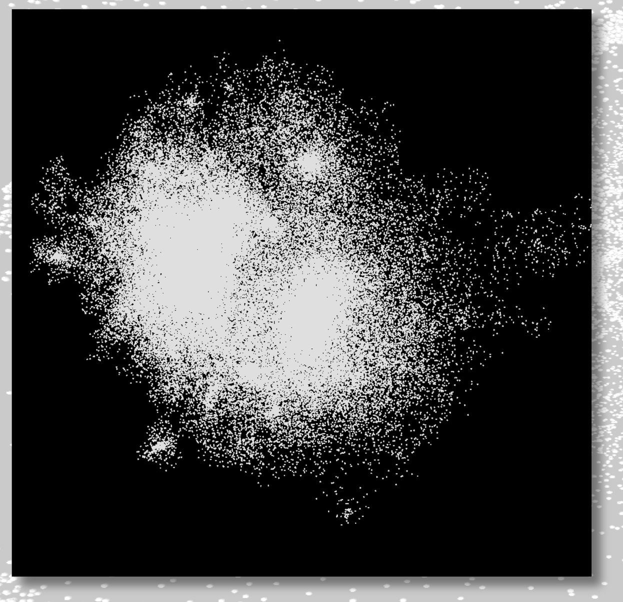
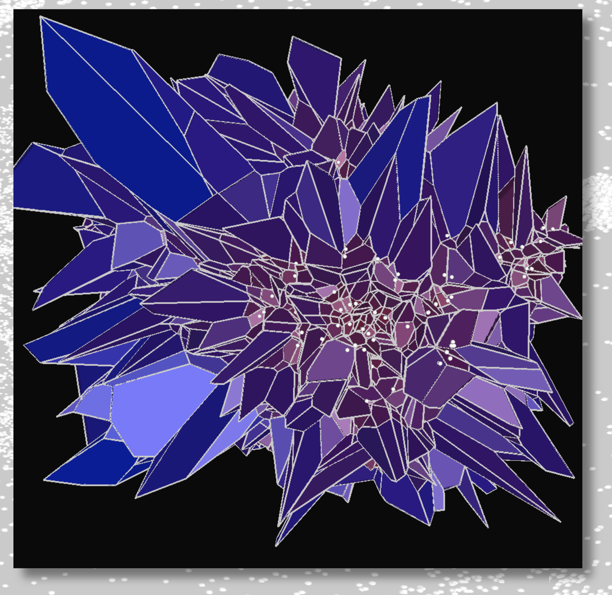
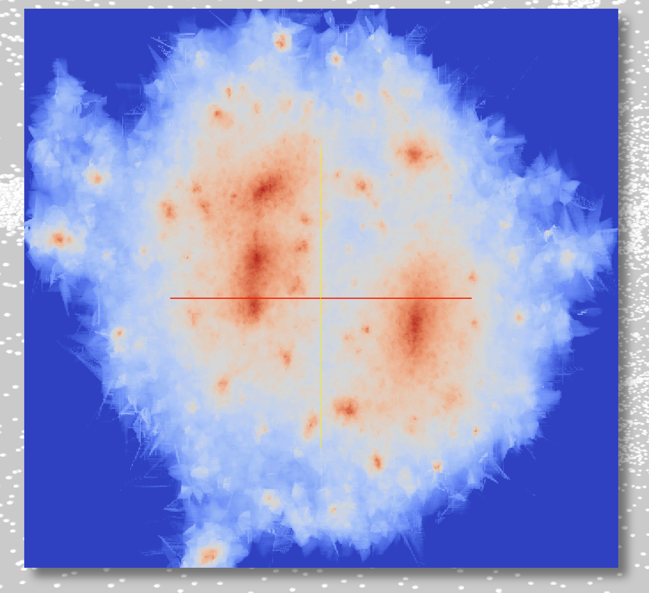
Meshing the Universe: In Situ Voronoi and Delaunay Tessellation

