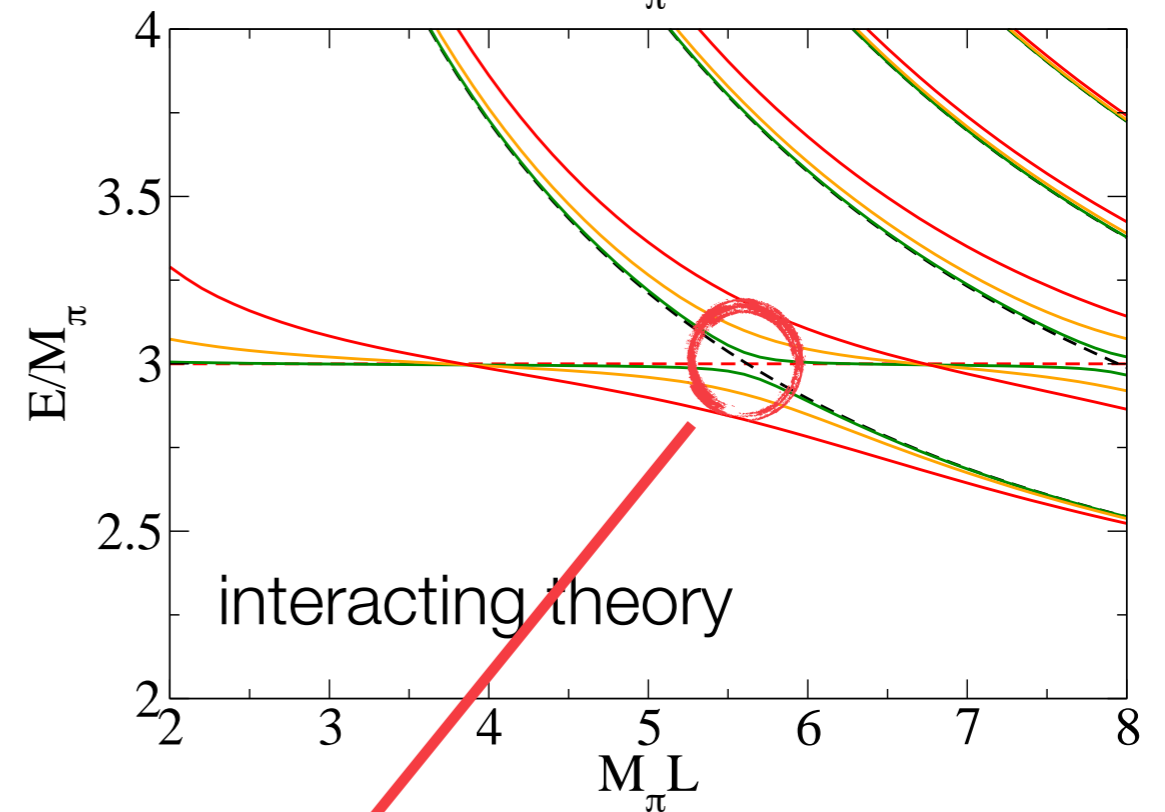
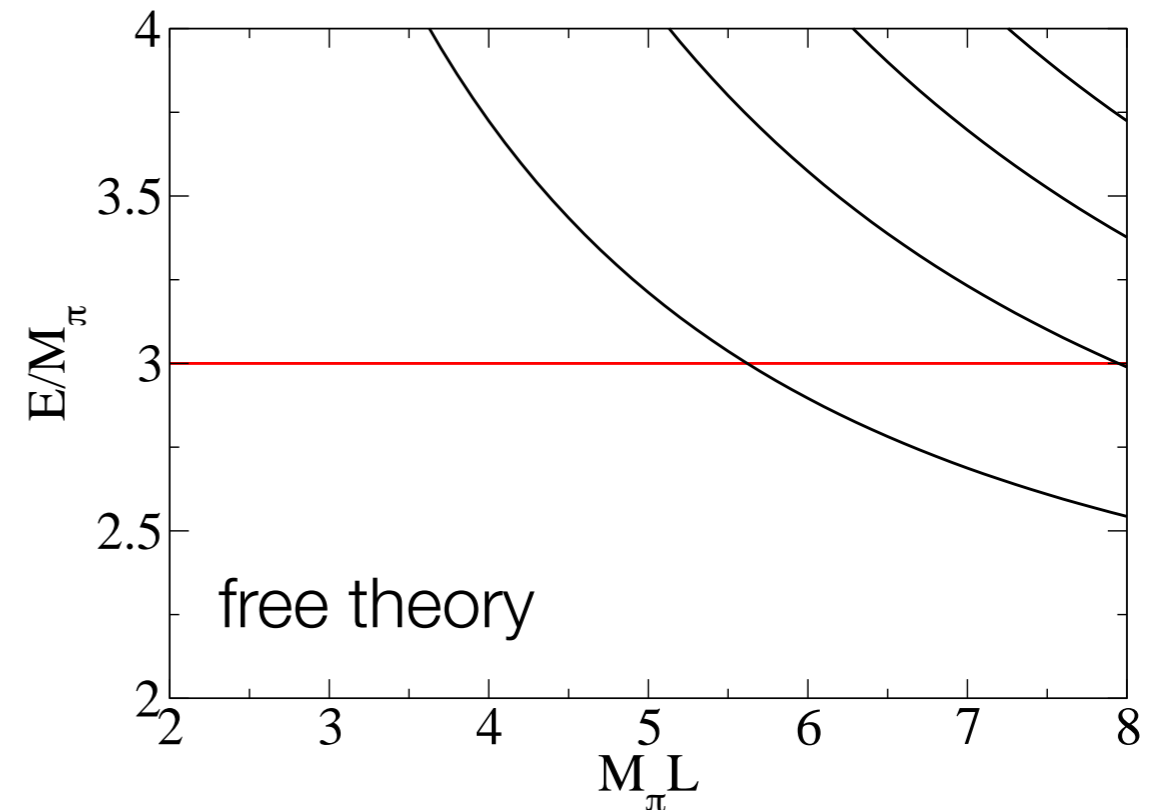
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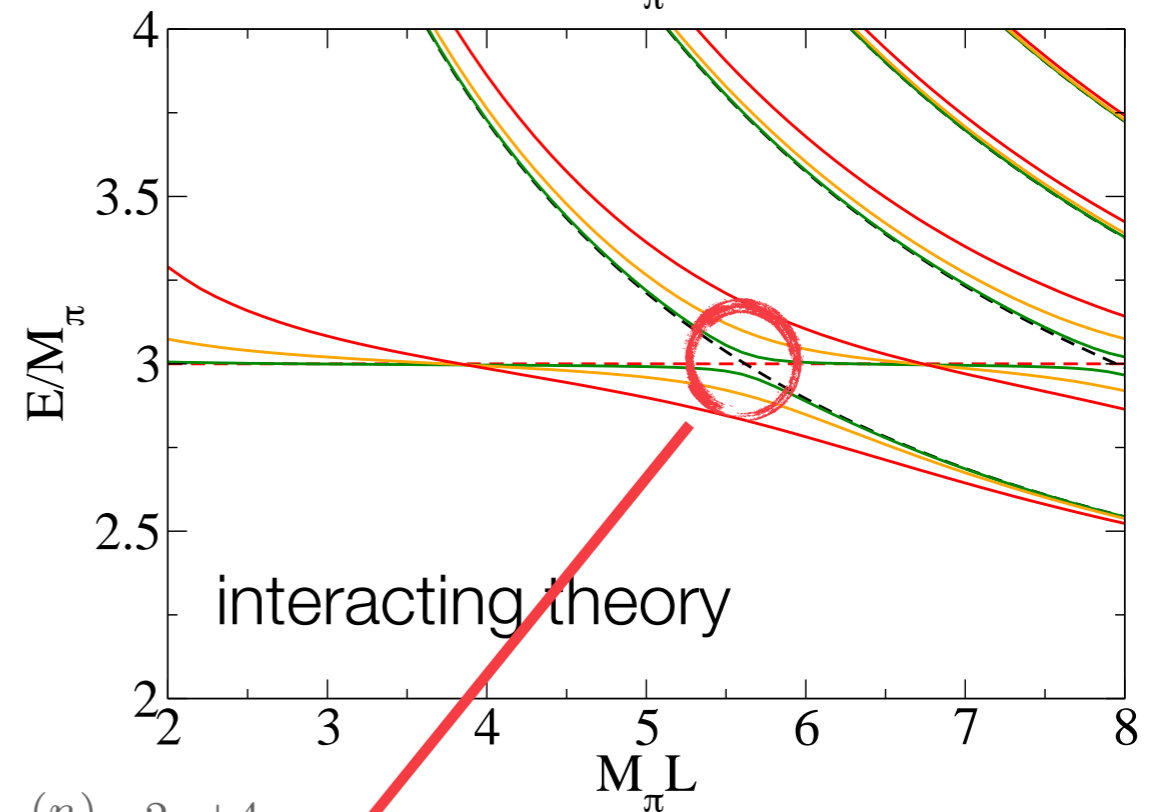
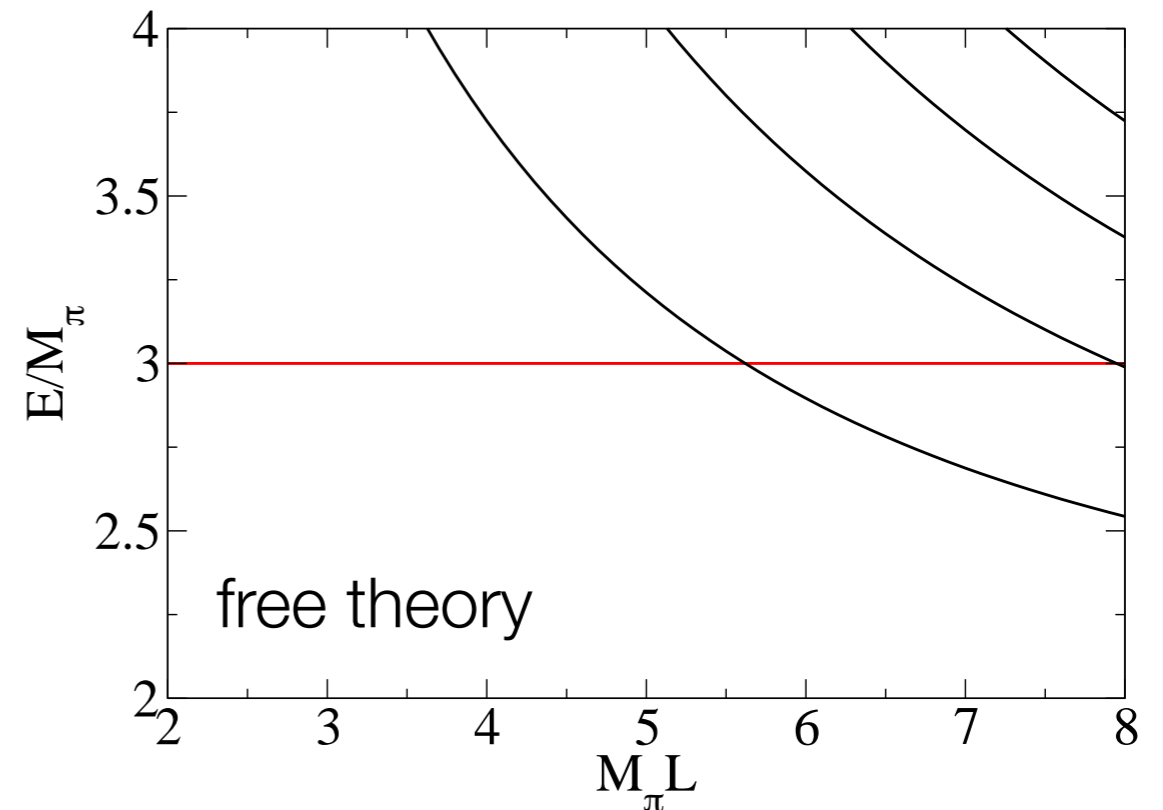


$$\Delta E \rightarrow k^2 \xrightarrow{\text{LFV}} \delta(k^2)$$

Nuclear Physics from Bottom Up

- compute multi-nucleon correlation functions
- measure energy difference between interacting and non-interacting system
- use Lüscher's finite volume formula to relate measured energies to phase shifts $\delta(k)$
- compute low energy observables from effective range expansion

$$k^{2l+1} \cot \delta_l(k^2) = -\frac{1}{a_l} + \frac{1}{2} r_l k^2 + \sum_{n=0}^{\infty} (-)^{n+1} P_l^{(n)} k^{2n+4}$$



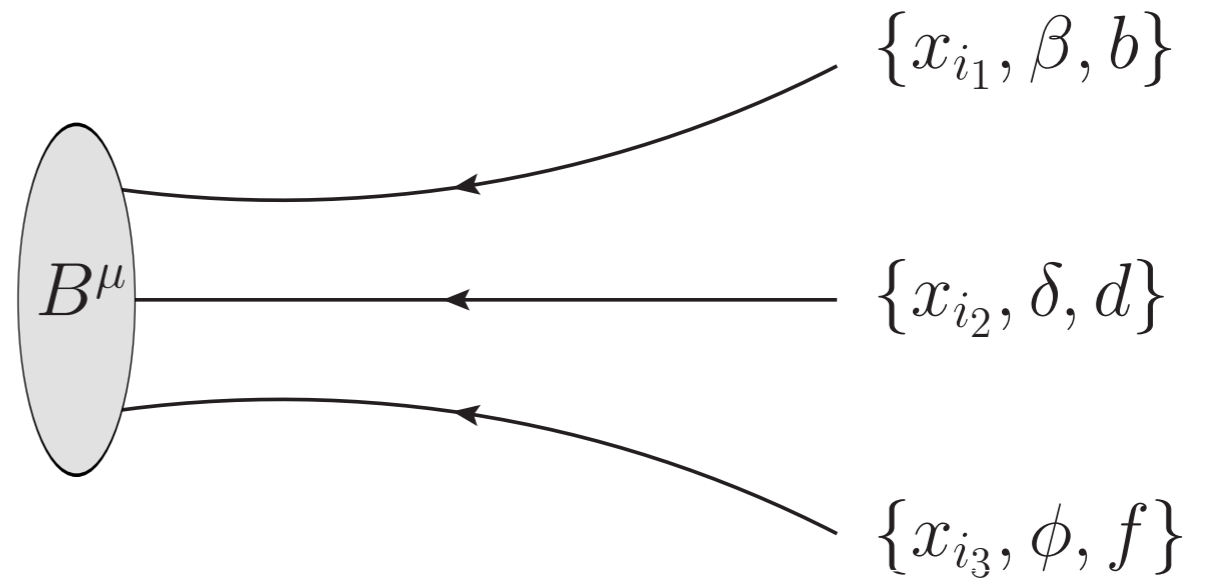
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Challenges

