



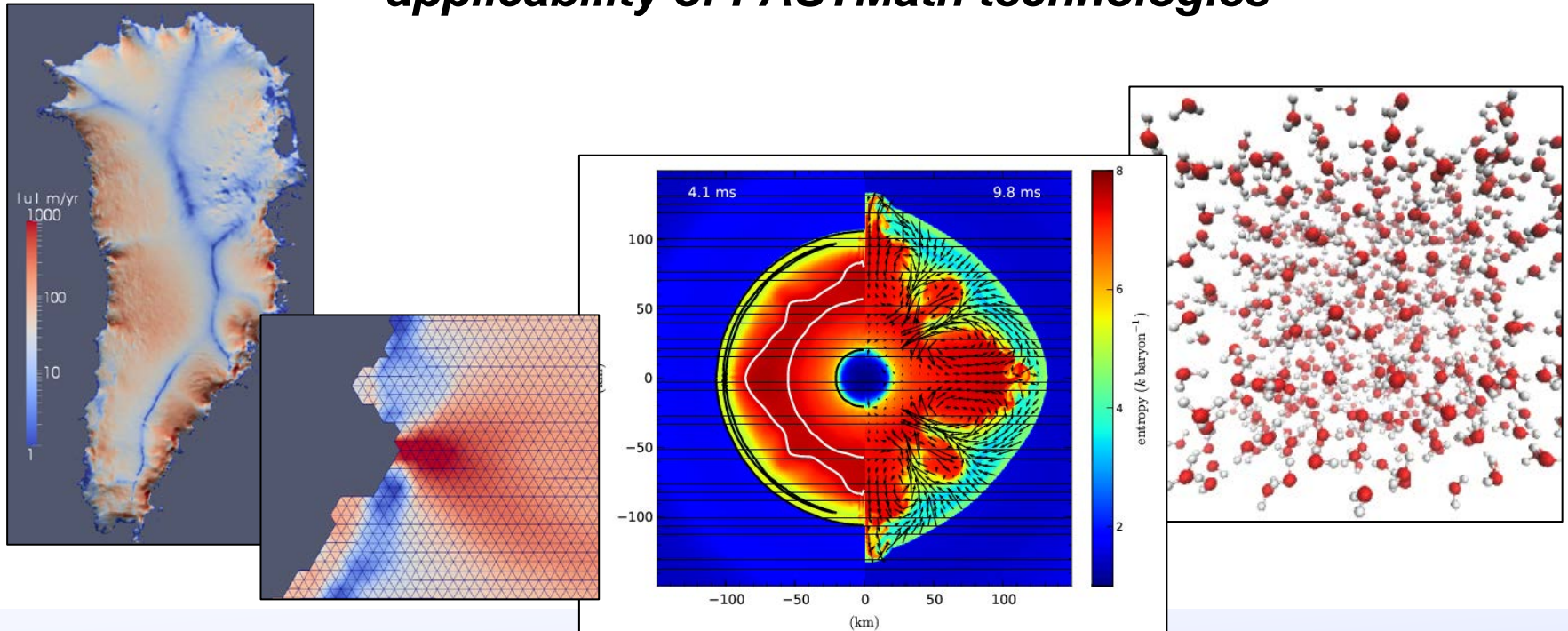
# FASTMath: Frameworks, Algorithms and Scalable Technologies for Mathematics

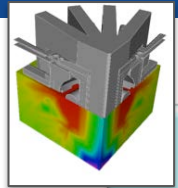
**FASTMath Team**  
**Lori Diachin, Institute Director**

FASTMath SciDAC Institute



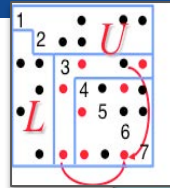
***The FASTMath SciDAC Institute develops and deploys scalable mathematical algorithms and software tools for reliable simulation of complex physical phenomena and collaborates with DOE domain scientists to ensure the usefulness and applicability of FASTMath technologies***





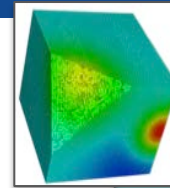
## Tools for Problem Discretization

- Structured grid technologies
- Unstructured grid technologies
- Adaptive mesh refinement
- Complex geometry
- High-order discretizations
- Particle methods
- Time integration



## Solution of Algebraic Systems

- Iterative solution of linear systems
- Direct solution of linear systems
- Nonlinear systems
- Eigensystems
- Differential variational inequalities