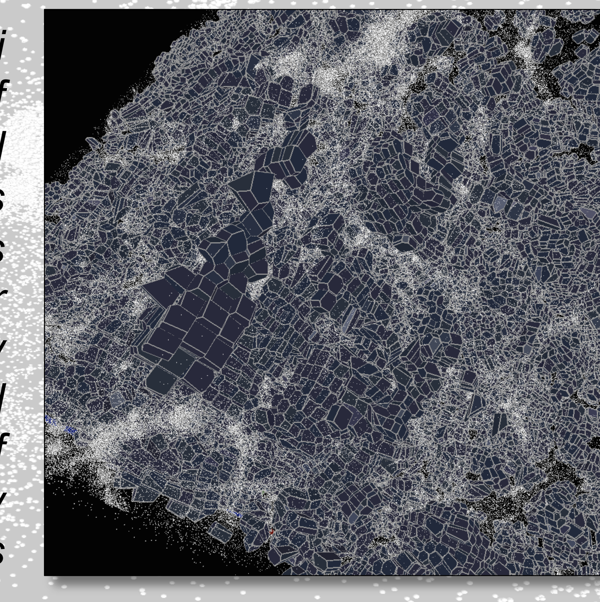




Three representations of the same halo. From left to right: original raw particle data, Voronoi tessellation, and regular grid density sampling.

Overview

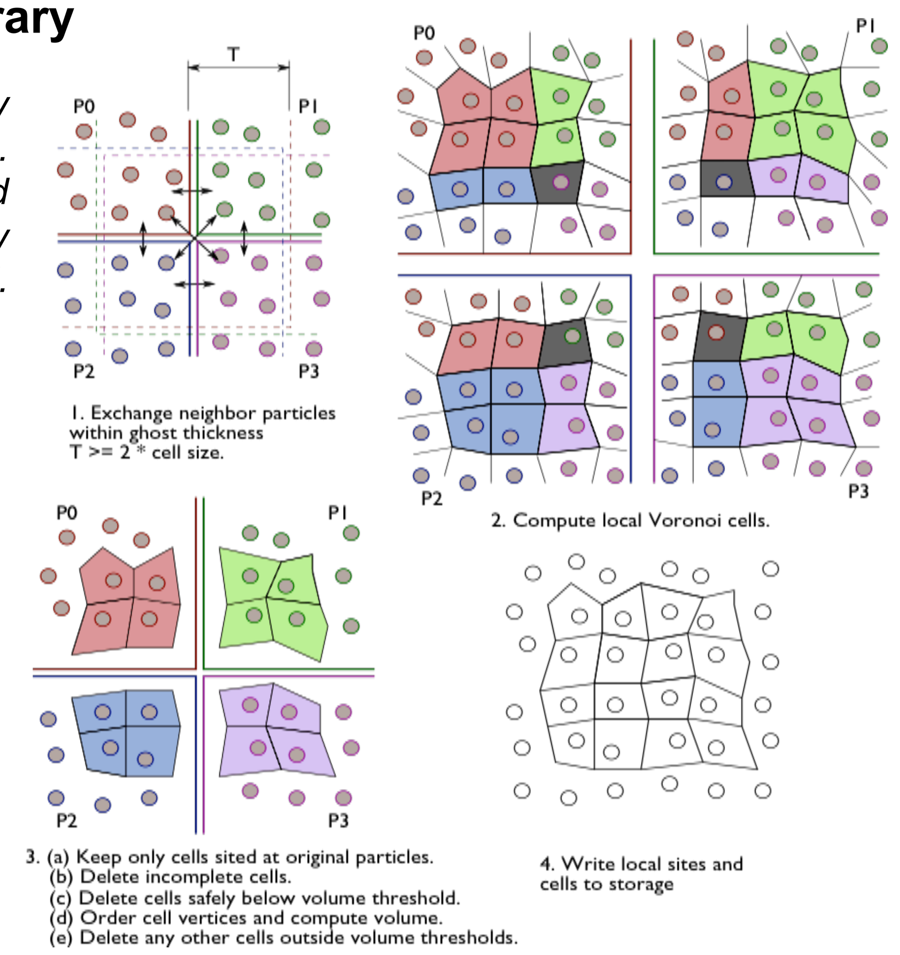
SDAV technologies aim to help cosmologists unravel the mysterious nature of dark matter and energy by transforming raw data into meaningful representations. For example, mesh tessellations help analyze point data because they transform sparse discrete samples into dense continuous functions. Similarly, large-scale structures such as halos and voids are extracted, tracked, and summarized in high-level models. The goal of SDAV's partnership with computational cosmology is to bring such methods to extreme scale.