- number of contractions/
diagrams grows
combinatorially

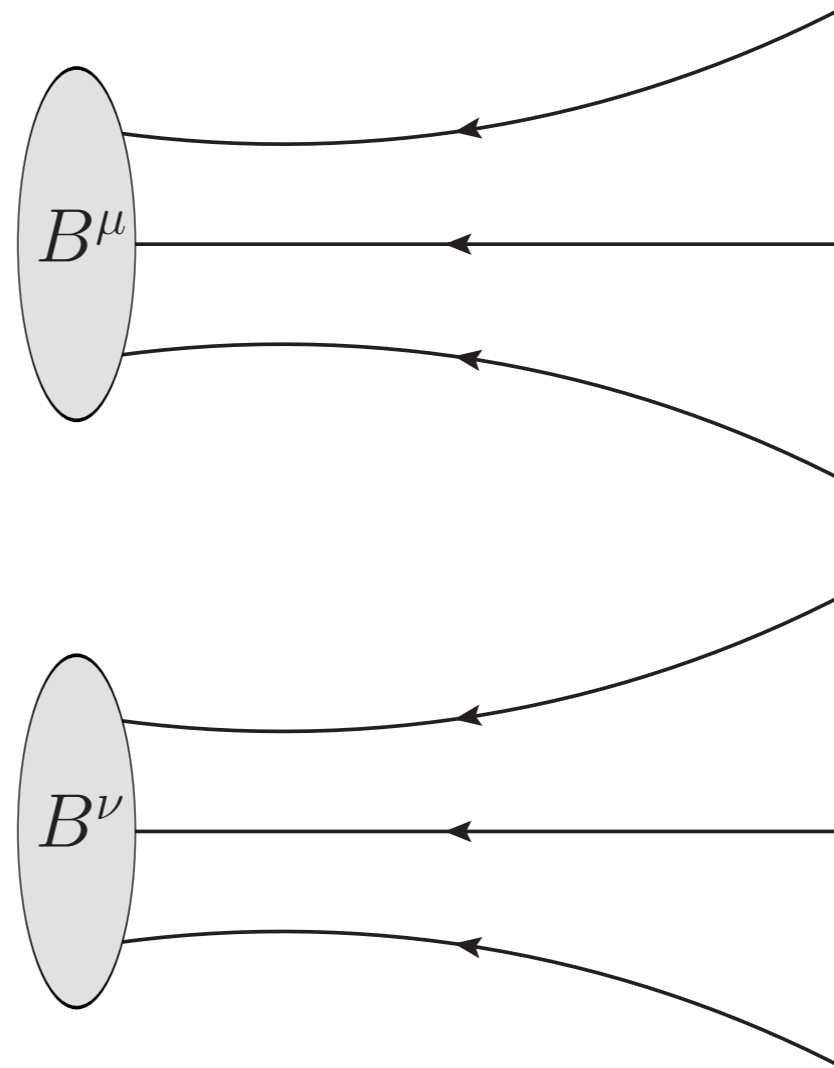
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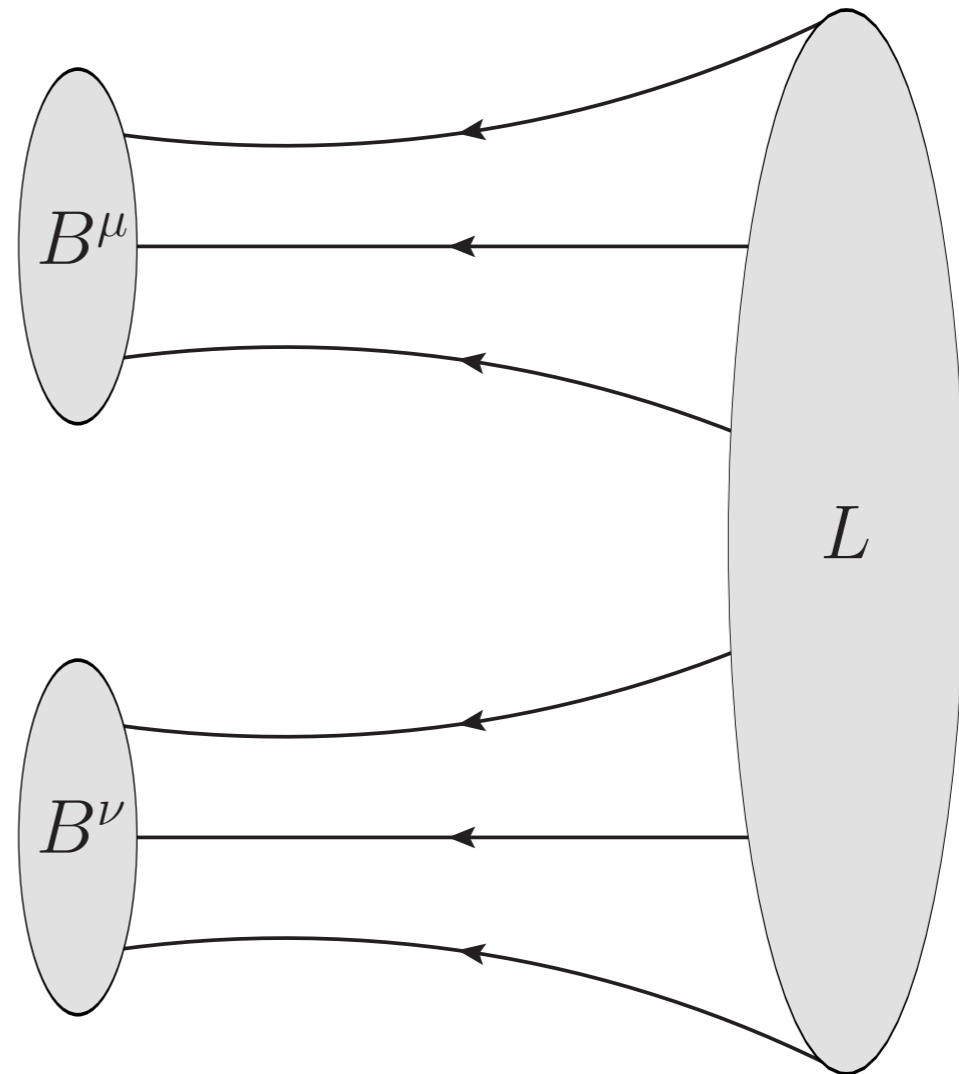
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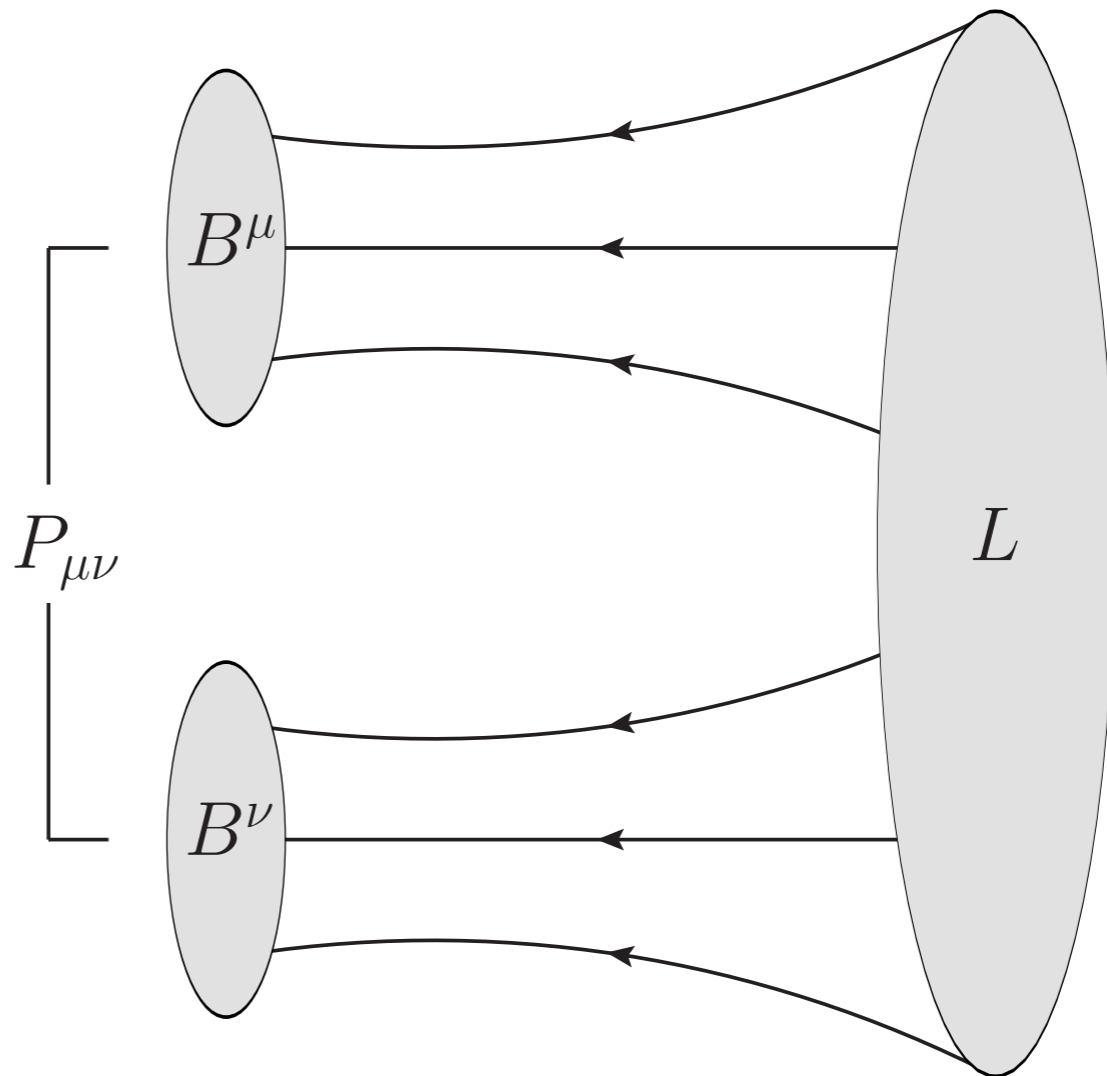
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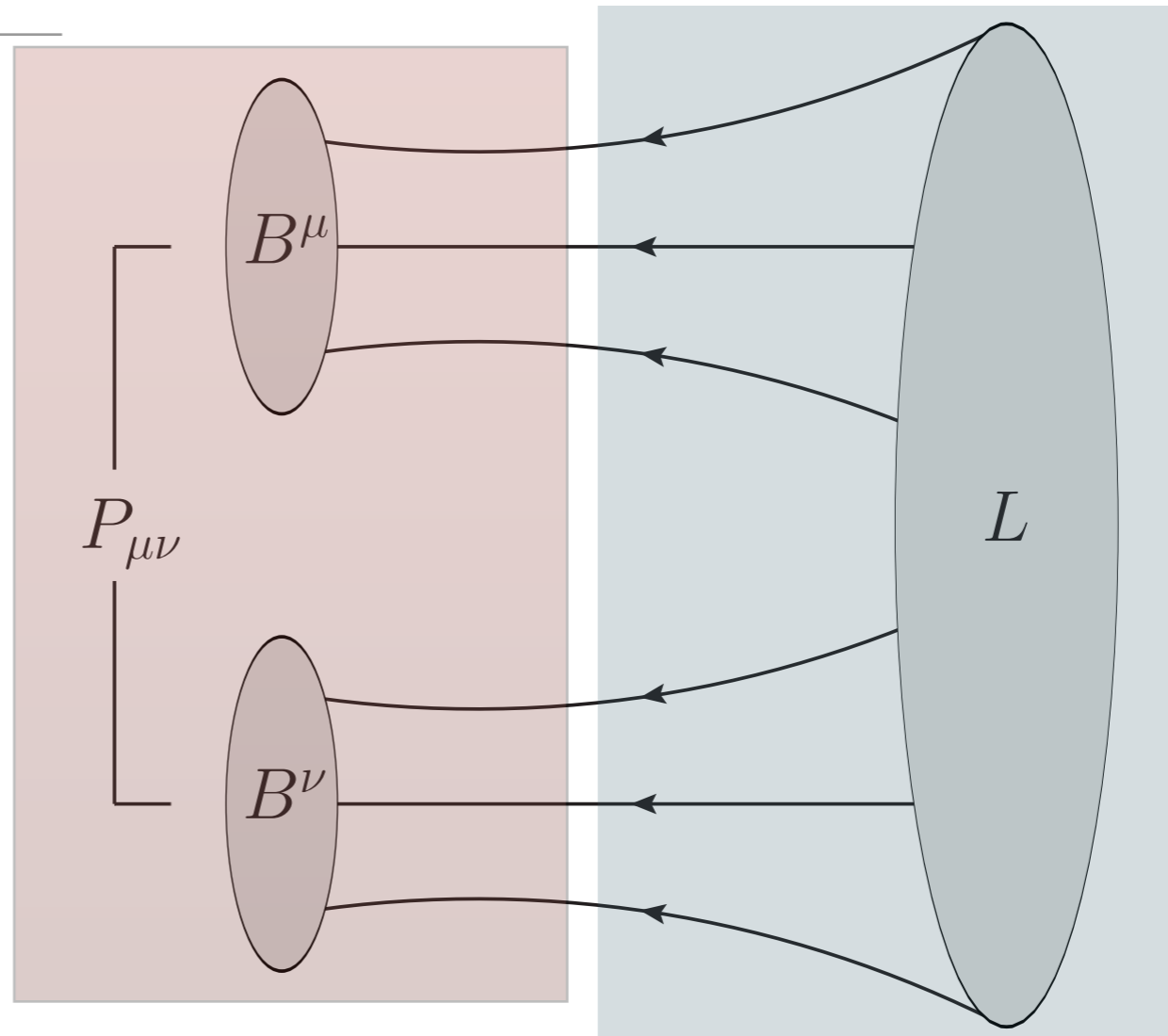
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Challenges

- number of contractions/
diagrams grows
combinatorially
- Baryon blocks



recompute for every
new measurement

compute once,
store to disk

Challenges

- number of contractions/
diagrams grows
combinatorially
- Baryon blocks
- use automatic code
generation

```
pp = Proton[sink][xf, μ] ** bar[Proton[source][xi, ν]] ** SpinProjector[Same, Spin[ν, μ]];
```

```
contractions = Contract[pp];
```

```
Notation[contractions]
```

```
Sameνμ Cγ5ν' ρ' Cγ5ν ρ S[down, xf - xi]ν'ν''b'b'' S[up, xf - xi]μρ''a'c'' S[up, xf - xi]ρ'νc'a'' εa''b''c'' εa'b'c' -
```

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```

```
Generate[BaryonBlocks][contractions] // Notation
```

```
-BaryonBlock[Color[b, d, f], Flavor[down, up, up], SpaceTime[xf, {xi, xi, xi}], Spin[ε, β, δ, φ]]  
(Sameφε Cγ5β δ + Sameδε Cγ5β φ) εbdf
```

Challenges

- number of contractions/
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- Baryon blocks
- use automatic code
generation
- calculating the contractions is
expensive: use GPUs

$$\sum_{\mu, \nu, A, B} P_{\mu\nu} B_A^\mu B_B^\nu L_{AB} = \mathbf{v}^T M \mathbf{v}$$

vector-Matrix-vector product

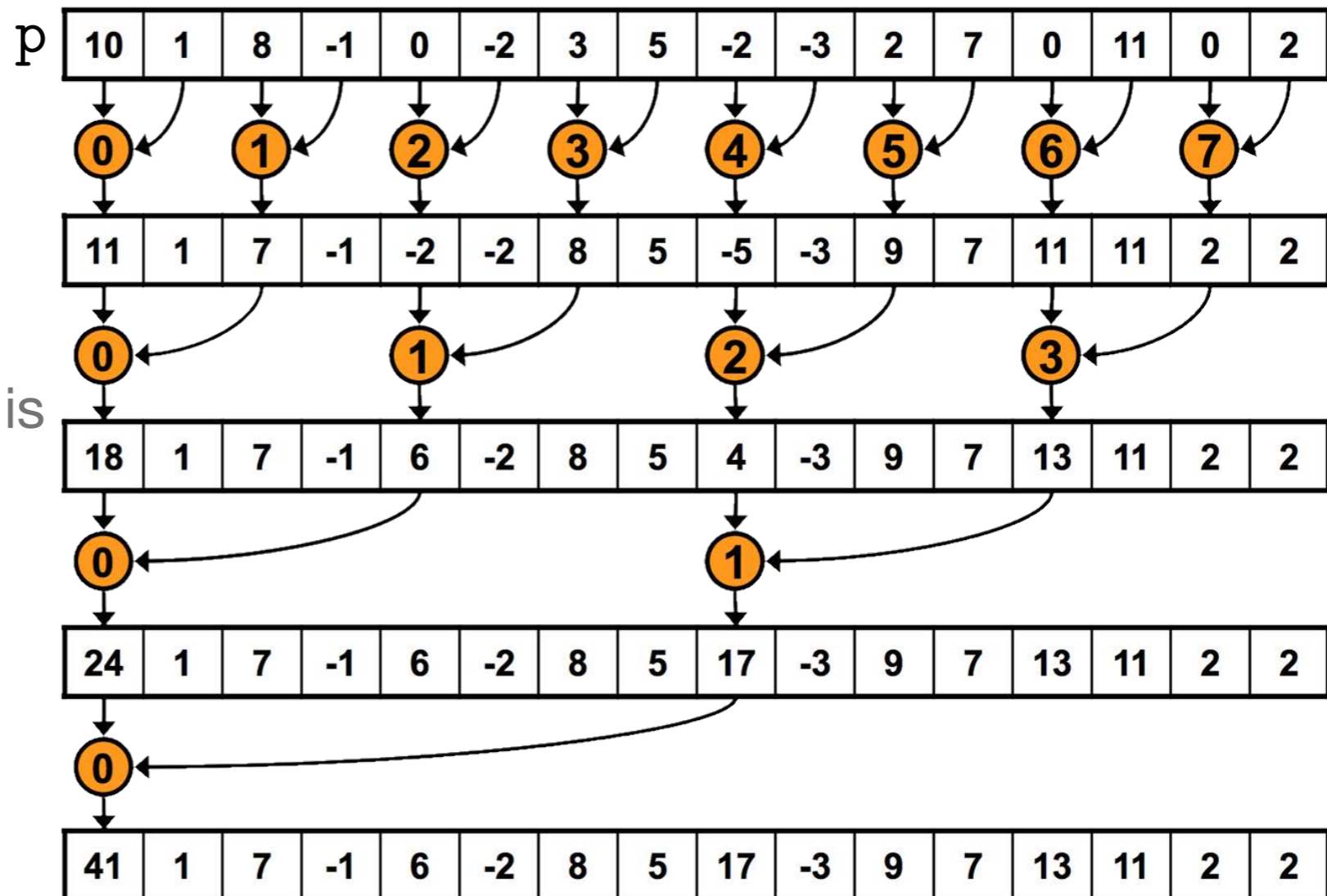
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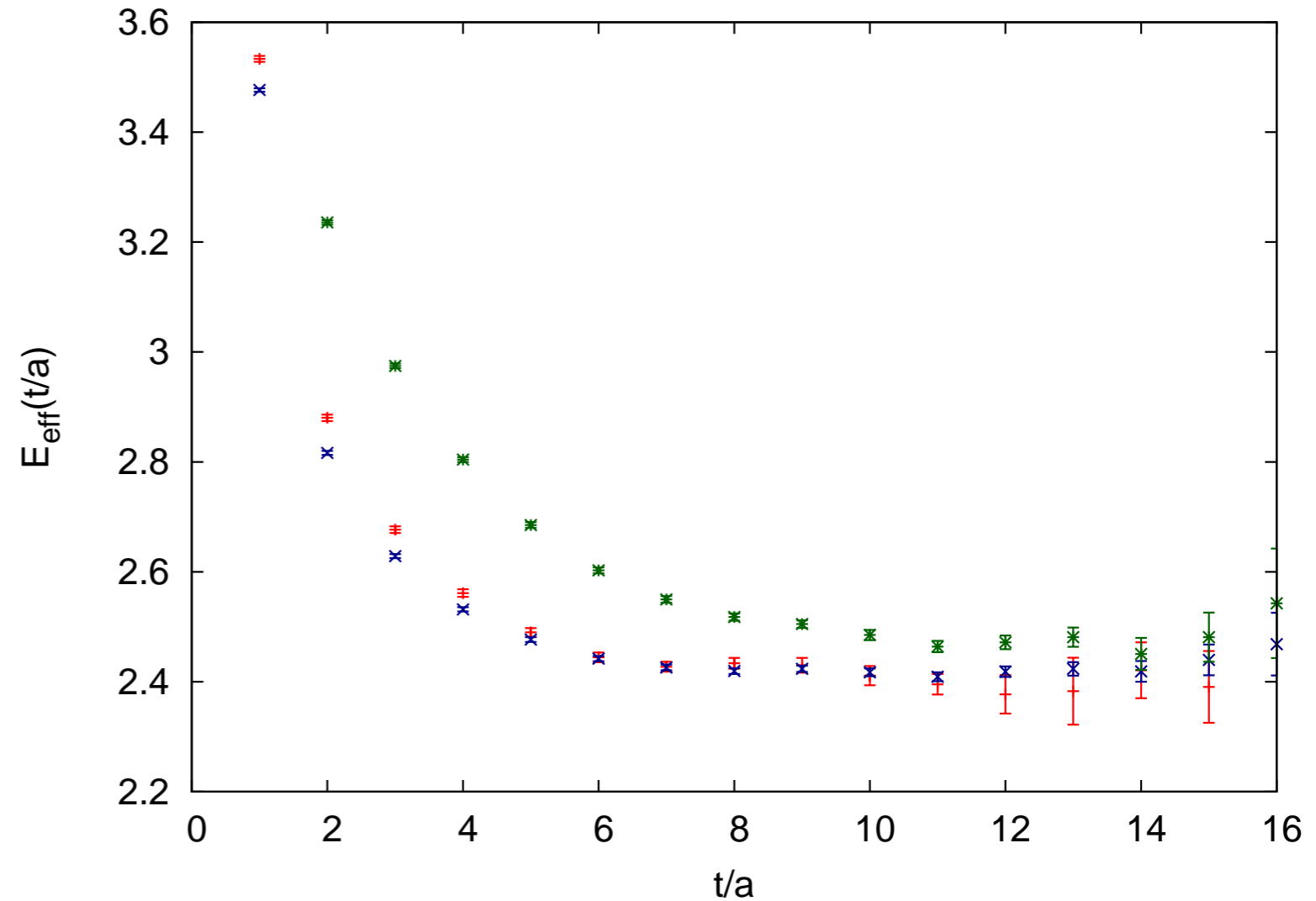
`p[tid] = v[I1[tid]] * M[tid] * v[I2[tid]];`