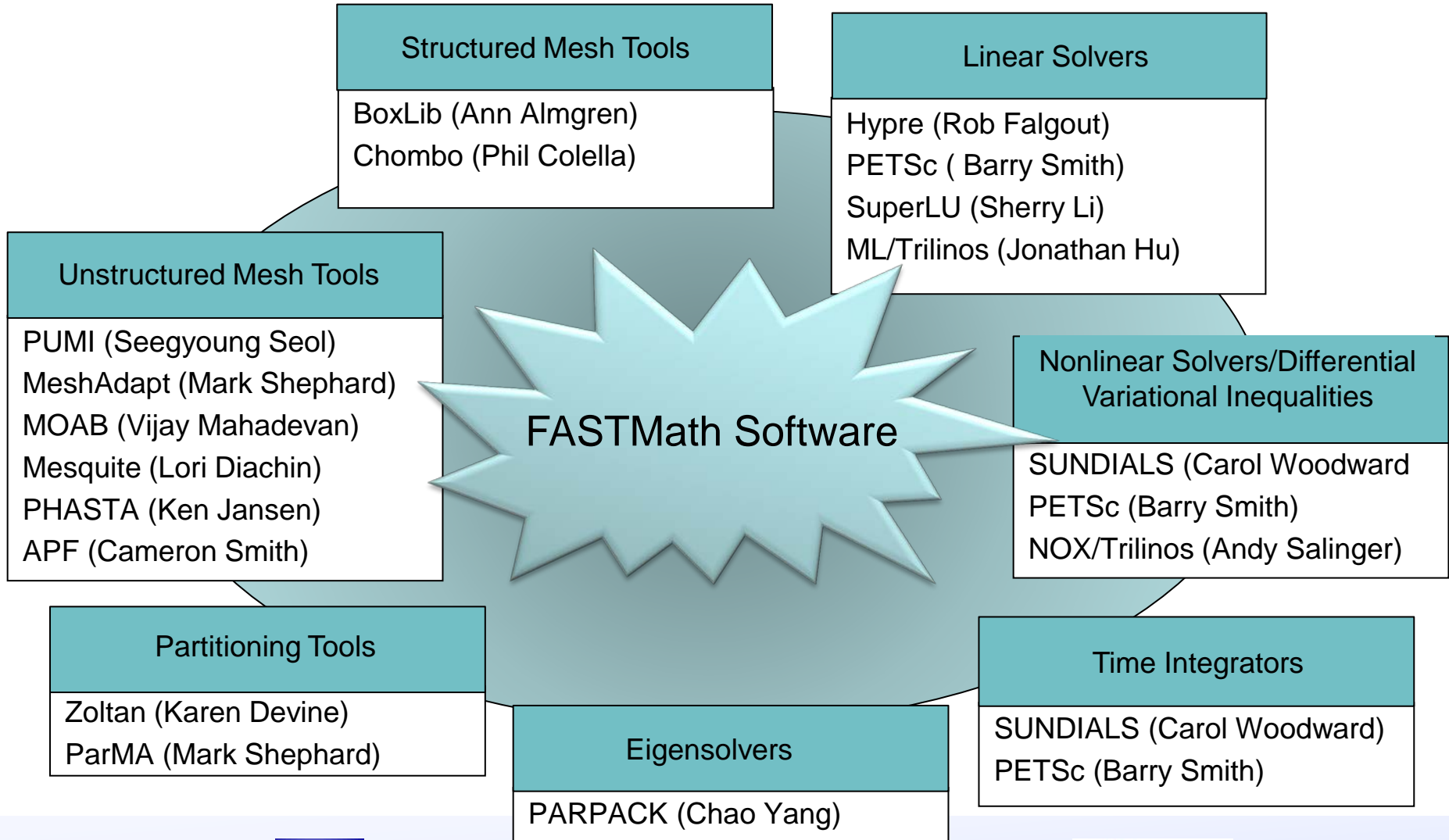


## High Level Integrated Capabilities

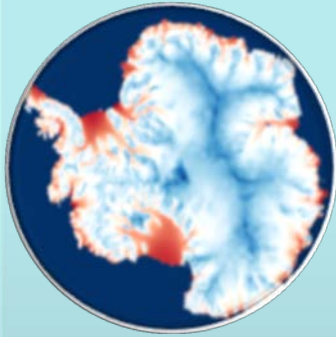
- Adaptivity through the software stack
- Management of field data
- Coupling different physics domains
- Mesh/particle coupling methods



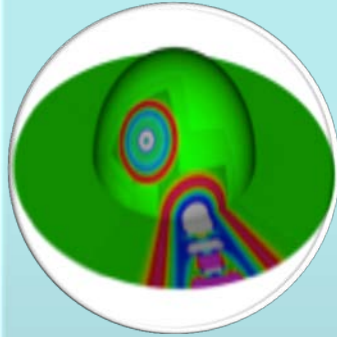
# FASTMath encompasses our algorithm development in widely used software



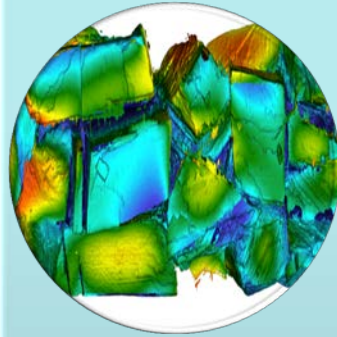
# Structured grid capabilities focus on high order, mapped grids, embedded boundaries, AMR and particles