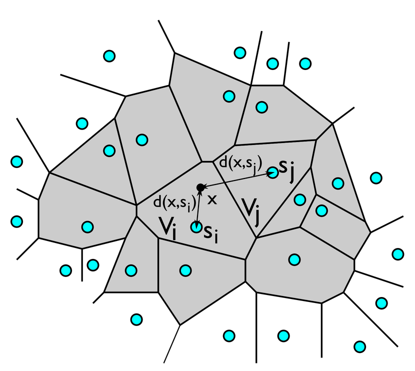
Voronoi tessellation of cosmological simulations reveals regions of irregular low-density voids amid clusters of high-density halos



Tess: In Situ Tessellation Library

Right: Overview of parallel algorithm by using an example with four processes. Particles and Voronoi cells are colored according to the process where they originated.





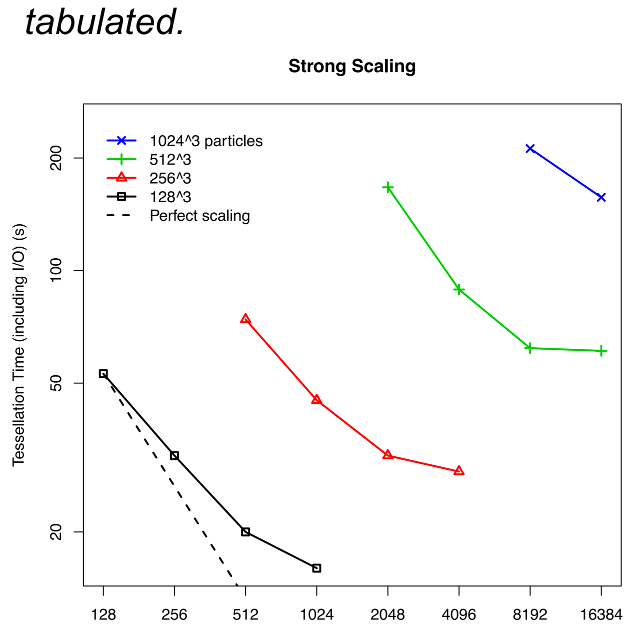
Above: Each Voronoi cell is associated with one input particle, the site of the cell. A cell consists all points closer to the site of that cell than to any other site

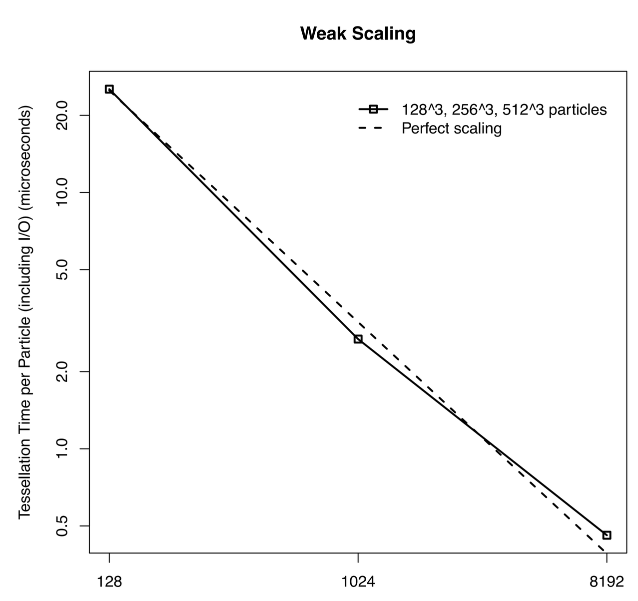
Vi = { x | d(x, si) < d(x, sk) } ∀ k≠ i

In 3D, Voronoi cells are polyhedra; dual is Delaunay tetrahedralization.