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- excited states contributions:
sink/source engineering

$$C(t, t_0) = \sum_{n=0}^{\infty} \tilde{Z}_n^* Z_n e^{-m_n(t-t_0)}$$

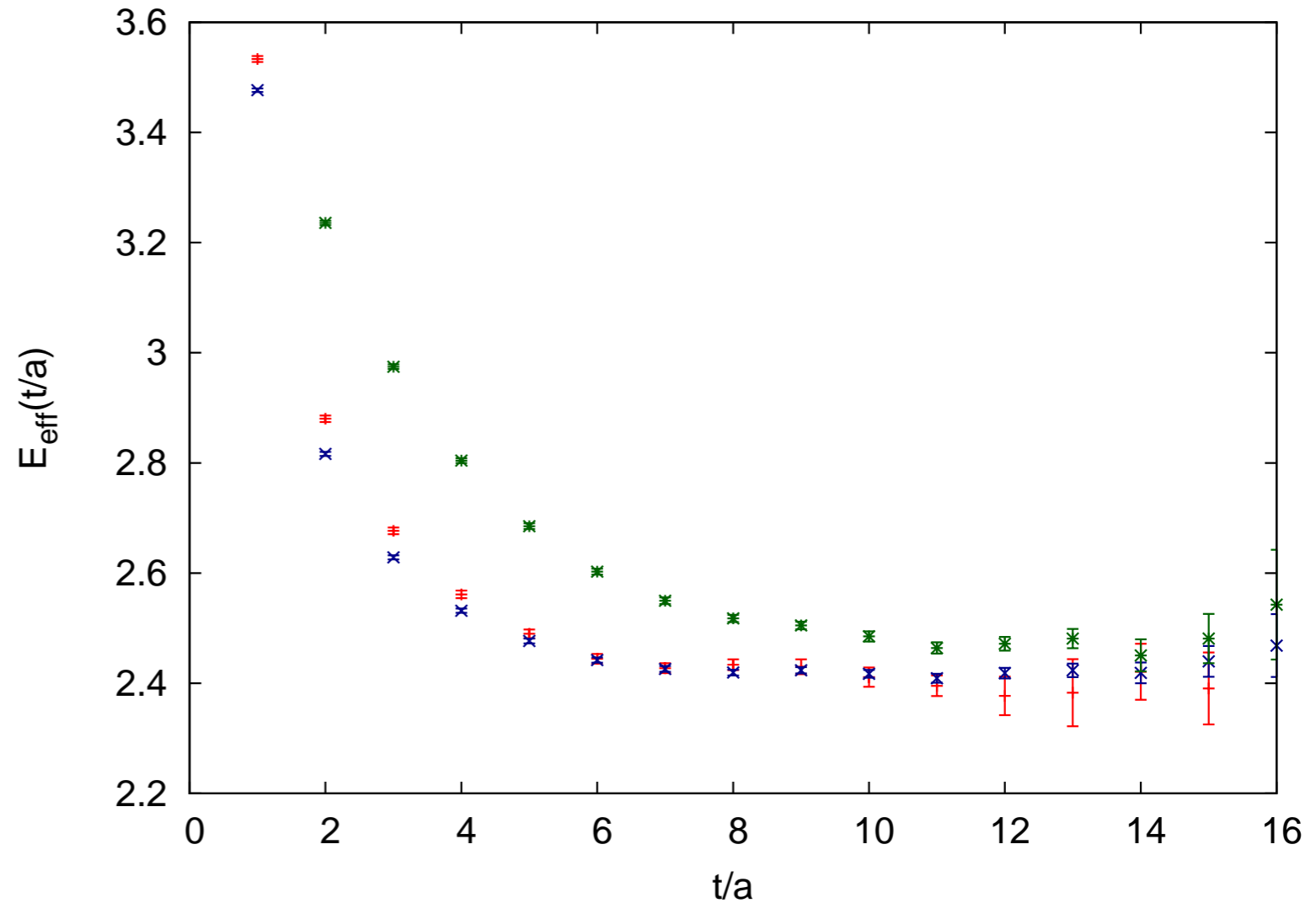


use sources/sinks with
spatially displaced nuclei

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- Lepage argument

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use sources/sinks with
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Lepage Argument

- signal-to-noise ratio of correlation functions

$$\text{SNR} \sim \frac{\langle N \bar{N} \rangle}{\sqrt{\langle (N \bar{N})(N \bar{N})^\dagger \rangle - \langle N \bar{N} \rangle^2}}$$

Lepage Argument

- signal-to-noise ratio of correlation functions
- numerator
 $\sim \exp(-m_N t)$

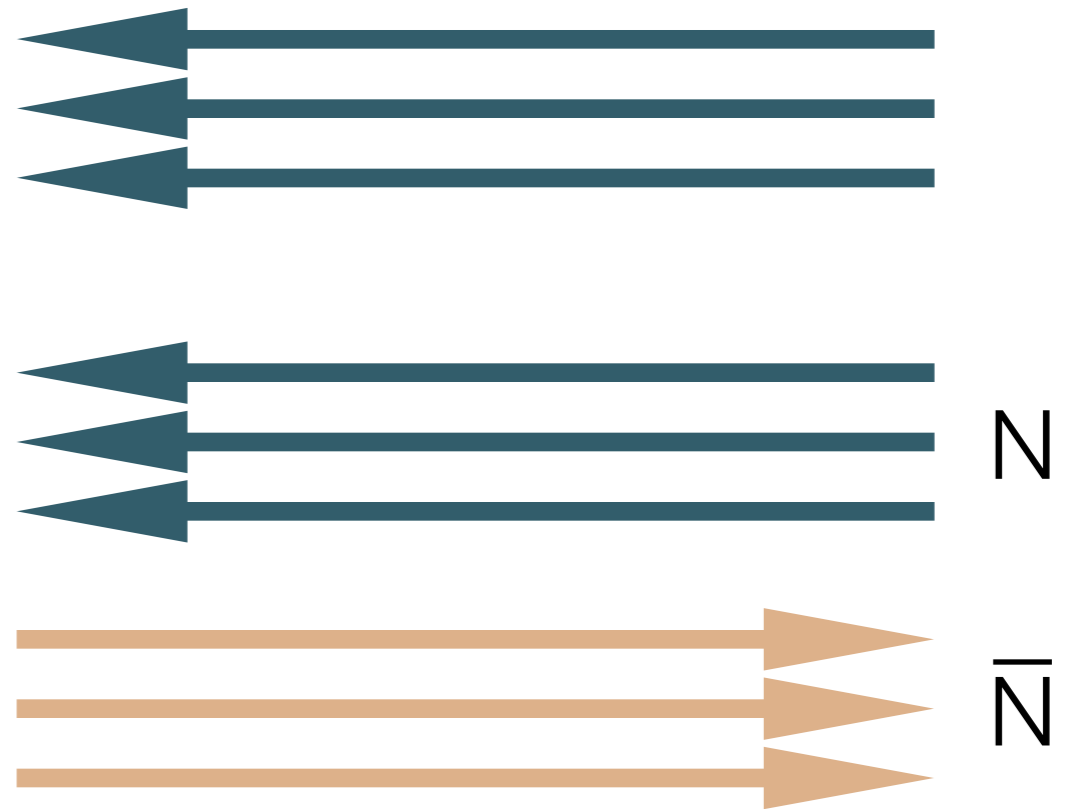
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Lepage Argument

- signal-to-noise ratio of correlation functions
- numerator
 $\sim \exp(-m_N t)$
- denominator

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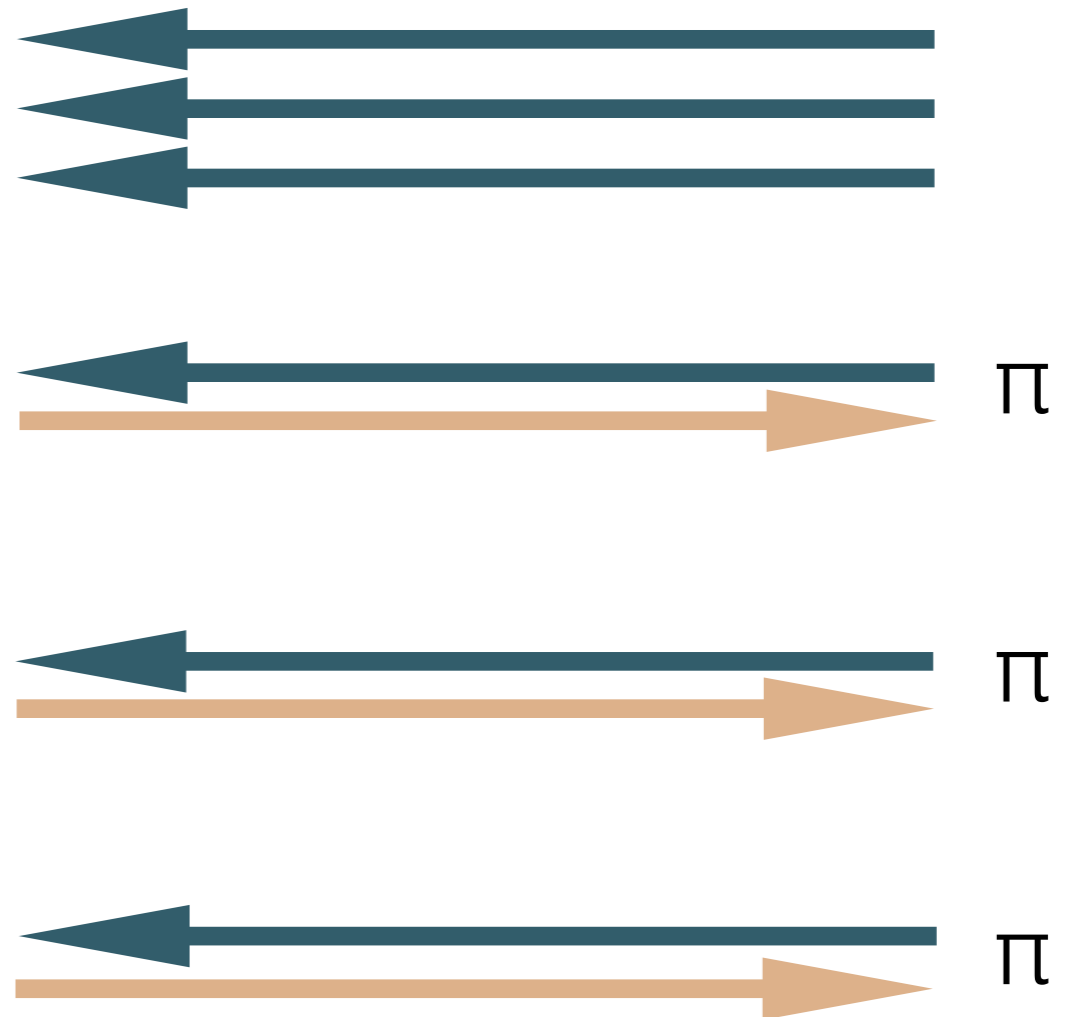
Lepage Argument

- signal-to-noise ratio of correlation functions

- numerator
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- denominator
 $\sim \exp\left(-\frac{3}{2}m_\pi t\right)$

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Lepage Argument

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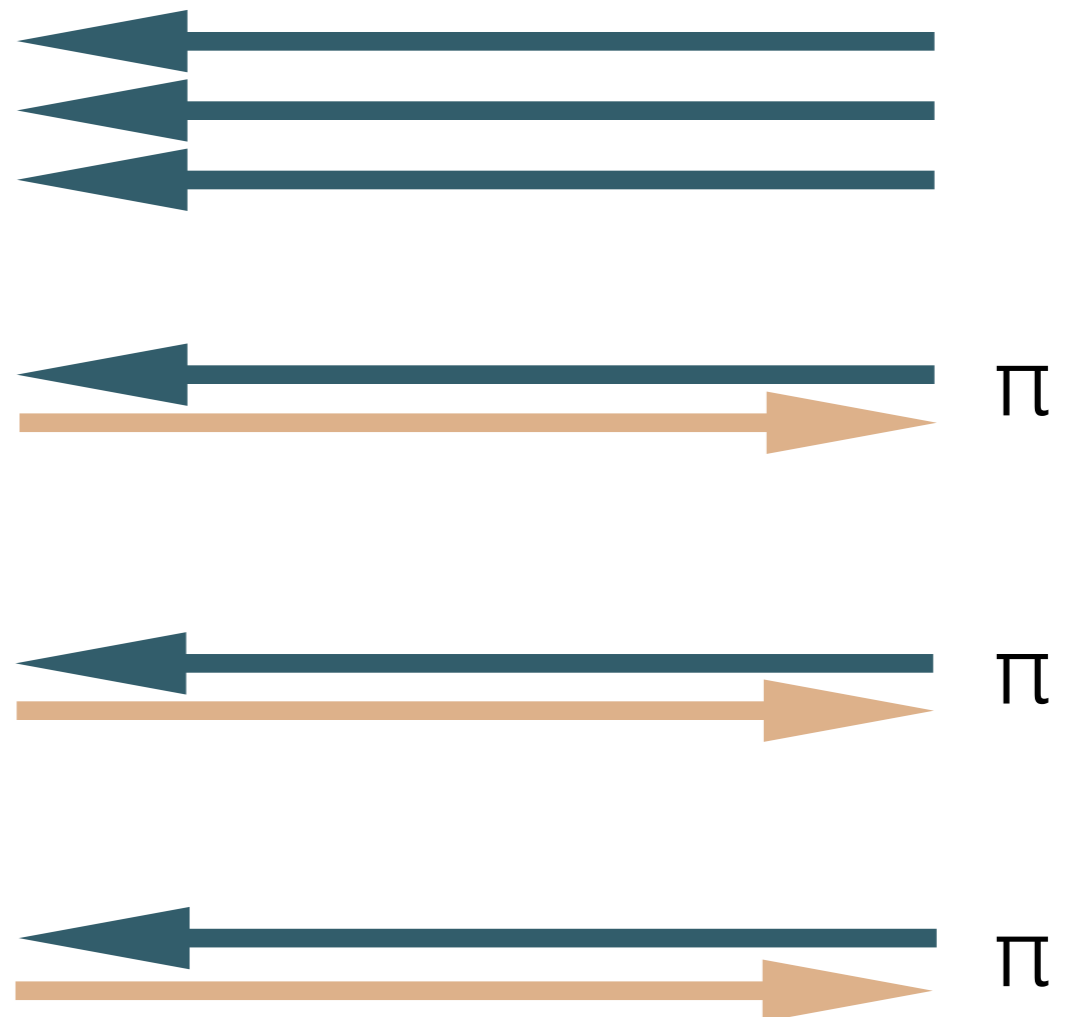
- numerator
 $\sim \exp(-m_N t)$

- denominator
 $\sim \exp\left(-\frac{3}{2}m_\pi t\right)$

- time-dependence of SNR

$$\sim \sqrt{N} \exp\left[-A \left(m_N - \frac{3}{2}m_\pi\right) t\right]$$

$$\text{SNR} \sim \frac{\langle N \bar{N} \rangle}{\sqrt{\langle (N \bar{N})(N \bar{N})^\dagger \rangle - \langle N \bar{N} \rangle^2}}$$



huge statistics needed (MG)
 huge amount of storage needed (HDF5)
 large pion mass