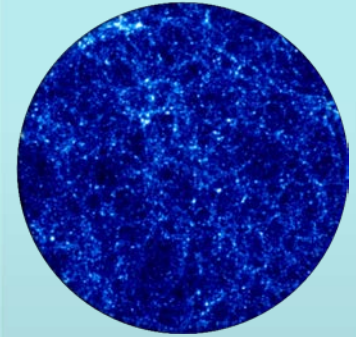
Structured AMR



Mapped-multiblock grids



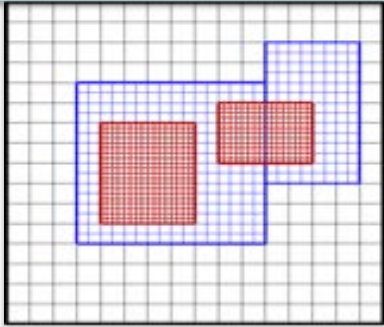
Embedded boundary methods



Particle-based methods

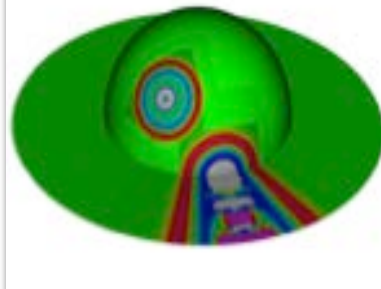
Application to cosmology, astrophysics, accelerator modeling, fusion, climate, subsurface reacting flows, low mach number combustion, etc.

## New AMR methods



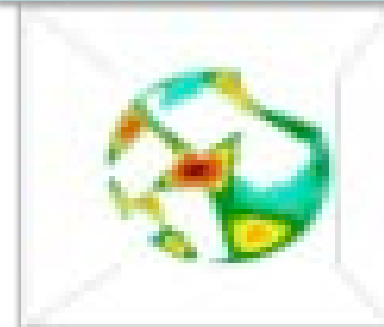
- Region based AMR for time stepping
- Fourth-order AMR for hyperbolic equations
- Allow mixed dimensionality

## Mapped-multiblock FV Methods



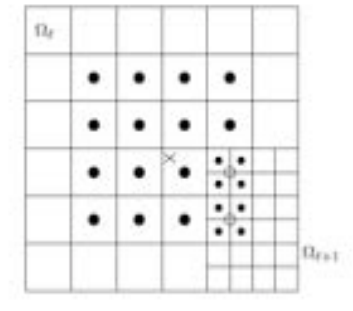
- Developing 4<sup>th</sup> order schemes
- Anisotropic refinement
- Mixed dimensions
- Extensions to PDEs on manifolds

## Embedded Boundaries



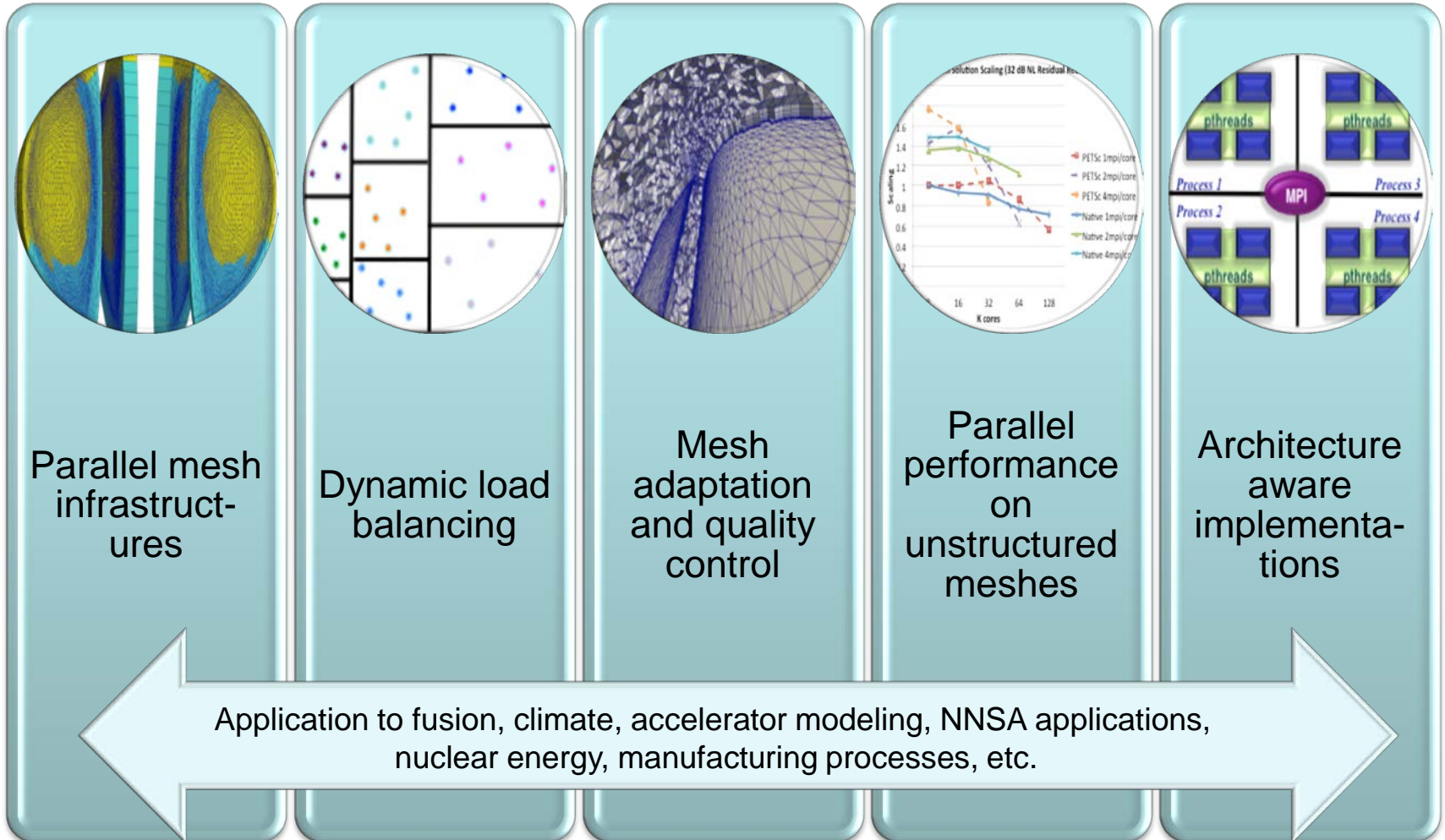
- Grid generation from light source image data
- Interface to PETSc solvers
- Discretization for tensor operators