Particles	Time steps	Processes	Tot Time (s)	Sim Time (s)	Tess Tot Time (s)	Exchange Time	Tess Time (s)	Output Time (s)	Output Size(GB)
128^3	100	128	1862	1809	53	1	50	2	0.3
		256	1354	1322	32	1	29	2	
		512	1116	1096	20	1	17	2	
		1024	745	729	16	1	14	3	
256^3	100	512	3090	3016	74	2	69	3	1.7
		1024	2391	2346	45	2	39	4	
		2048	1861	1830	32	2	26	4	
		4096	1334	1305	29	2	15	12	
512^3	50	2048	3852	3684	167	4	157	6	14
		4096	2008	1918	89	3	77	9	
		8192	1784	1722	62	3	48	11	
		16384	1406	1344	61	2	32	27	
1024^3	25	8192	2331	2119	212	6	186	20	101
		16384	1446	1289	157	4	113	40	

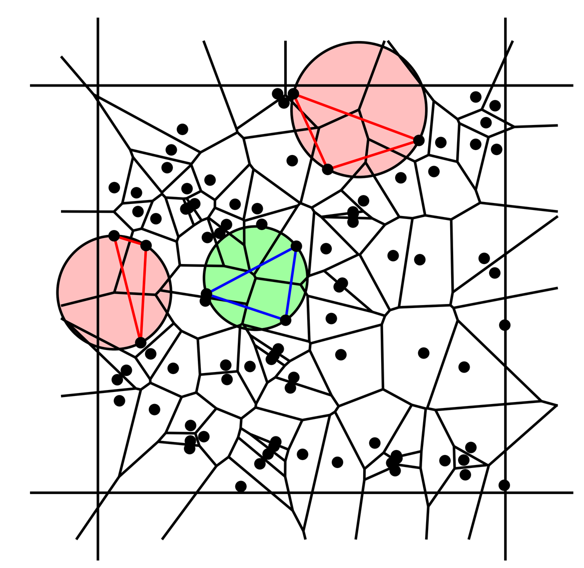
Bottom: In situ strong scaling (left) and weak scaling (right) are plotted on a log-log scale. Weak scaling time is normalized by the number of particles. Plots represent the total tessellation time, including the time to write the result to storage. Strong scaling efficiency is 41%; weak scaling efficiency is 86%. Right: raw performance is tabulated.





Algorithm

1. Compute local Voronoi tessellation.
2. Examine all circumspheres and identify those that extend past region boundaries.
3. Exchange points that support the circumspheres + points on the convex hull.
4. Update the local diagram with newly received points.
5. Exchange all points on the convex hull that are not finalized.
6. Finalize the Voronoi tessellation.

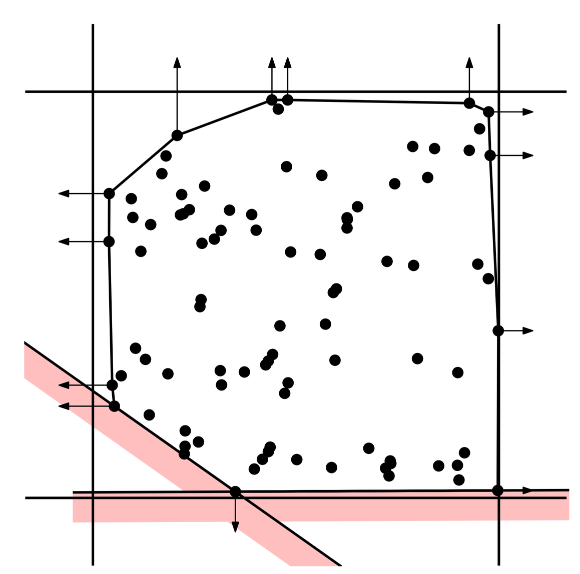


Circumspheres: the green empty circumsphere is contained entirely inside the local region and, therefore, cannot change (the dual triangle is guaranteed to be in the Delaunay triangulation); the red circumspheres extend past the local region, their dual triangles are not finalized.