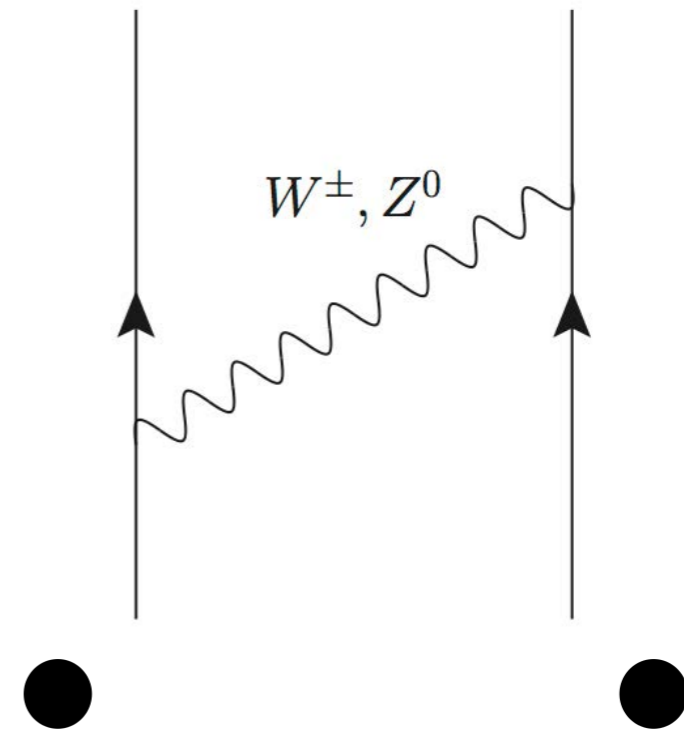
Parity Violation - Contractions $\Delta I=2$

- 2 Baryon s-wave source



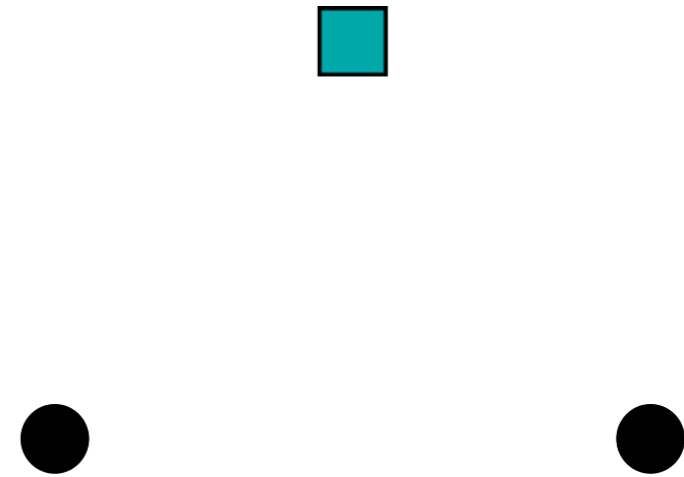
Parity Violation - Contractions $\Delta I=2$

- 2 Baryon s-wave source
- EW vertices \Rightarrow 4-quark operator insertion



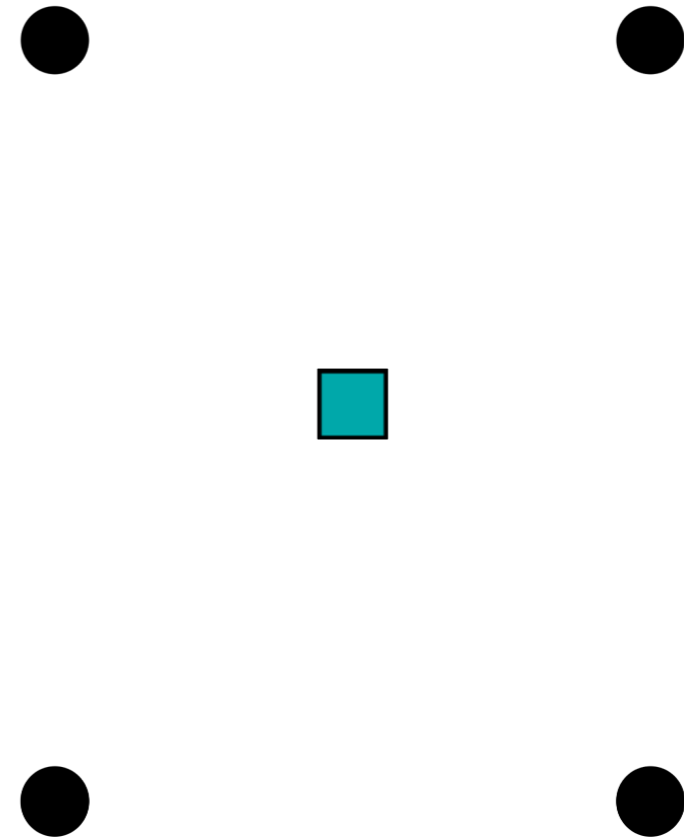
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Parity Violation - Contractions $\Delta I=2$

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- 2 Baryon p-wave sink



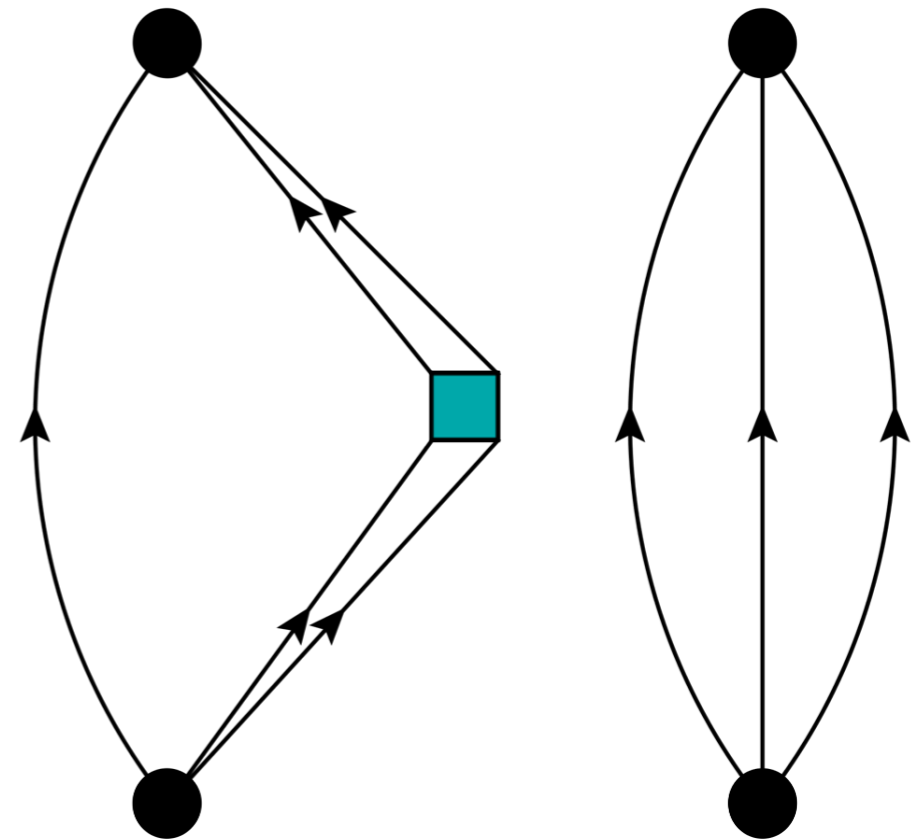
Parity Violation - Contractions $\Delta I=2$

- 2 Baryon s-wave source
- EW vertices \Rightarrow 4-quark operator insertion
- 2 Baryon p-wave sink
- In total there are 4896 contractions.
- Degenerate light quarks reduce this to 2208.
- Requires 15 tensors.



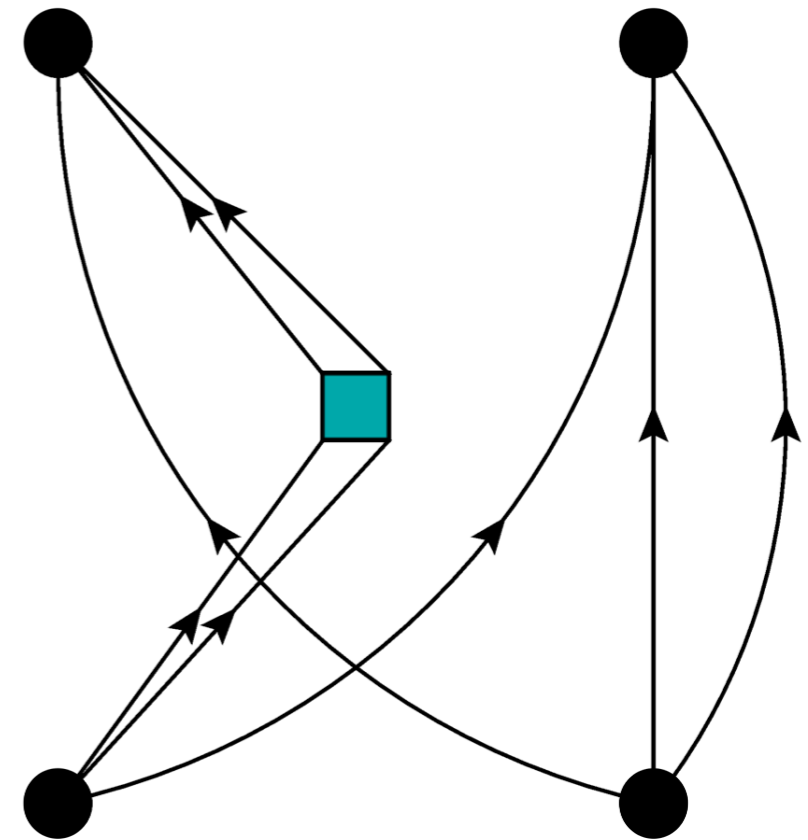
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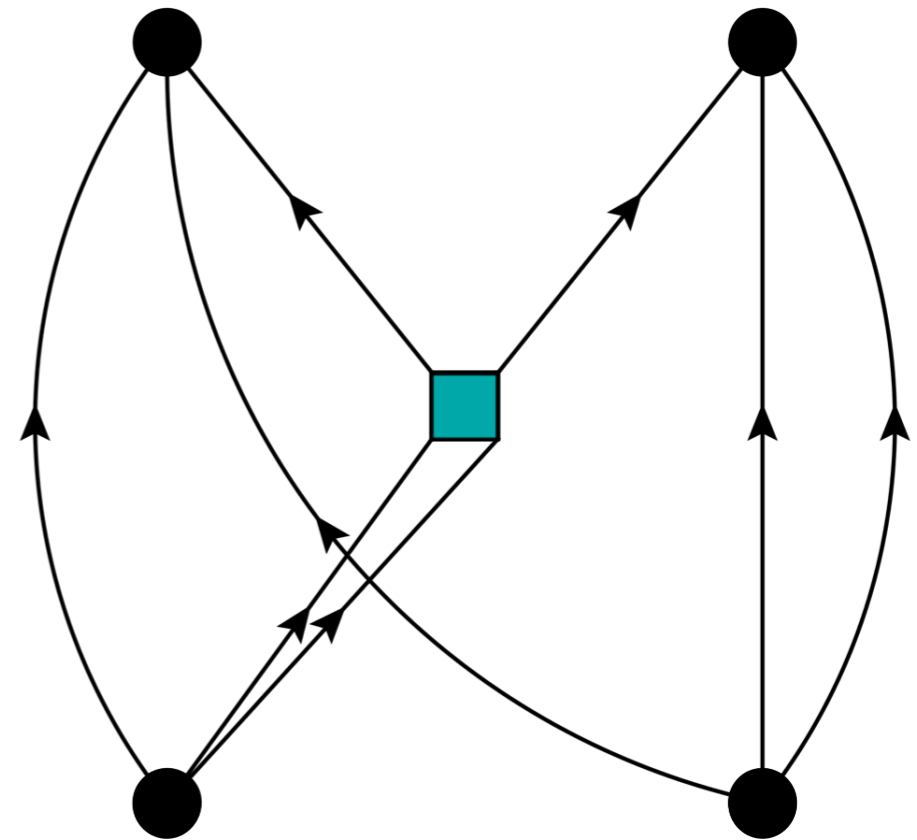
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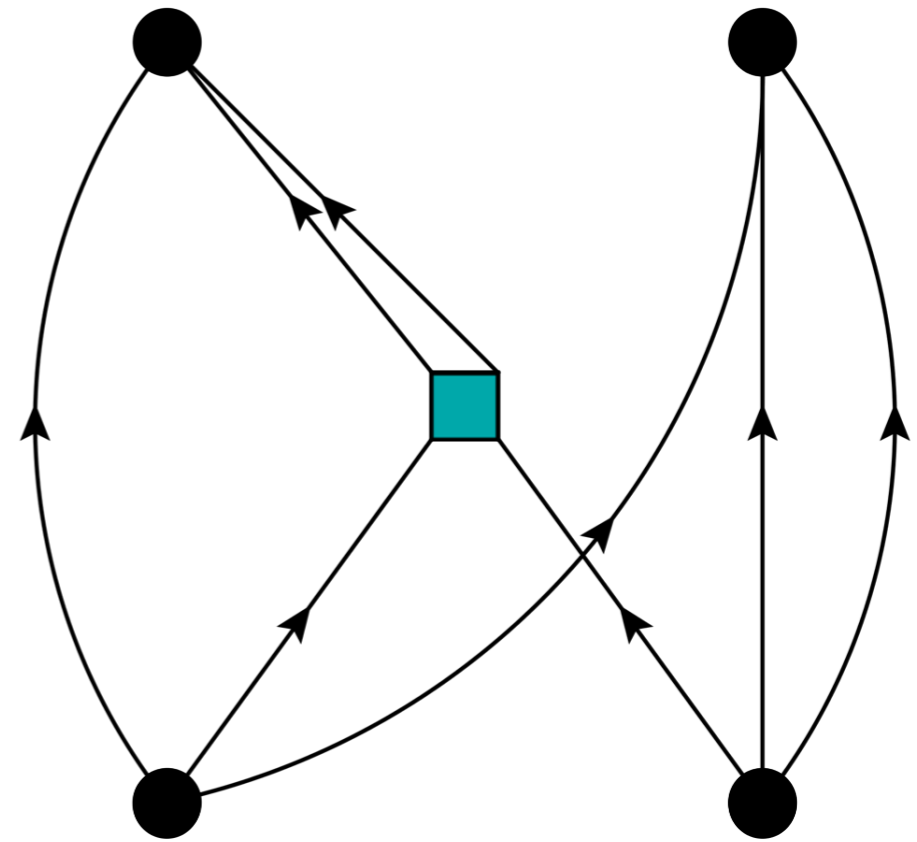
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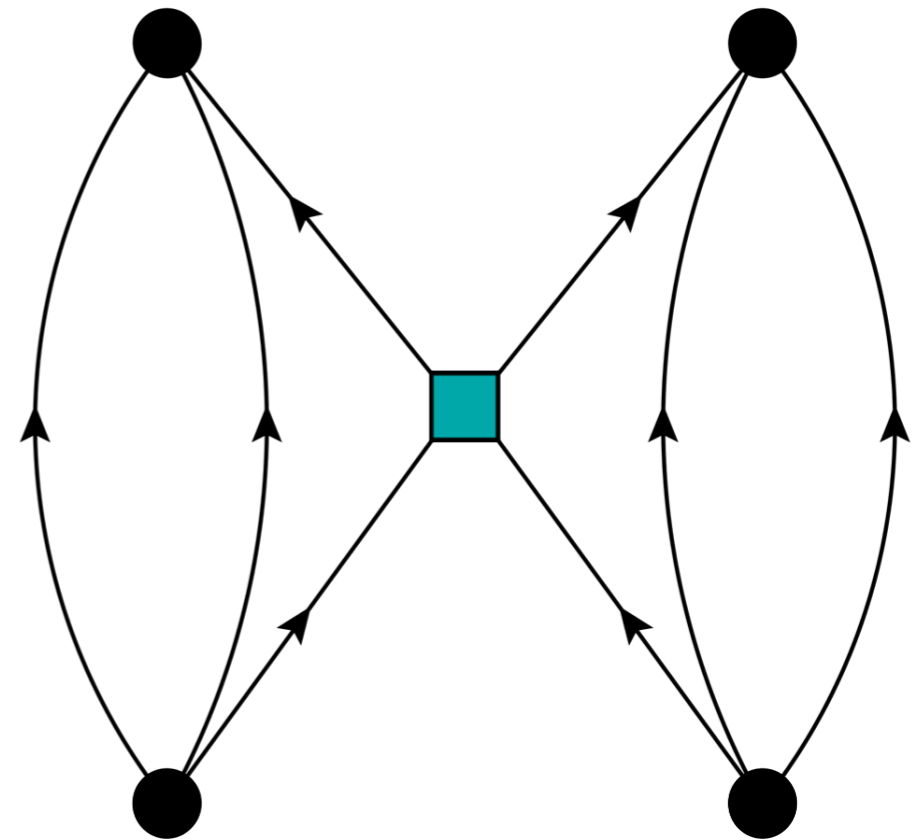
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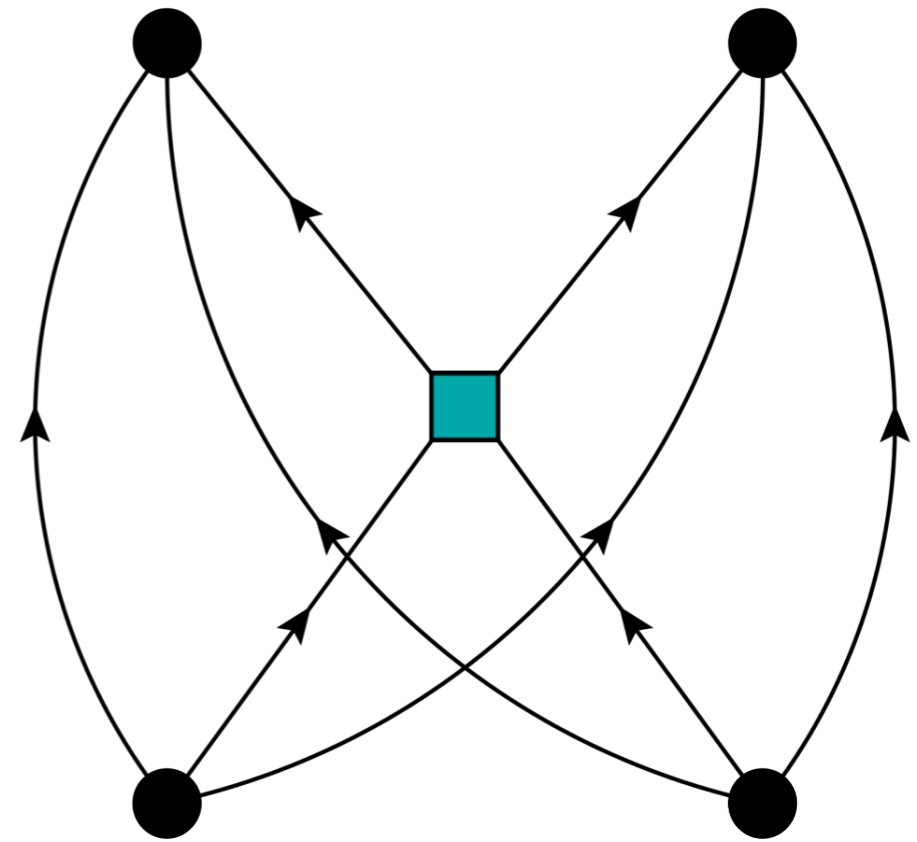
Parity Violation - Contractions $\Delta I=2$