## Particle methods

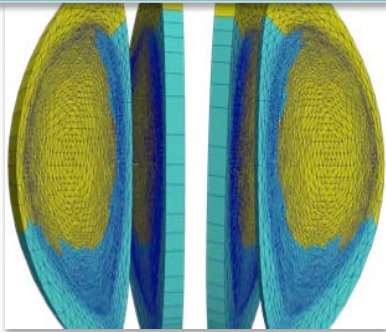


- Two-grid methods to improve scaling
- Space filling curve to assign particles to processors
- Load balancing schemes that separate particle and field calculations

# Our unstructured grid capabilities focus on adaptivity, high order, and the tools needed for extreme scaling

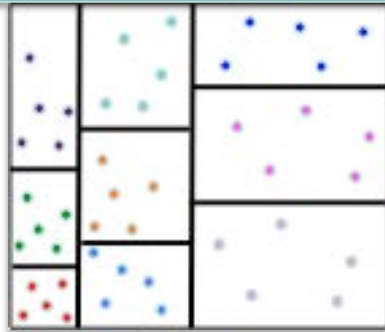


## Parallel Unstructured Mesh Infrastructures



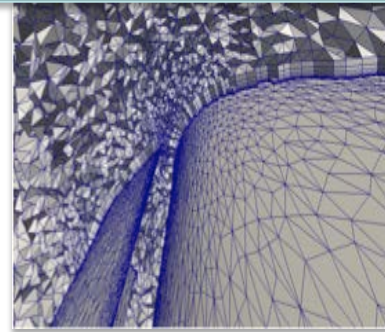
- PUMI scales to 750K cores and 92B elements
- MOAB scaling to 32K cores with improved memory access and Parallel I/O

## Dynamic Load Balancing



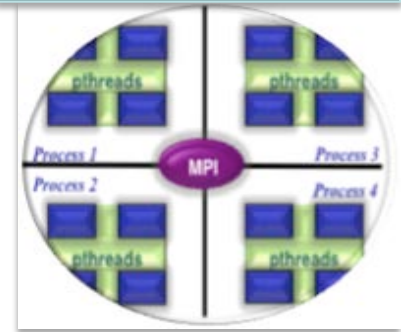
- Partitioning tools for hybrid programming
- Predictive load balancing to avoid memory problems
- Partitioning for multiple entity types
- Multilevel partitioning for large core counts (750K cores, 3.1M parts)

## Adaptive Mesh Refinement



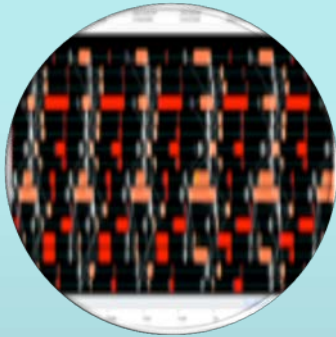
- Boundary layer adaptation (mixed meshes and boundary layer thickness)
- Support for curved geometries
- In-memory integration of MeshAdapt

## Architecture-aware Implementations

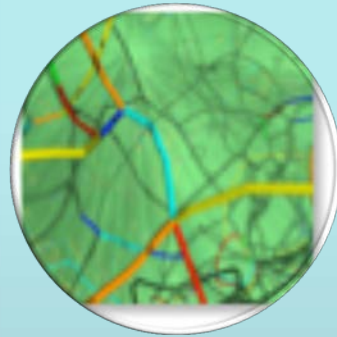


- Arch-aware task placement to reduce communication costs results in 34% reduction in stencil-based computational cost
- Parallel control utility developed

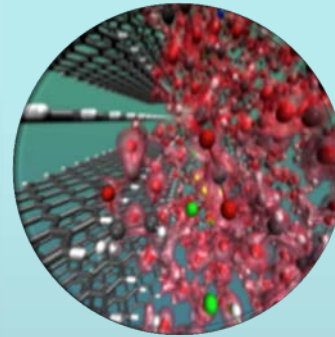




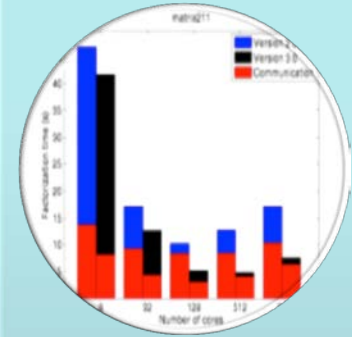
Linear system solution using direct and iterative solvers