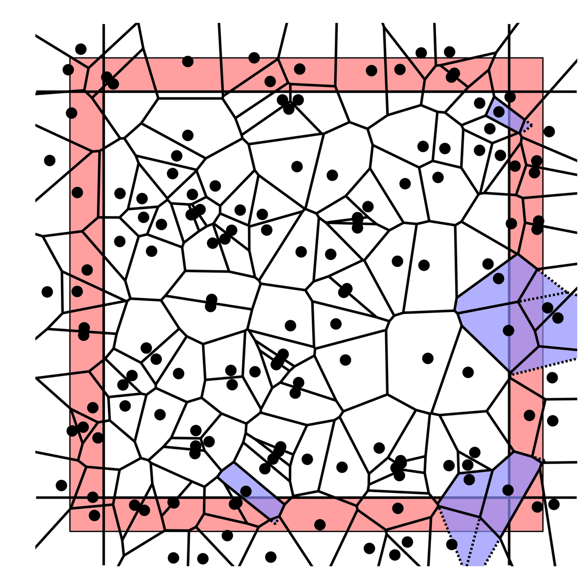
Points on the **convex hull** require special attention. Three solutions:

1. Send them everywhere;
2. Use supported halfspaces (see figure below); the halfspaces are the limits of circumspheres;
3. **Use a heuristic**: send to nearest neighbors, followed by a second phase, where still active points get sent everywhere.

Key idea to reduce communication: we are affected by a point in a neighbor if and only if they are affected by our point. Therefore, we do not need to request points, all the destinations can be determined locally.

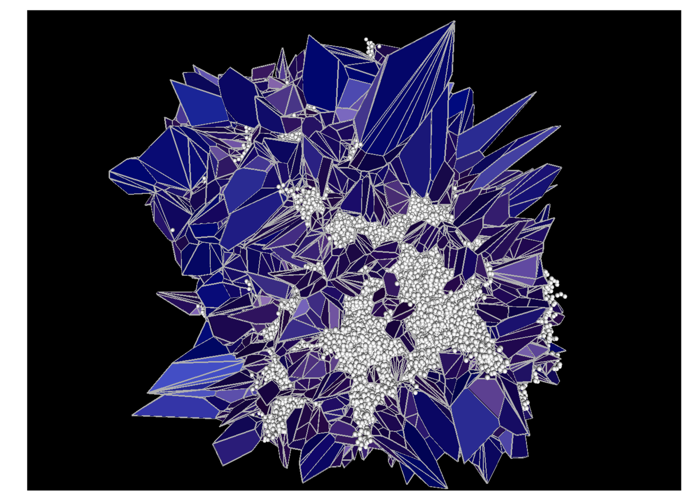


Points on the convex hull, the external halfspaces they support (pink), and their nearest neighbor blocks.