- 2 Baryon s-wave source
- EW vertices \Rightarrow 4-quark operator insertion
- 2 Baryon p-wave sink
- In total there are 4896 contractions.
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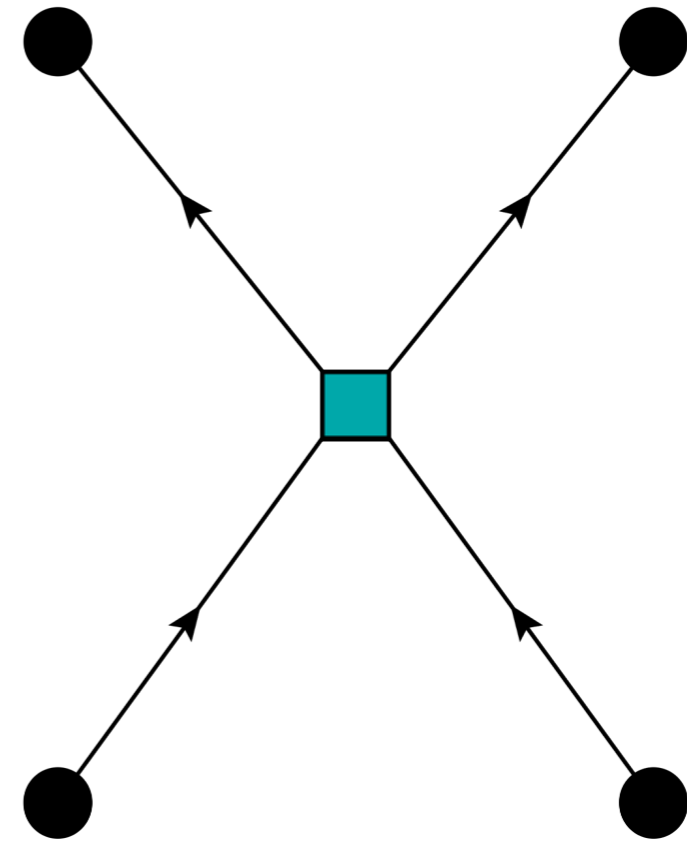
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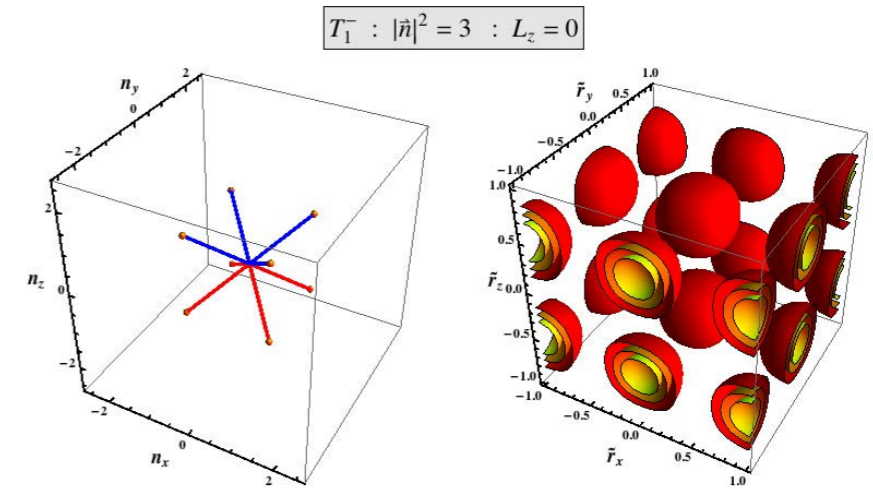
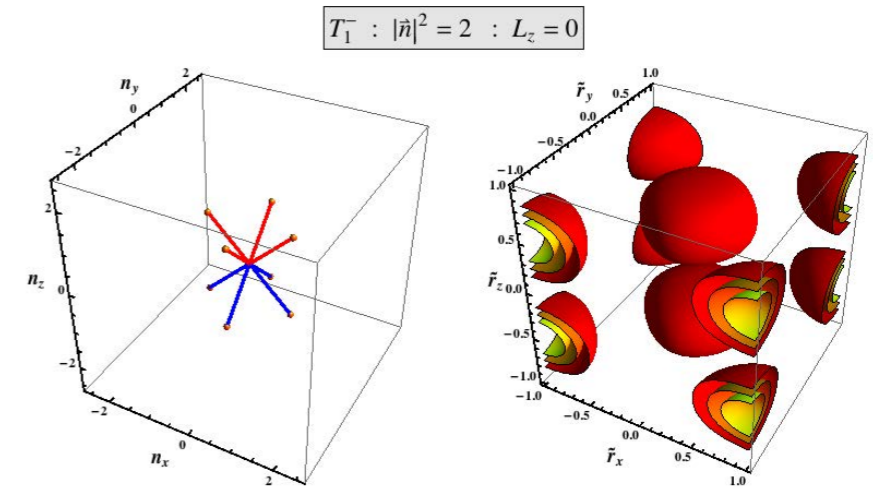
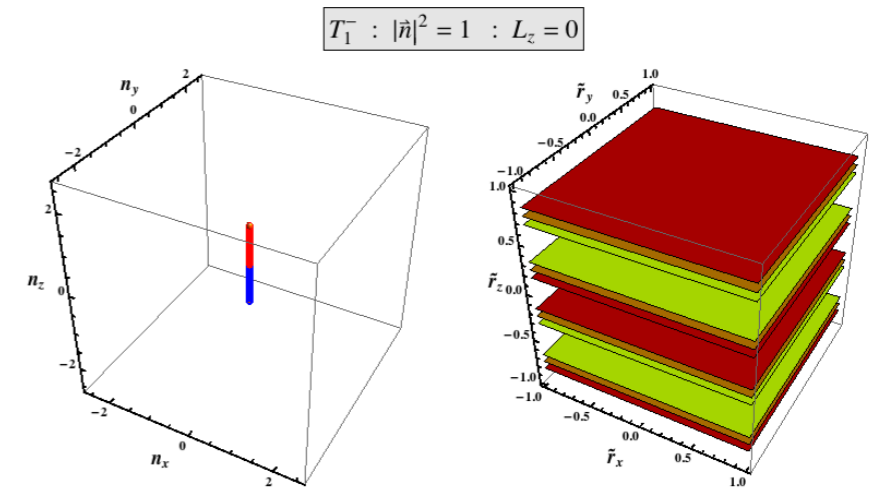
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- 2 baryon blocks & 1 four-quark object



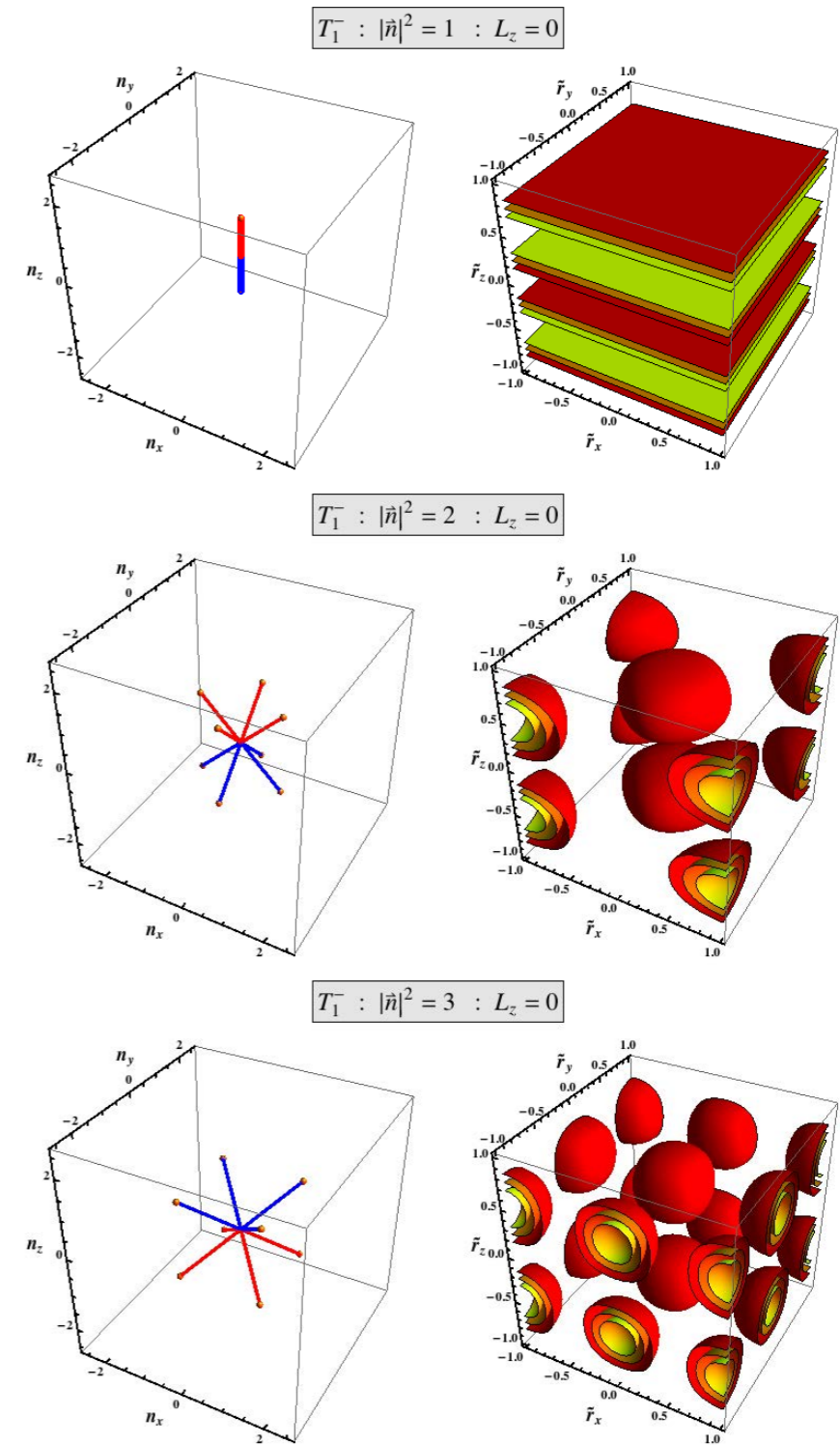
Source and Sink Construction

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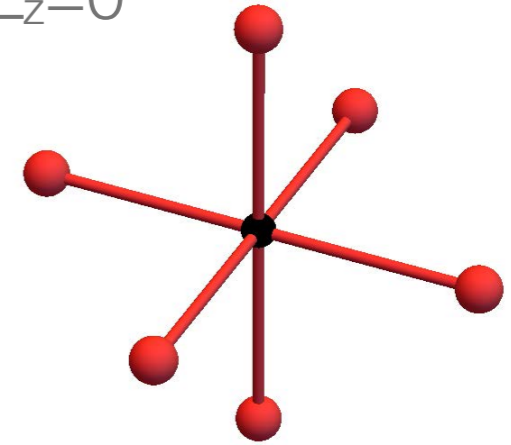
we wish to use:
$$\bar{N}(p) = \sum_x e^{ipx} \bar{q}_1(x) \bar{q}_2(x) \bar{q}_3(x)$$

we are limited to:
$$\bar{N}'(p; p_1, p_2) = \sum_{x_1, x_2, x_3} e^{ip_1 x_1} \bar{q}_1(x_1) e^{ip_2 x_2} \bar{q}_2(x_2) e^{i(p-p_1-p_2)x_3} \bar{q}_3(x_3)$$

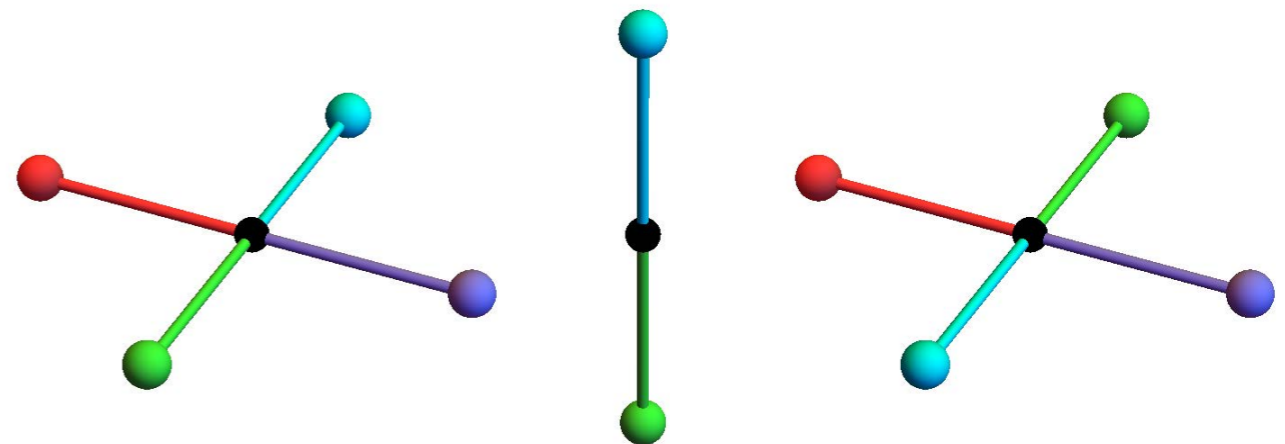
Source and Sink Construction

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- we use coordinate-space sources (and sinks) instead