Nonlinear system solution using acceleration techniques and globalized Newton methods



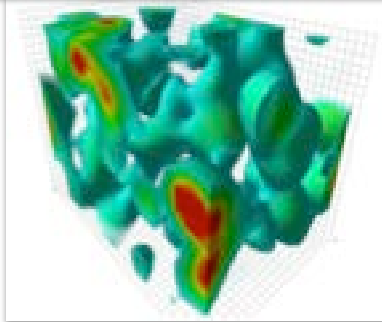
Eigensolvers using iterative techniques and optimization



Architecture aware implementations

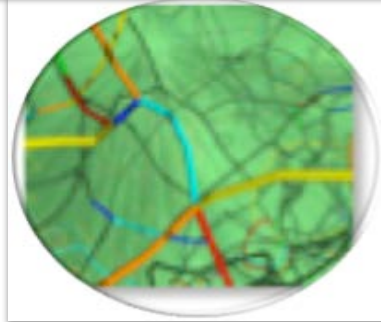
Application to fusion, nuclear structure calculation, quantum chemistry, accelerator modeling, climate, dislocation dynamics etc,

## Linear System Solution



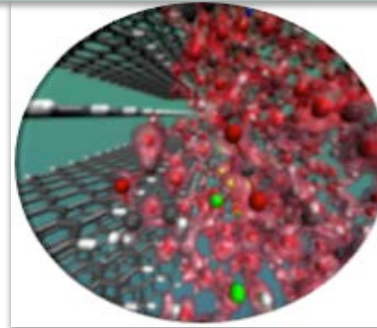
- Constrained energy minimization multigrid implemented
- HSS schemes for solving structured linear systems is 2X faster and uses 1/5 the memory of original

## Nonlinear System Solution



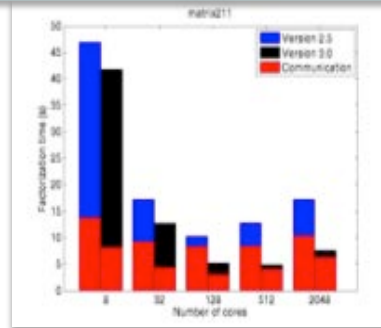
- Developed ARKode solver in SUNDIALS (IMEX method)
- SUNDIALS interfaced to SuperLU\_MT
- Developed fixed point and Anderson accelerated solvers for SUNDIALS and NOX

## Eigen solvers

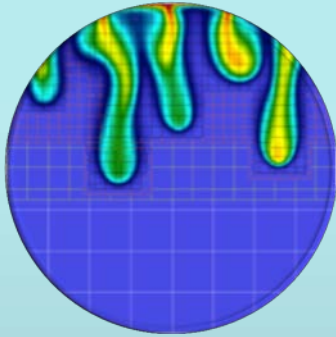


- Many eigenpairs of Hermitian matrices by reducing number of Rayleigh-Ritz calculations
- Topology aware data distributions for scalable eigensolver for nuclear configuration calcs
- Interior eigenvalues of non-Hermitian matrices

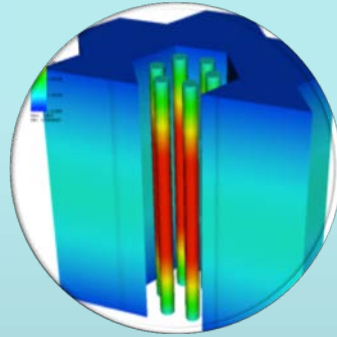
## Architecture-aware Implementations



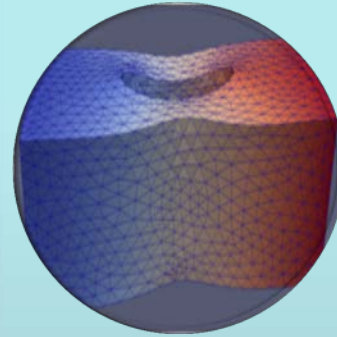
- Communication reduction in multigrid
- Communication bounds in iterative and direct methods
- Pipelined Krylov solvers to reduce global reductions
- New algorithms for DAG execution



Mesh/solver interactions



Mesh-to-mesh coupling methods



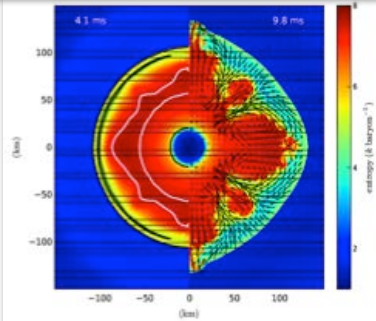
Unstructured mesh technologies into simulation workflows



Software unification strategies

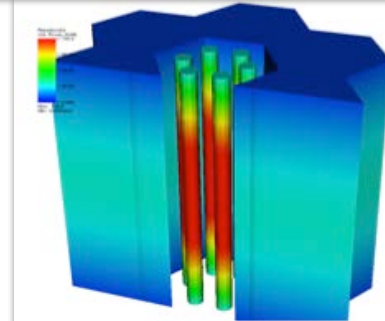
Application to climate, plasma surface interactions, structural mechanics, nuclear energy, cosmology, fluid flow, etc.

## Mesh Solver Interactions



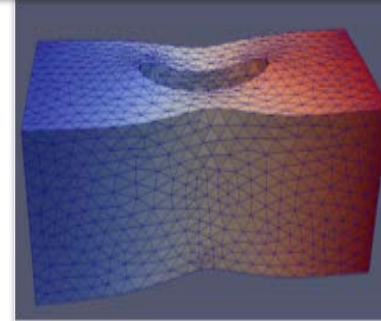
- BoxLib/PETSc, hypra, Trilinos interfaces for porous media and radiation
- Chombo/PETSc for climate and pore scale modeling
- PHASTA-PUMI-PETSc for compressible flow
- MOAB/PETSc for plasma surface interactions

## Mesh-to-mesh coupling



- Developed CouPE for tight multi-physics coupling with loose coupling interfaces
- Strong scalability up to 512K cores for coupling

## Simulation Workflows



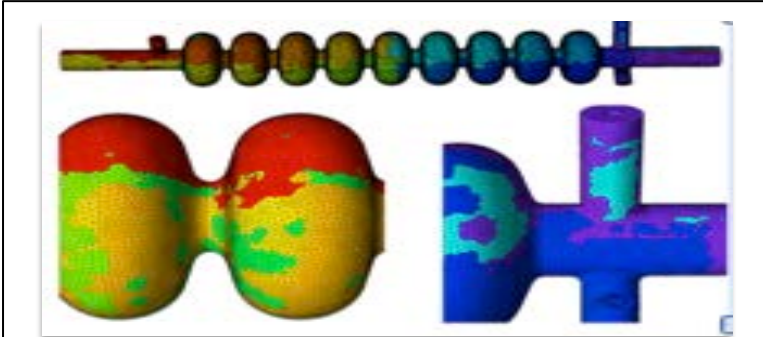
- Eliminate file I/O in parallel adaptive simulations
- Attached parallel fields interface to support data and information transfer
- Integrated PUMI, Albany, MeshAdapt, Zoltan

## Software unification

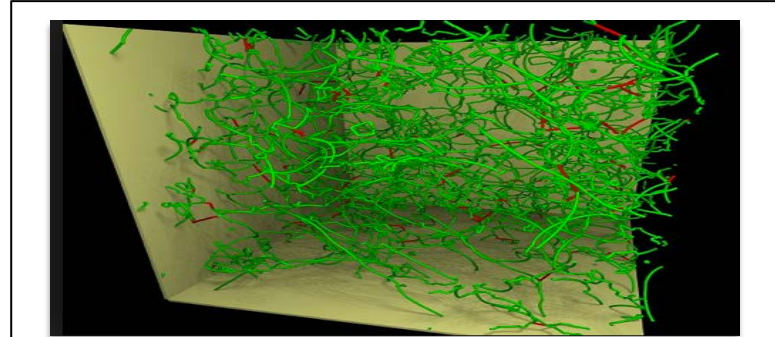


- Tutorials lower barriers to SW adoption
- Common build/configure practices
- One stop shopping web site
- Reference installations at ANL and LBNL

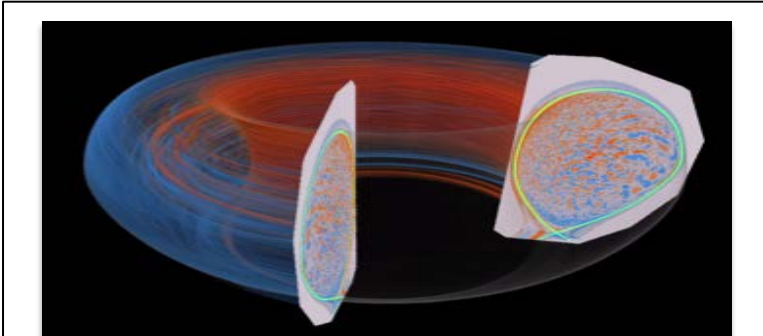
# We have helped the application teams significantly reduce time to solution in their simulations



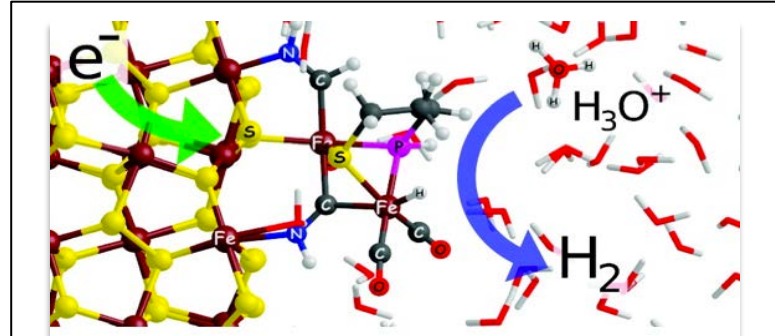
Sparse direct solves improve time to solution 20X for accelerators allowing 8 cavity simulation (Spentzouris)



Acceleration-based nonlinear solvers speed up dislocation dynamics 35-50%; multistage Runge-Kutta methods reduce time steps by 94% (Arsenlis)