$L=0, L_z=0$



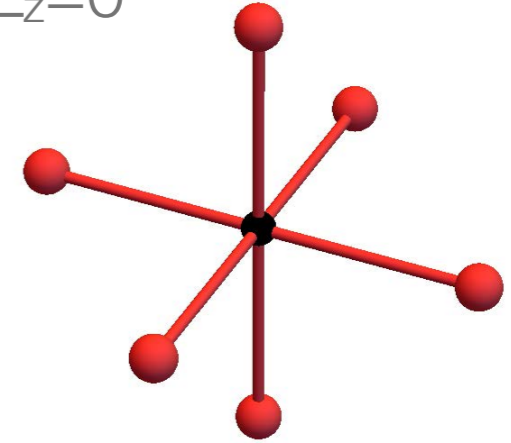
$L=1, L_z=+1, 0, -1$



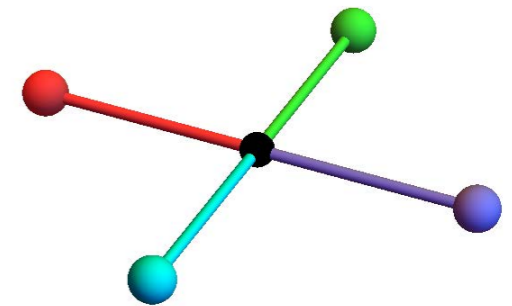
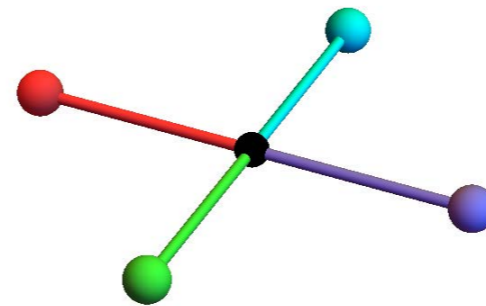
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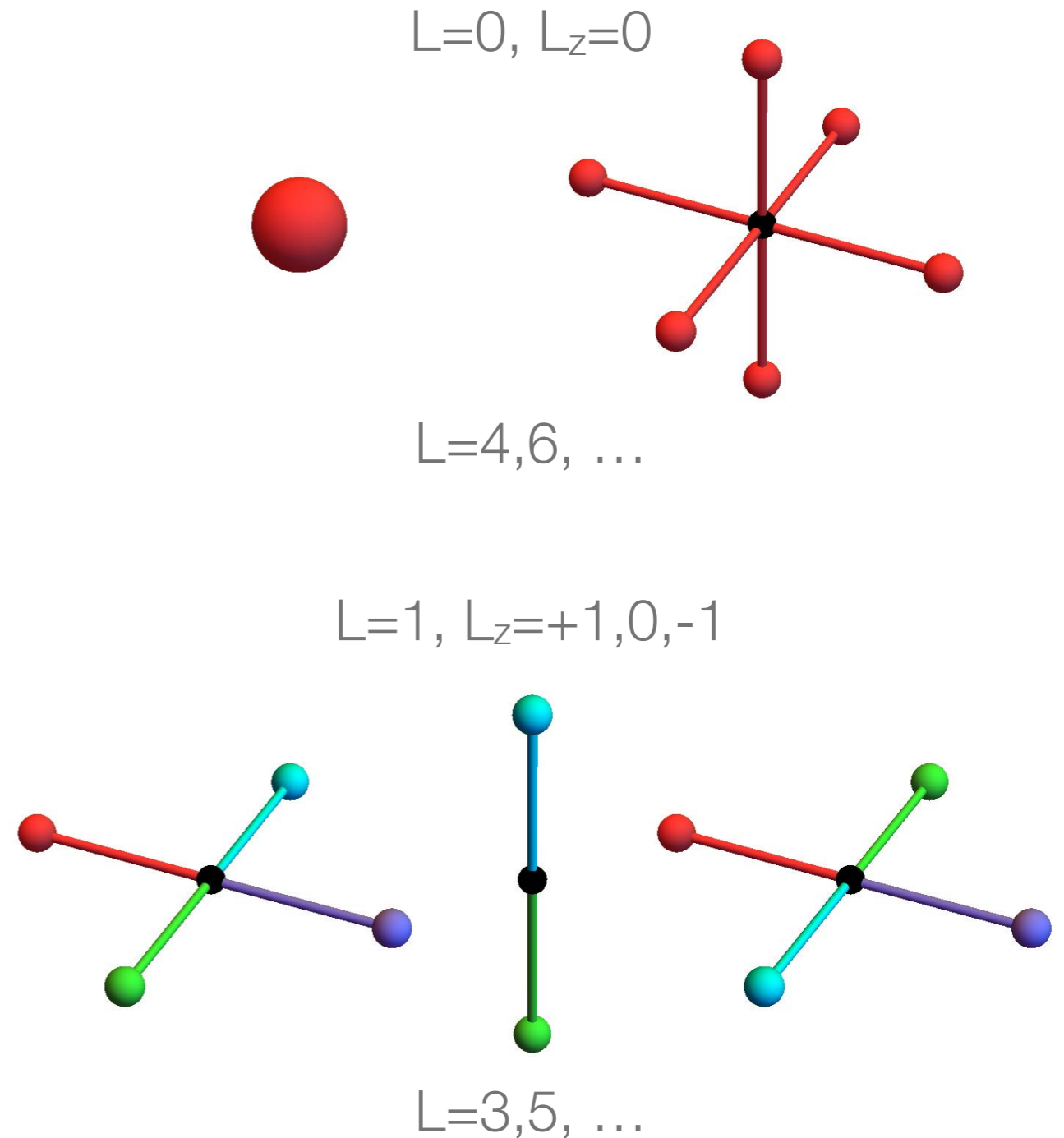


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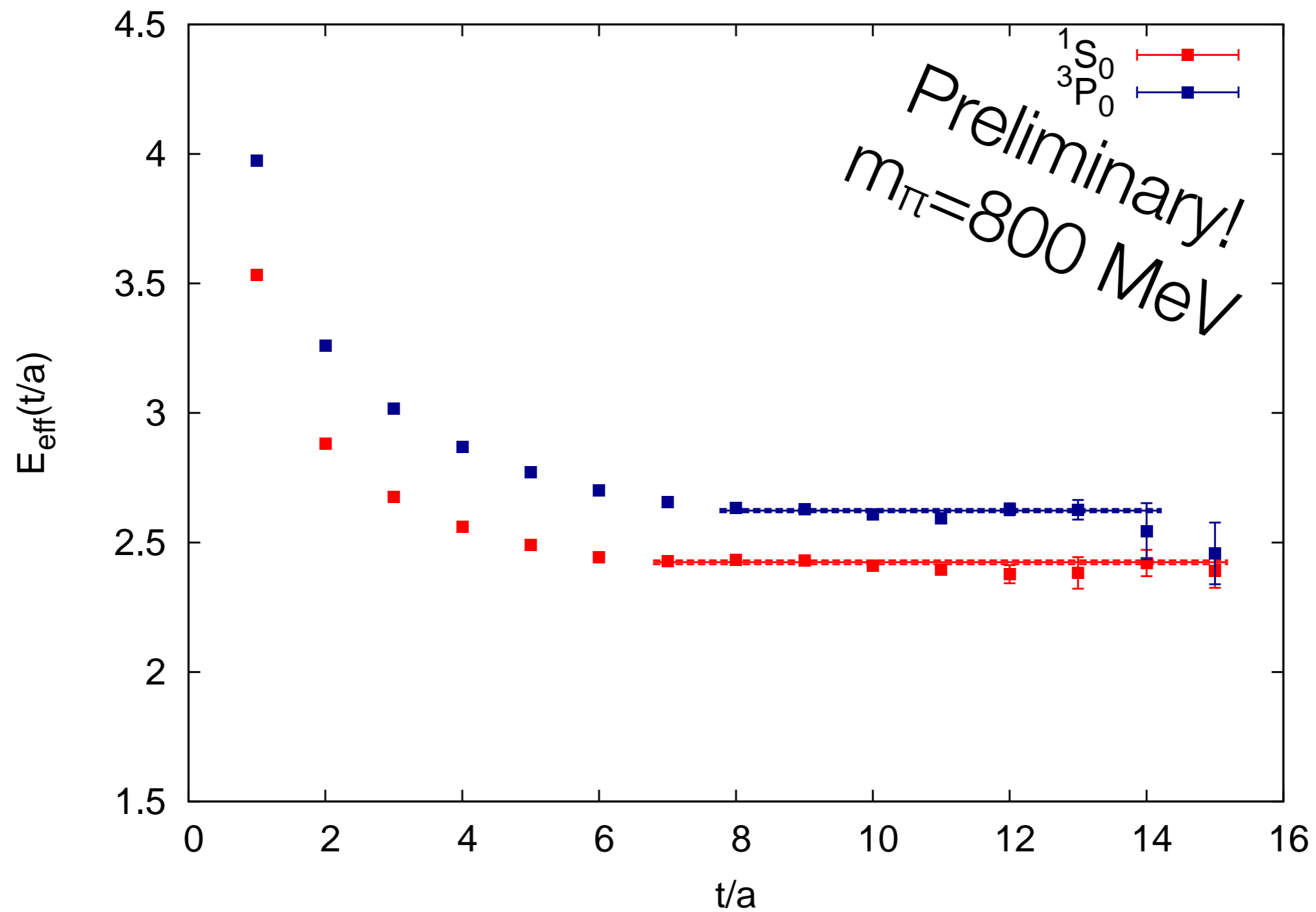
Source and Sink Construction

- use of momentum sources and sinks (non-local in coordinate space) would be optimal
- we use coordinate-space sources (and sinks) instead
- mix different lattice irreducible representations \Rightarrow excited states
- rotational symmetry breaking: mixing of different continuum multiplets



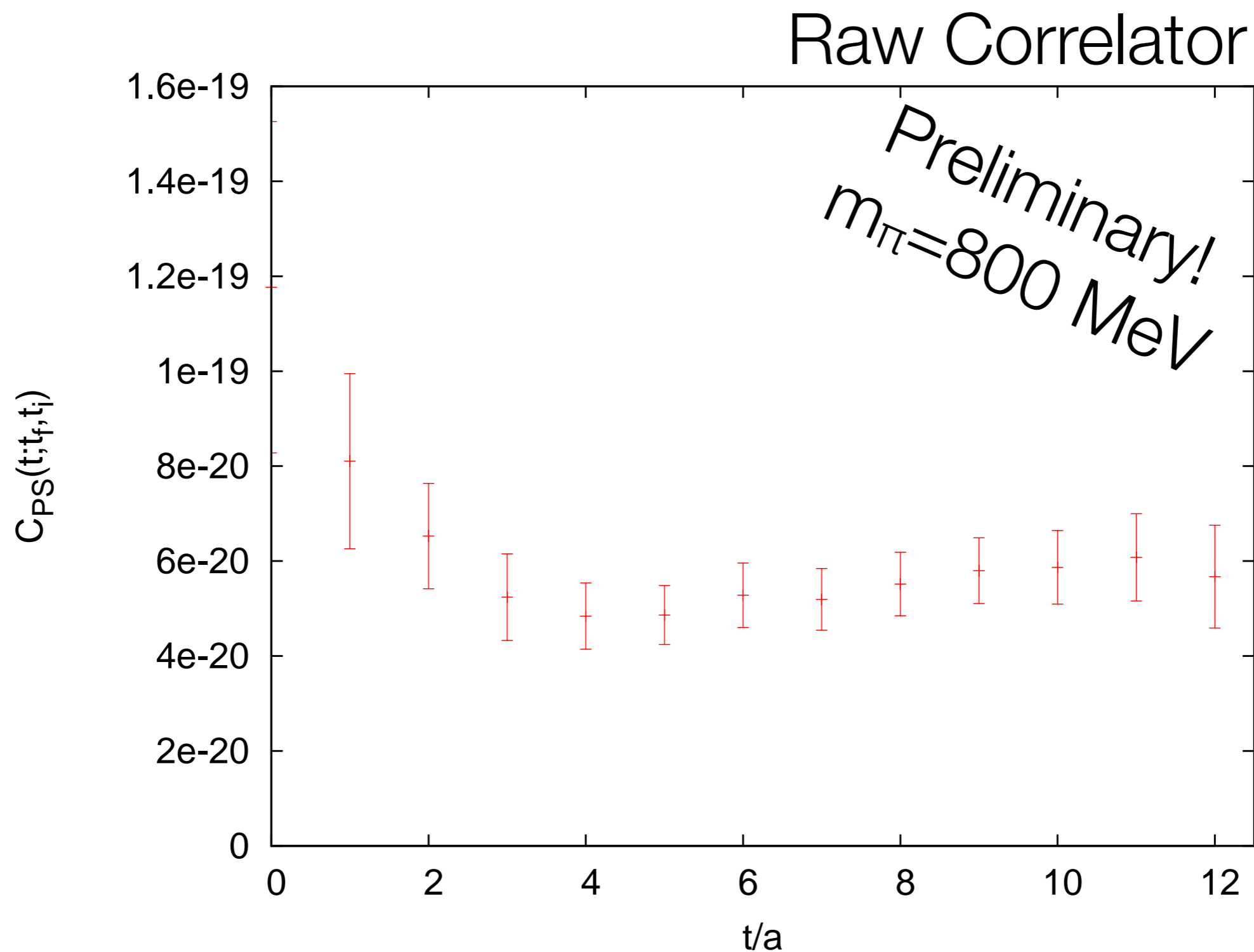
Preliminary Results:

Partial Wave Scattering at $m_\pi=800$ MeV



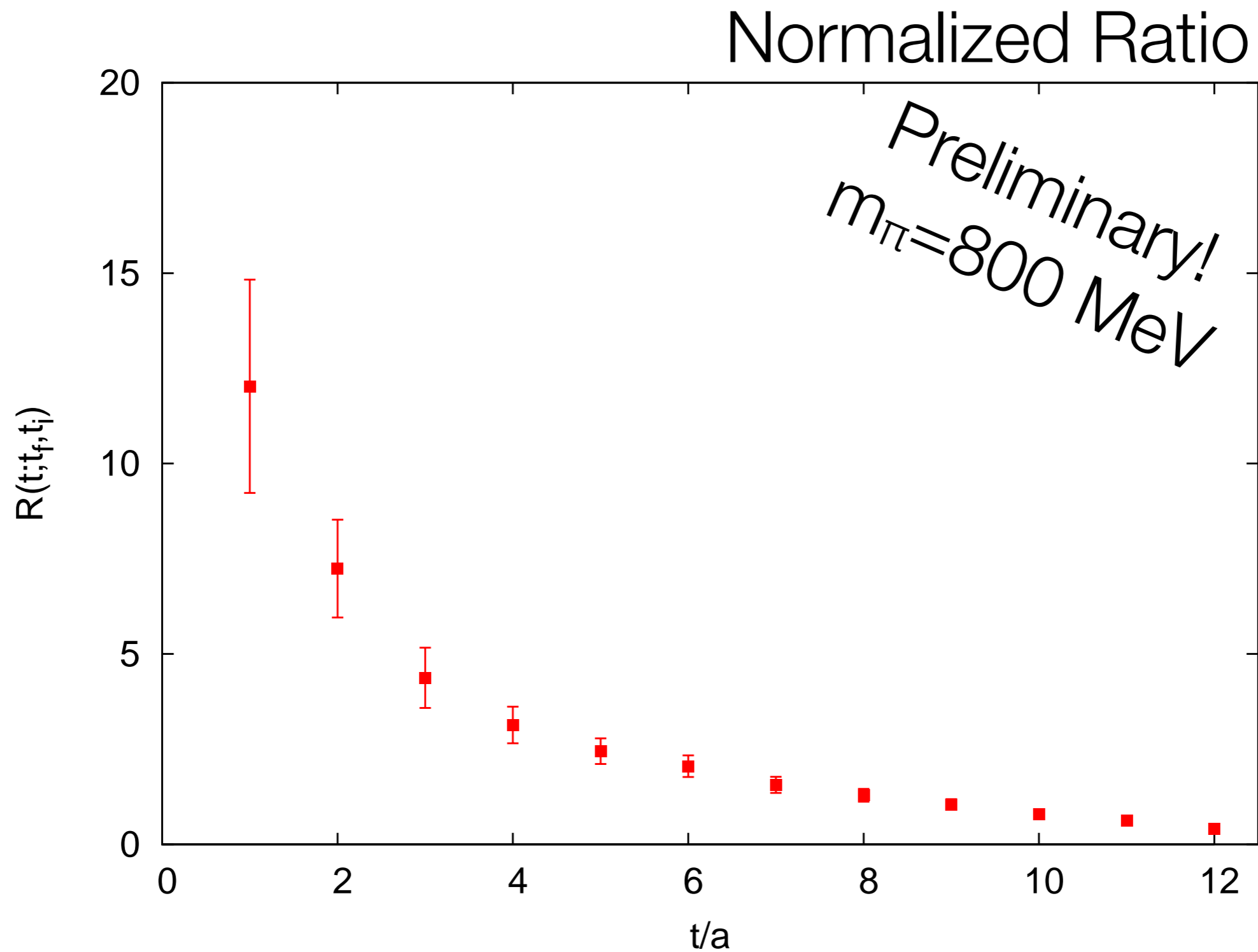
Preliminary Results:

Parity Violation Matrix Element at $m_\pi=800$ MeV

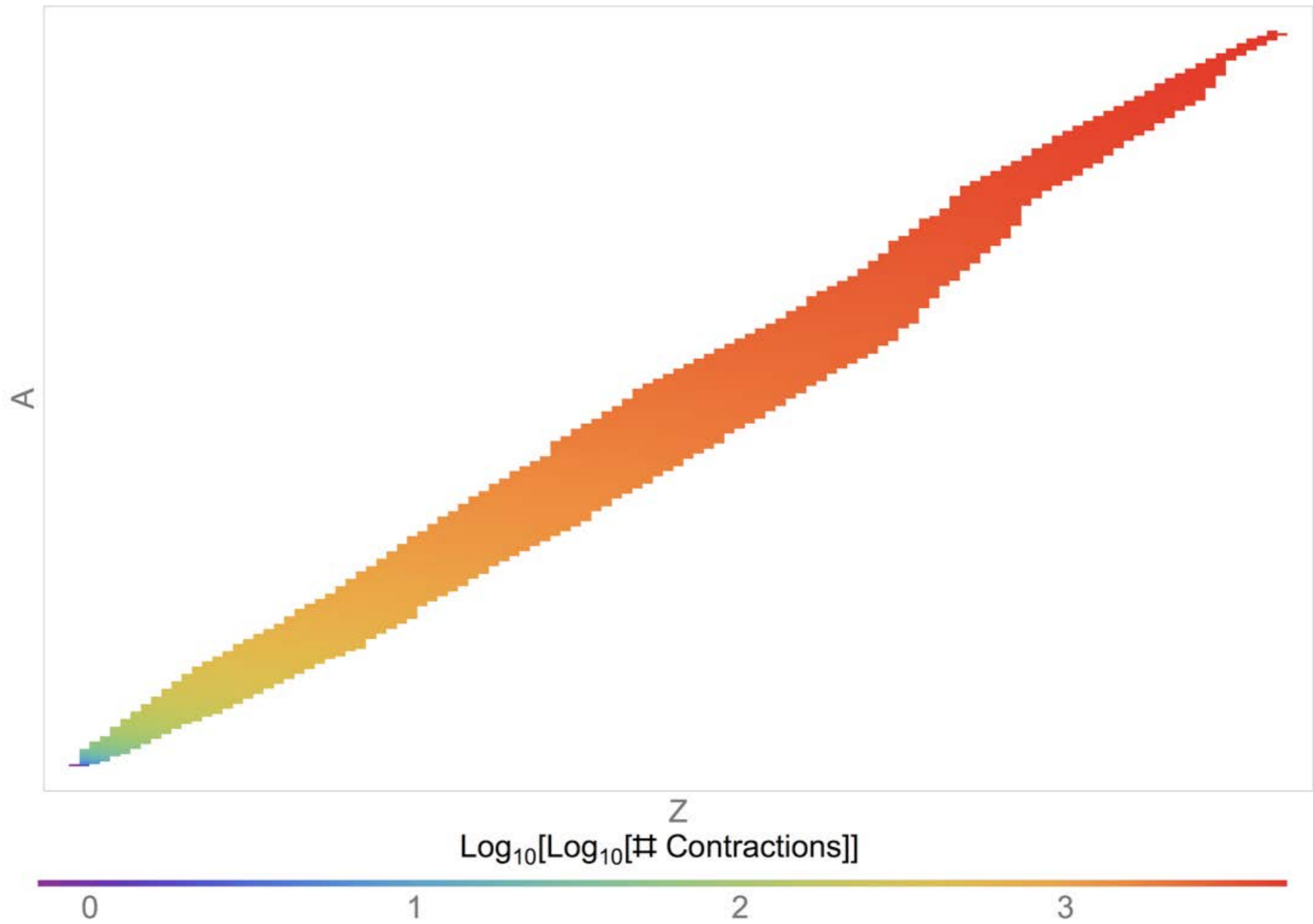


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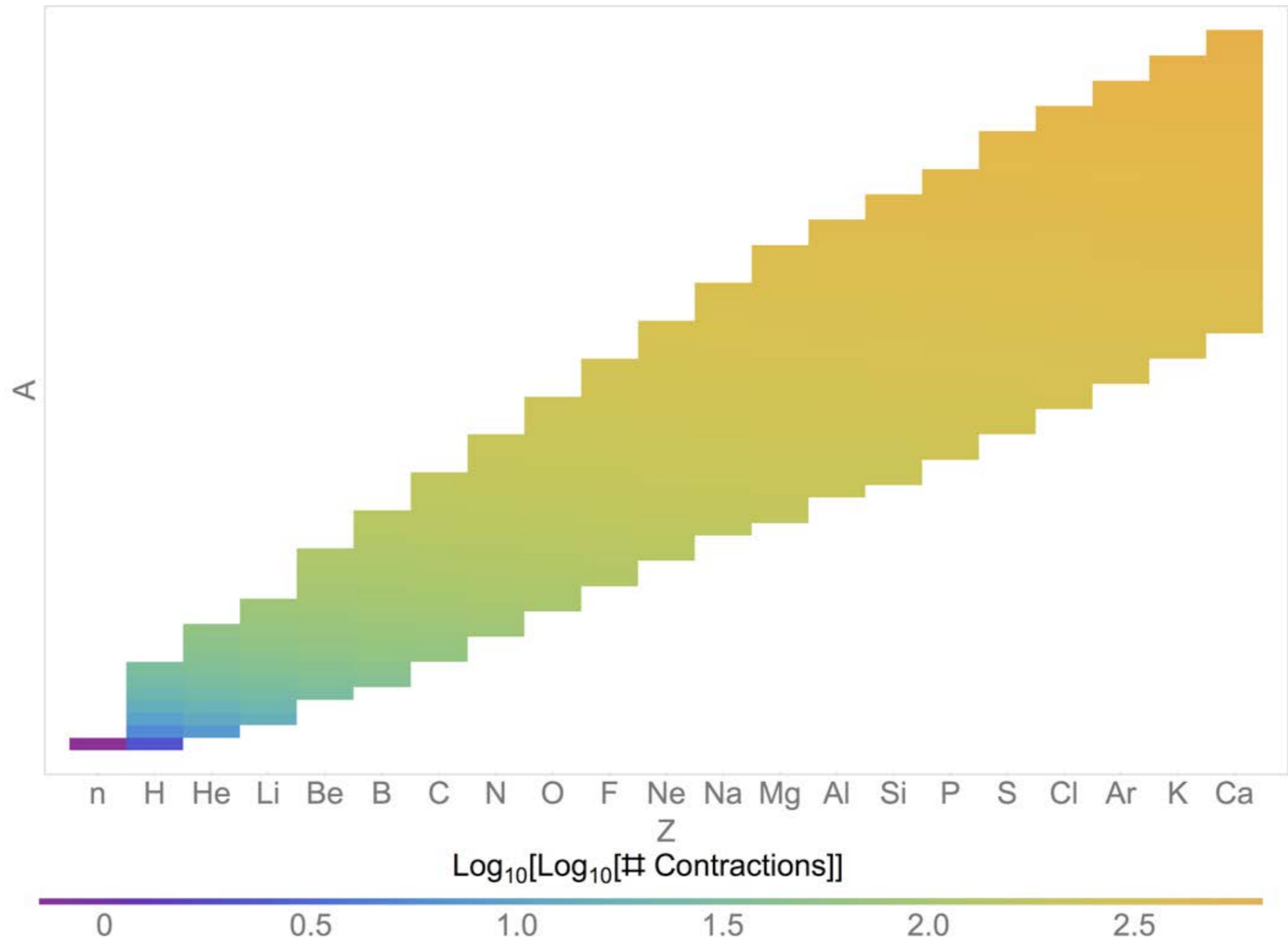
Parity Violation Matrix Element at $m_\pi=800$ MeV



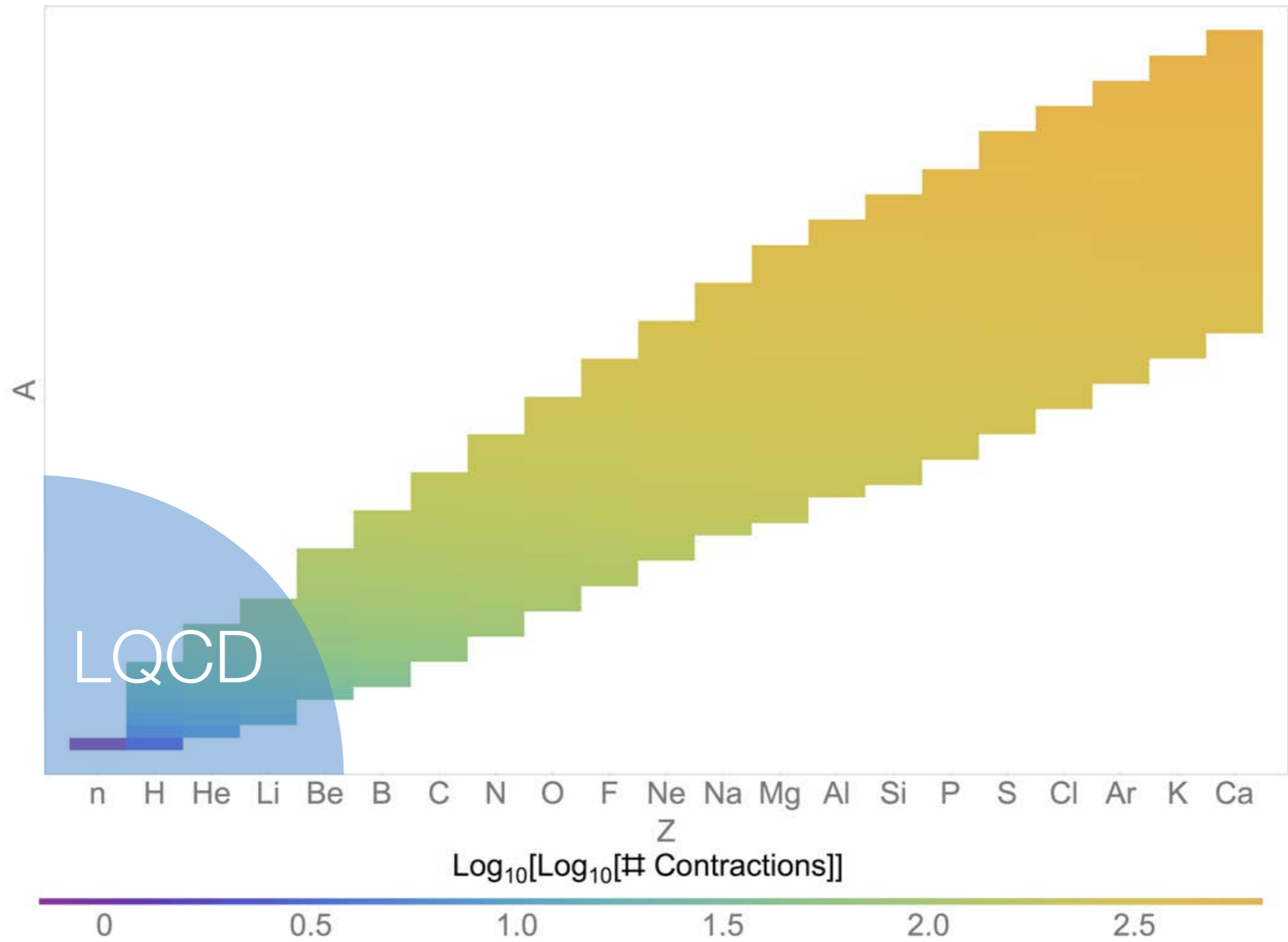
Reach



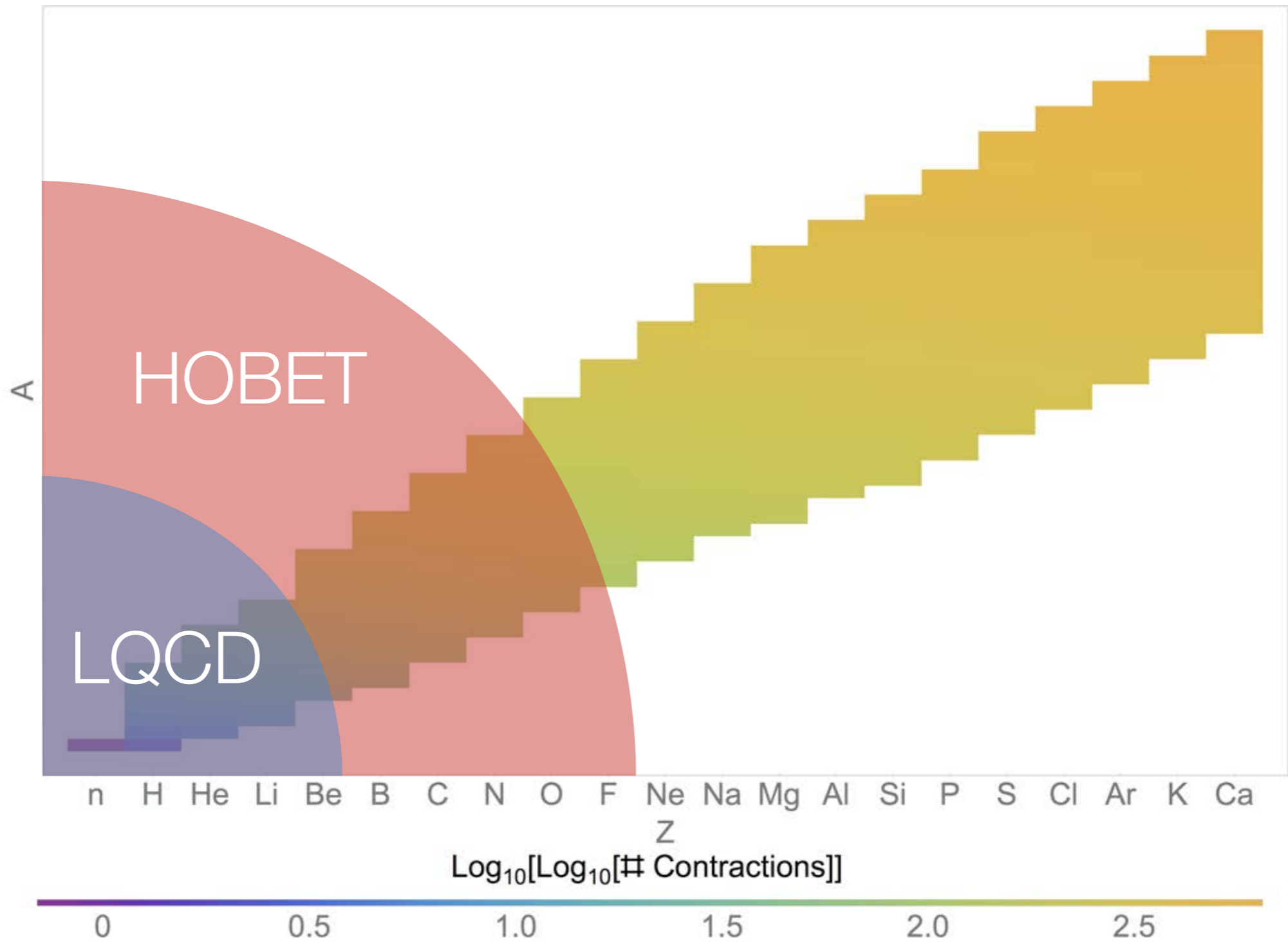
Reach



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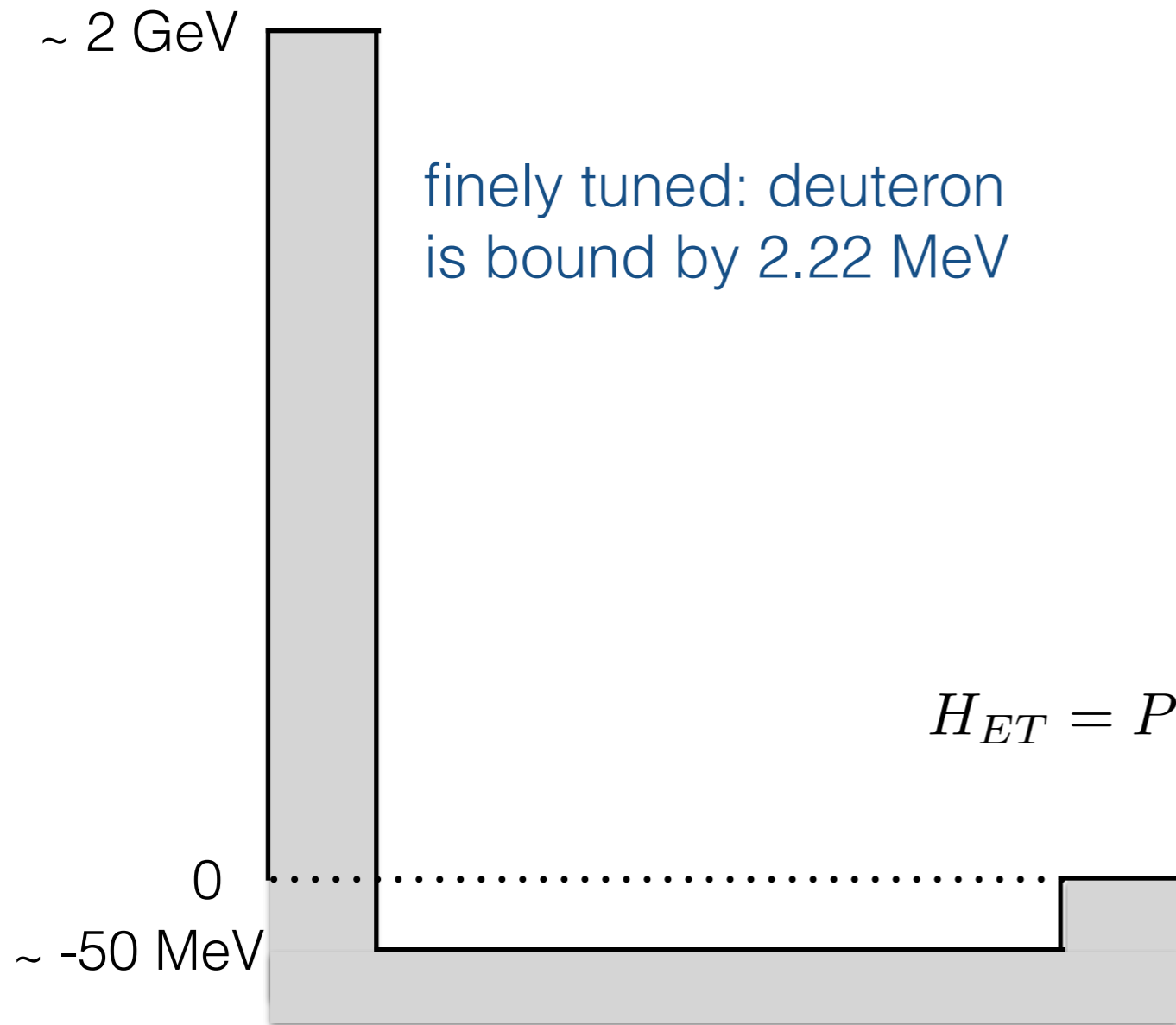
Reach



HOBET: Connecting a non-relativistic effective theory to the lattice

- Idea: Build an effective theory that can utilize lattice input to fix parameters that are unknown, while taking other parameters from experiment
- This includes utilizing either experiment or lattice input on the NN interaction to predict properties of light nuclei
- HOBET is a true ET: the effective interaction is constructed directly in the soft — or “included” — P-space
- Usual nuclear physics approach:
QCD → singular NN potential → P-space effective interaction
- HOBET: QCD → P-space effective interaction
- The difficult effective interactions problem is avoided
- How is this done? HOBET’s unique infra-red/ultraviolet separation allows us to utilize NN phase shifts to fix ET parameters