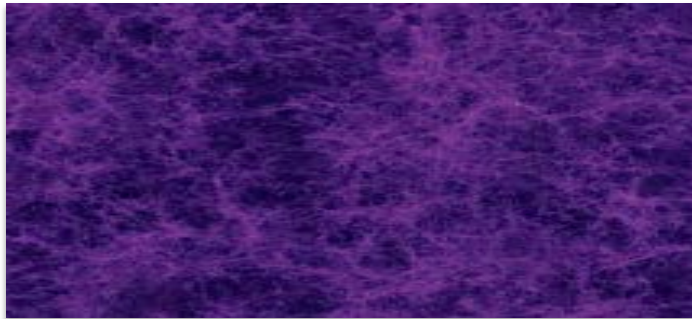


Sped up flux surface creation to improve 2D mesh generation in fusion application from 11.5 hours to 1 minute (Chang)

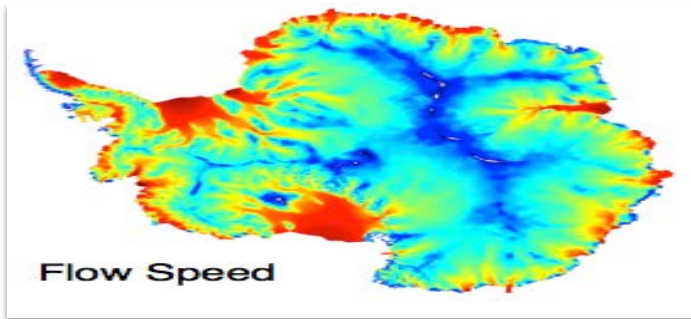


Sophisticated eigensolvers significantly improve materials calculations in many domains including ions in solution (Car), excited state phenomenon (Chelikowsky, Head-Gordon)

# We have helped the application teams achieve unprecedented resolution and increased reliability

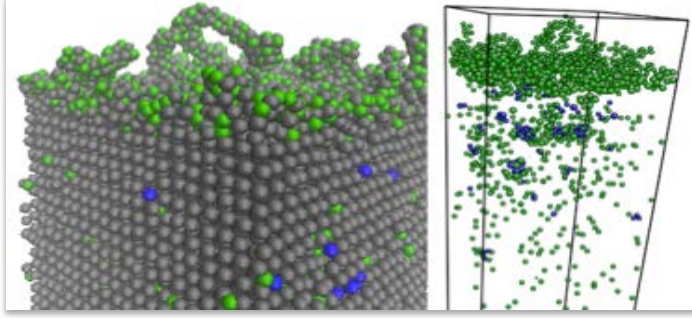


Astrophysics Lyman- $\alpha$  forest simulation at  $4096^3$  in an 80Mpc/h box; produced statistics at 1% accuracy for first time (Habib)




Flow Speed

Predictions of grounding line match experiment for first time in ice sheet modeling due to AMR (Price)



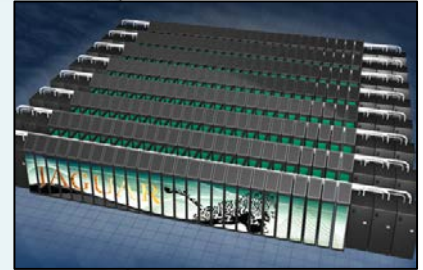
Implicit ODE integrators combined with AMG linear solvers enables solution of 4D reaction-diffusion eqns for plasma surface interactions (Wirth)



High-order unstructured meshes for particle accelerators overcome mesh generation/ adaptation bottlenecks (Spentzouris)

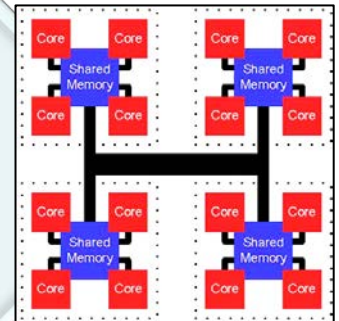
## Inter-node: Massive Concurrency

- Reduce communication
- Increase concurrency
- Reduce synchronization
- Address memory footprint
- Enable large communication/computation overlap



## Intra-node: Deep NUMA

- MPI + threads for many packages
- Compare task and data parallelism
- Thread communicator to allow passing of thread information among libraries
- Low-level kernels for vector operations that support hybrid programming models



## Reduce communication

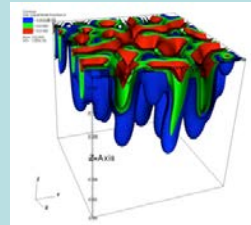
- AMG: develop non-Galerkin approaches, use redundancy or agglomeration on coarse grids, develop additive AMG variants (hypr) (2X improvement)
- Hierarchical partitioning optimizes communication at each level (Zoltan) (27% improvement in matrix-vector multiply)
- Relaxation and bottom solve in AMR multigrid (Chombo) (2.5X improvement in solver, 40% overall)
- HSS methods

## Increase concurrency

- New spectrum slicing eigensolver in PARPACK (Computes 10s of thousands of eigenvalues in small amounts of time)
- New pole expansion and selected inversion schemes (PEXSI) (now scales to over 100K cores)
- Utilize BG/Q architecture for extreme scaling demonstrations (PHASTA) (3.1M processes on 768K cores unstructured mesh calculation)

## Reduce synchronization points

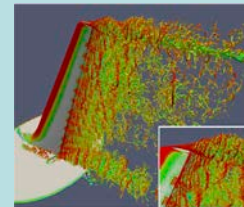
- Implemented pipelined versions of CG and conjugate residual methods; 4X improvement in speed (PETSc) (30% speed up on 32K cores)



Used in PFLOTRAN applications

## Address memory footprint issues

- Predictive load balancing schemes for AMR (Zoltan) (Allows AMR runs to complete by maintaining memory footprint)
- Hybrid programming models



Used in PHASTA extreme scale applications

## Increase communication and computation overlap

- Improved and stabilized look-ahead algorithms (SuperLU) (3X run time improvement)



Used in Omega3P accelerator simulations



# We are refactoring our software to support hybrid programming models

Block structured AMR in Chombo (MPI + OpenMP)

- Adding course grained thread loop over blocks and micro-blocking reduced communication costs and memory footprint; performance improvements limited

Unstructured grids using Parallel Control Utility (MPI and threads)

- Utility layer that allows support of both MPI and threads; showed 30% efficiency improvement and 10% memory reduction on BG/Q

Partitioning with Zoltan2 (MPI + OpenMP)

- Implementation in multi-dimensional jagged geometric partitioning scaled to 8B elements on 64K nodes

Direct linear solvers in SuperLU\_DIST (MPI + OpenMP + CUDA)

- Aggregation of small BLAS operations into larger ones to hide long-latency operations resulted in 2.7X faster performance and 5X reduction in memory costs on 100-node GPU cluster

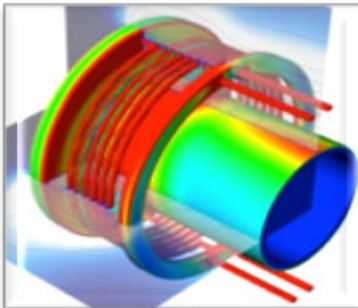
Linear solver kernels (Pthreads and OpenMP)

- Introduction of a thread communicator allows passing this information among libraries for portable performance with maintainable kernels

Time integrators in SUNDIALS (MPI + OpenMP)

- New threaded kernels and integration with SuperLU\_MT provide speed up and flexibility

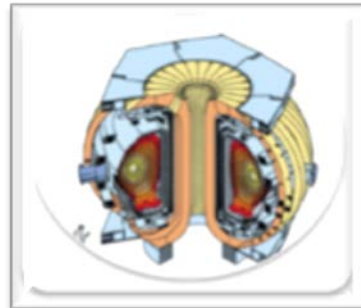
FASTMath technologies leverage Applied Math base



Examples:  
Algorithms in Chombo, BoxLib, PETSc, SUNDIALS, hypr, ML and Mesquite

Image courtesy of hypr Project

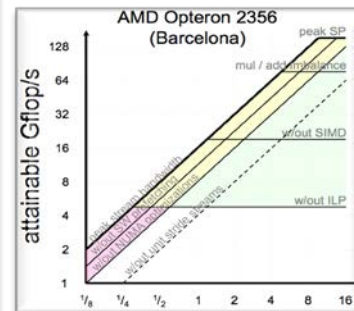
Driving requirements in the Applied Math base program



Example: Fusion collaboration led to a new base math project in high-order discretization methods

Image courtesy of ESL Project

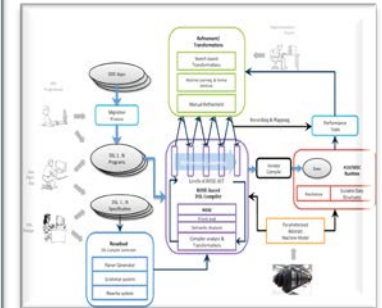
New joint math/cs projects in Exascale solvers initiative



Example: Discussions between FASTMath and Super led to ExReDi Project

Image courtesy of Sam Williams, LBNL

Use of FASTMath capabilities in base CS projects



Example: DTEC X-Stack project is exploring use of domain specific languages in Chombo


Image courtesy of DTEC Project



# The FASTMath team includes experts from four national laboratories and six universities


 Lawrence Berkeley National Laboratory


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 (UC Berkeley)






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**2. FASTMath Unstructured Mesh Technologies**

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**3. FASTMath Partitioning and Task Placement**

POC: Karen Devine, SNL

**4. FASTMath Iterative Solver Technologies**

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**5. FASTMath Direct Solver Technologies**

POC: Sherry Li, LBNL

**6. FASTMath Nonlinear and ODE Solver Technologies**

POC: Carol Woodward, LLNL

**7. FASTMath Eigensolver Technologies**

POC: Esmond Ng, LBNL

**8. FASTMath Unstructured Mesh/Solver Interactions**

POC: Vijay Mahadevan, ANL

**9. Construction of Explicit (PETSc) sparse matrices from (Chombo) AMR Hierarchies**

POC: Mark Adams, LBNL

**10. FASTMath Component-Based Unstructured Mesh Simulation Workflows**

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<http://www.fastmath-scidac.org>