Simplified analytic example: hard-core s-wave interaction, chosen to reproduce deuteron binding energy



finely tuned: deuteron is bound by 2.22 MeV

P is the soft ET space

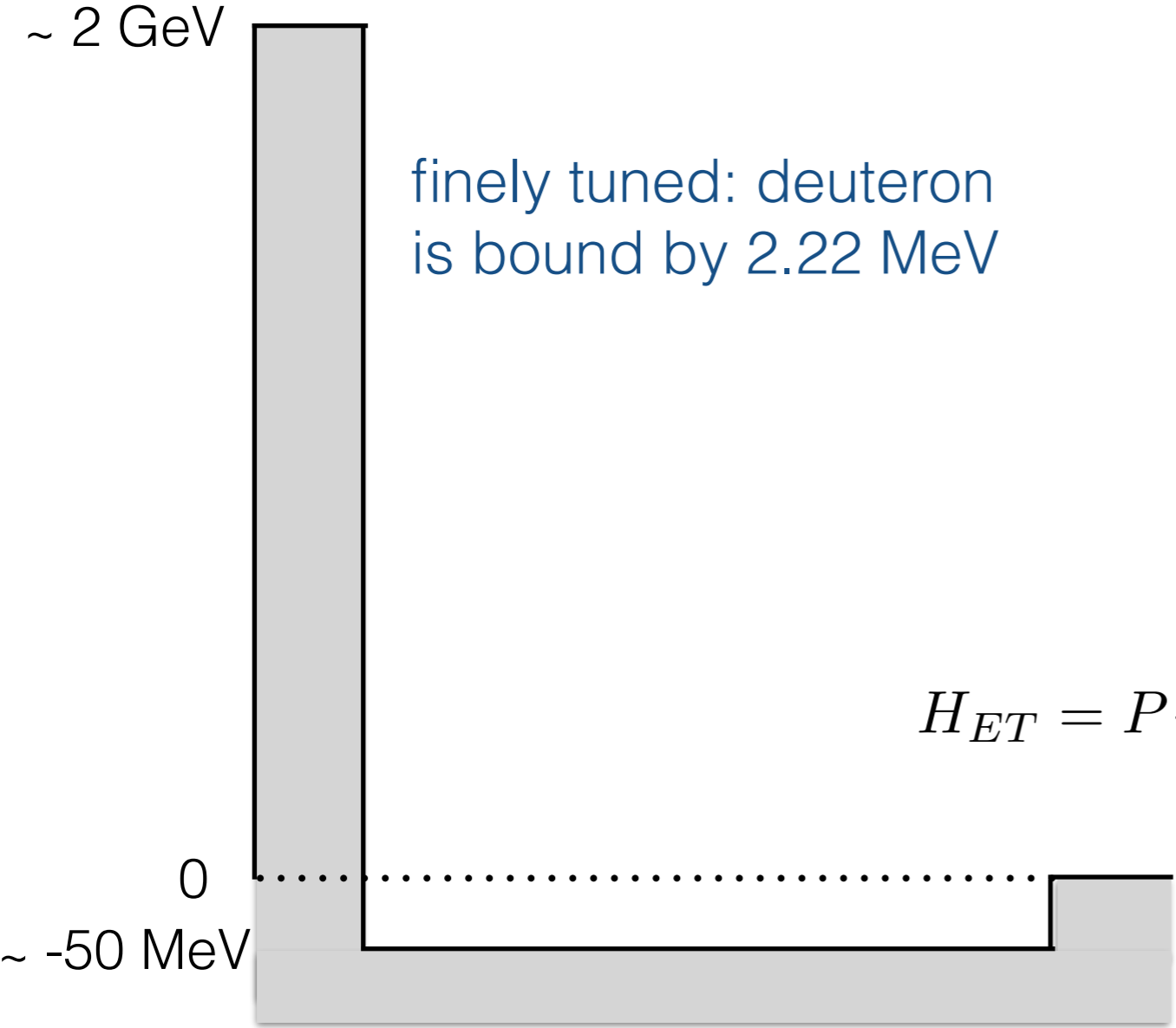
$Q=1-P$

$H = T+V$

use the Haxton/Luu factorization of the Bloch-Horowitz equation

$$H_{ET} = P \frac{E}{E - TQ} \left[T - T \frac{Q}{E} T + V + V_{\delta} \right] \frac{E}{E - QT} P$$

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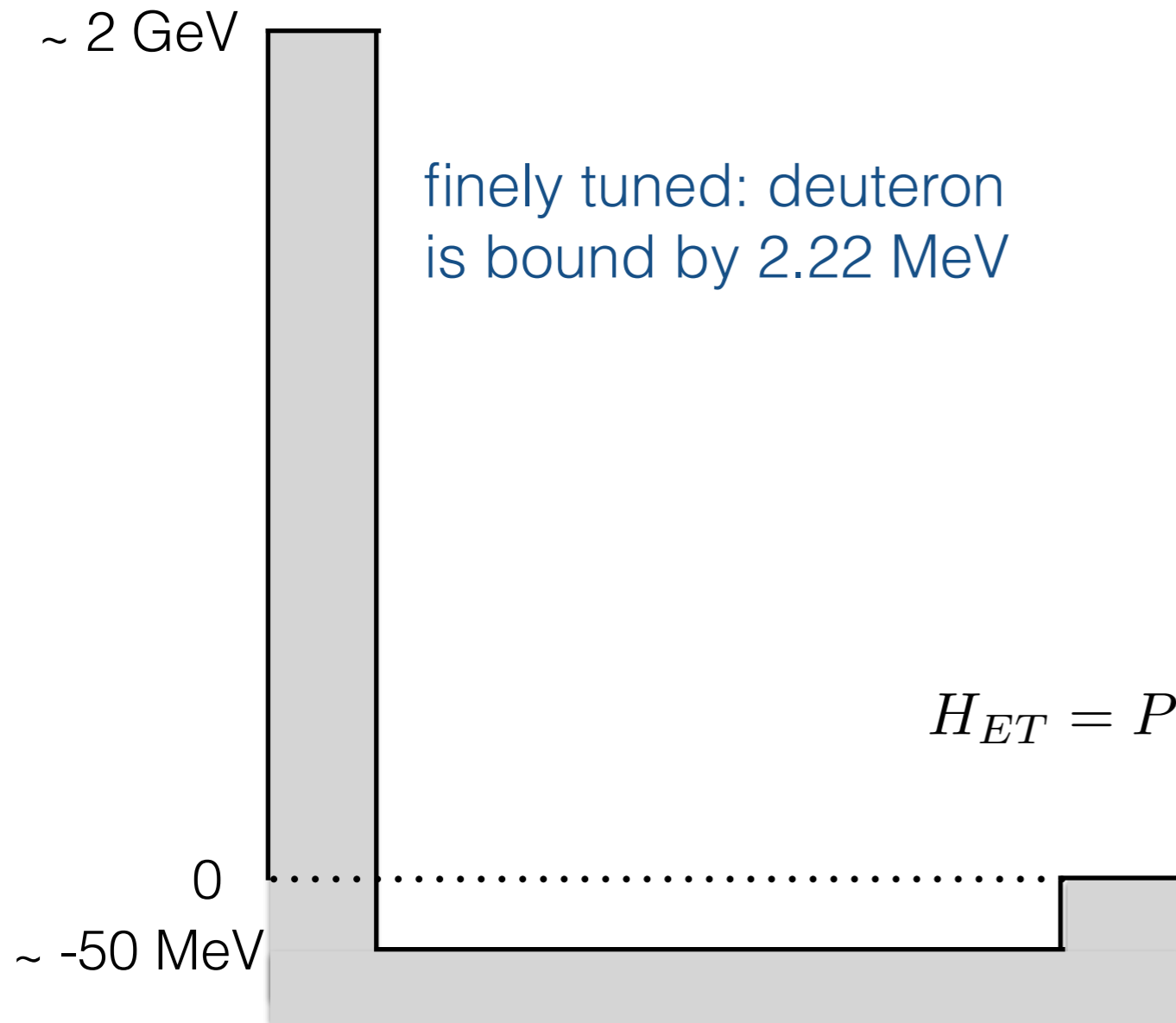
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sums kinetic energy T to all orders

outgoing solution for a specific E depends on the phase shift $\delta(E)$

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short-range operator that corrects
for the effects of V acting outside P :
can be efficiently expanded in
terms of contact operators

The BH equation must be solved self-consistently: $H_{ET} = H_{ET}(E)$

So one picks an E, supplies $\delta(E)$, calculates $H_{ET}(E)$, diagonalizes in P

A solution must exist at every $E > 0$

But the diagonalization does not give an eigenvalue at E

Thus we “dial” V_δ - this can be done order-by-order to reproduce $\delta(E)$ over a range of E

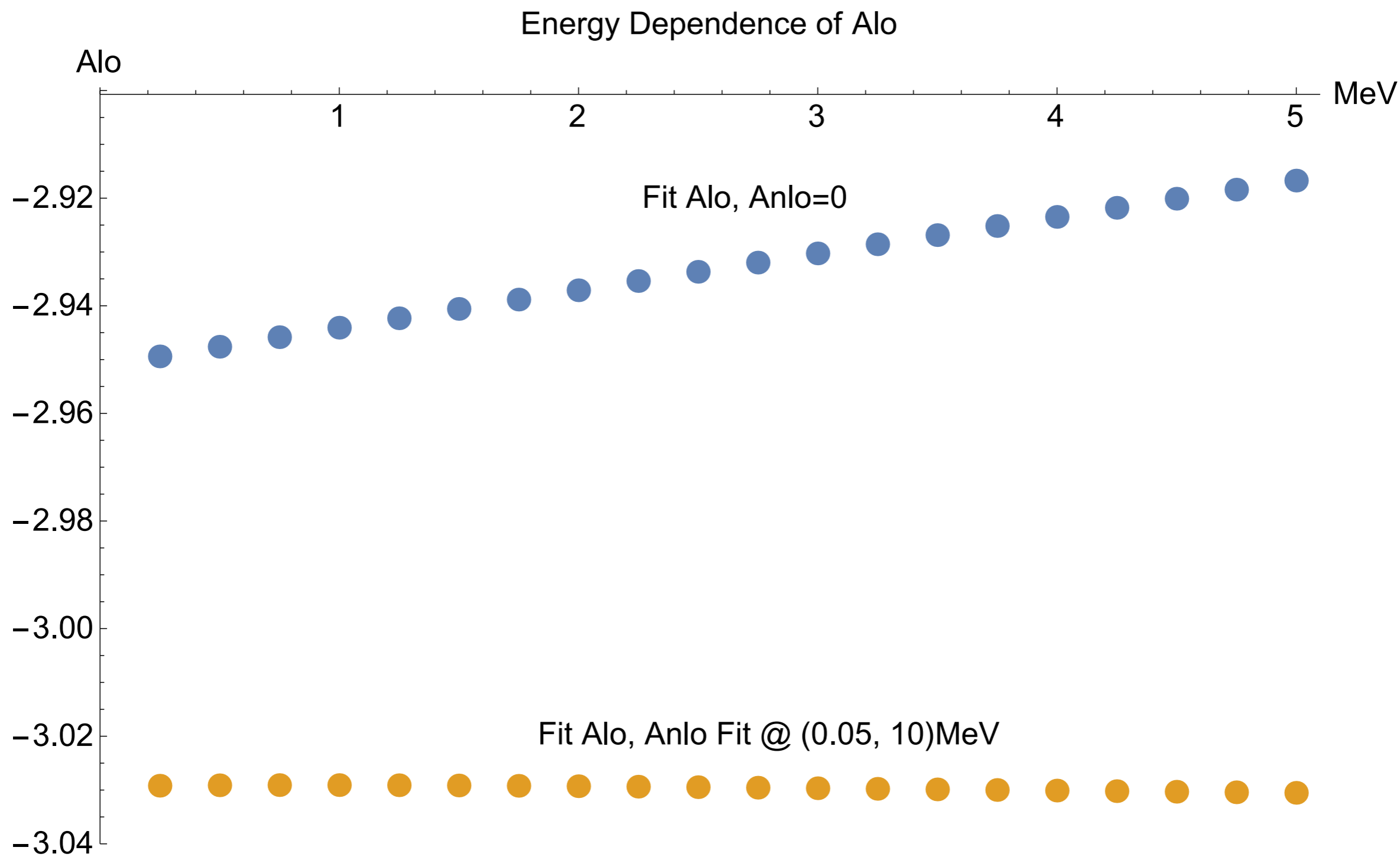
Through NNLO:

$$V_\delta = a_{LO} \delta(\vec{r}) + a_{NLO} \left(\overleftarrow{\nabla}^2 \delta(\vec{r}) + \delta(\vec{r}) \overrightarrow{\nabla}^2 \right) + a_{NNLO}^1 \overleftarrow{\nabla}^2 \delta(\vec{r}) \overrightarrow{\nabla}^2 + a_{NNLO}^2 \left(\overleftarrow{\nabla}^4 \delta(\vec{r}) + \delta(\vec{r}) \overrightarrow{\nabla}^4 \right)$$

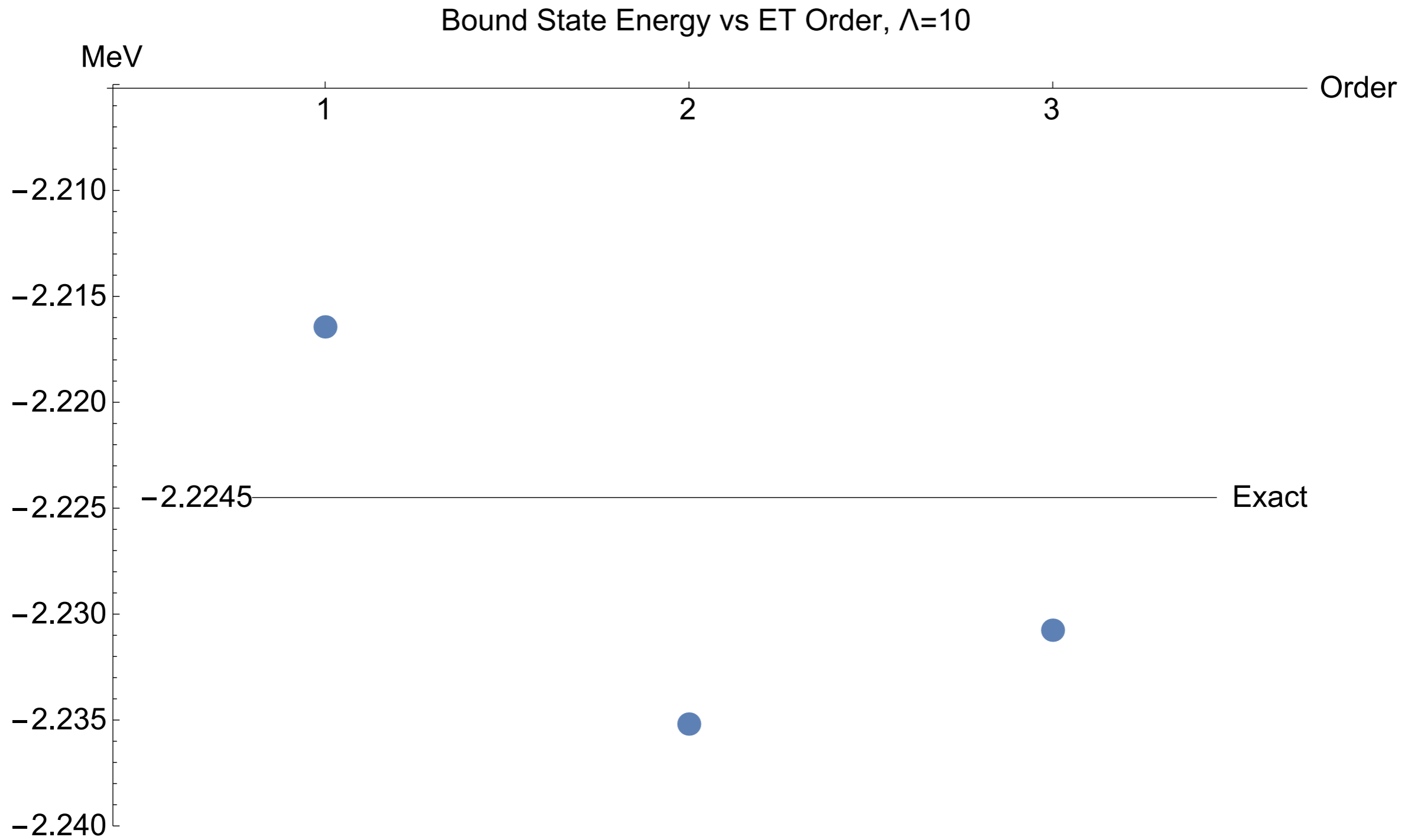
The larger the range in E over which $\delta(E)$ is fit, the more terms needed

Phase shifts yield the deuteron binding energy, in a calculation limited to P

Low-energy constants become constant, as higher orders are included



For $V_\delta = 0$ the predicted binding energy is -0.68 MeV



CalLat is applying these techniques to the parity-violating asymmetry in



Ongoing SNS experiment to measure the neutral current in the hadronic weak interaction (see poster)

We will use HOBET + lattice to fully determine this system, and to relate this observable to other parity-violating observables

Experimental strong phase shifts: fix all strong interaction low-energy constants

Lattice: provides the experimentally unknown parameter — the parity-violating weak phase shift

Conclusions

- Nuclear physics can be done on the Lattice
 - Baryon blocks and tensors tame Wick Contractions
⇒ automatic code generation, parallelization (GPUs)
 - sophisticated, expensive non-local sources/sinks for higher partial waves ⇒ treating excited states effects
 - exponential decrease of SNR ⇒ large m_π , huge statistics (MG), huge amount of data to store (HDF5)
 - preliminary higher partial wave and PV results look promising, but might need to disentangle angular momentum multiplets

Conclusions

- Expected computational costs:
 - $m_\pi \downarrow$: inversions \uparrow , SNR \downarrow
 - $L \uparrow$: inversions \uparrow , FFT \uparrow , contractions \uparrow , SNR \uparrow
- feed lattice results into HOBET \Rightarrow solve complex nuclear systems

Thank You