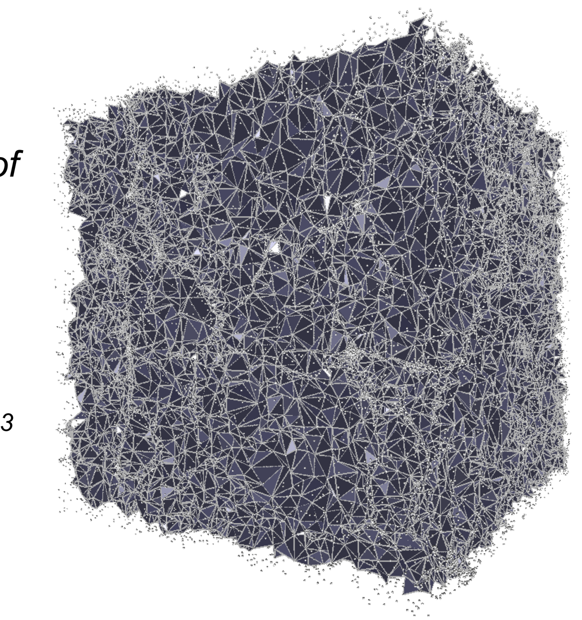


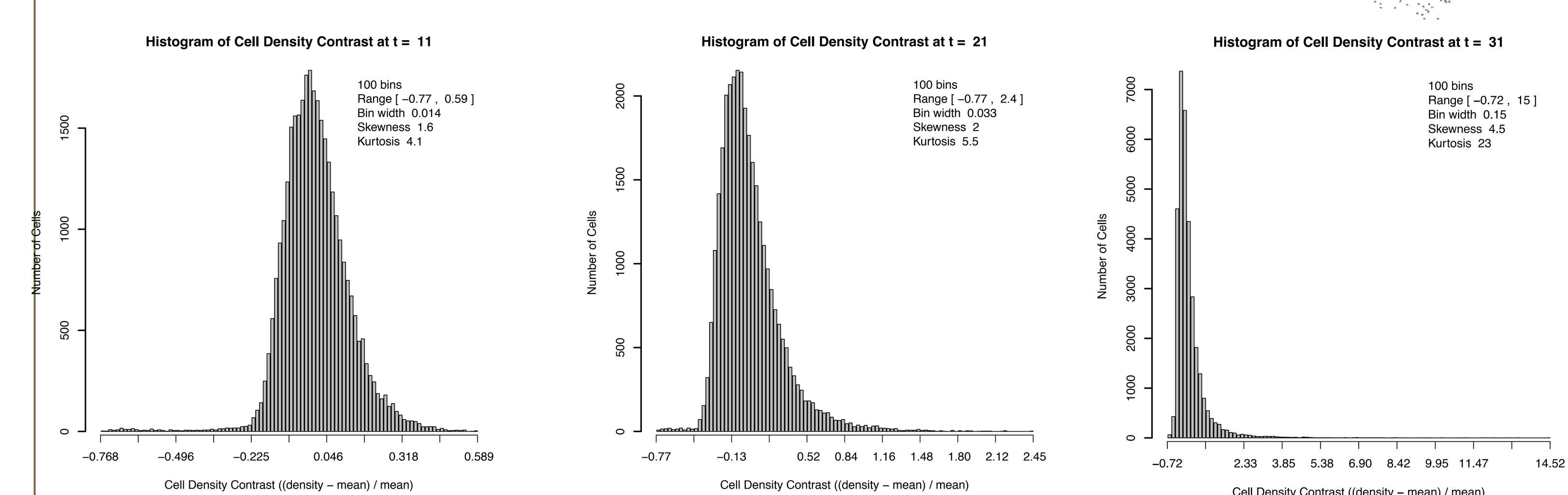
Problem with the old approach: fixed radius ghost regions do not guarantee correctness for too small radius; create large overhead for too large radius.

Cosmology Applications

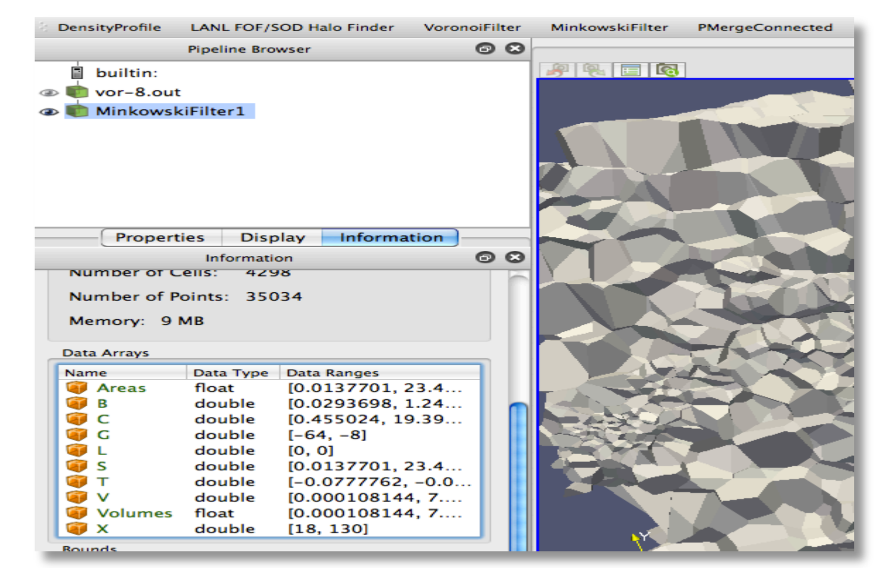


Left: Tessellation of halo 6606356352 shows substructures inside the halo. Right: Delaunay tessellation of 128^3 dark matter tracer particles.

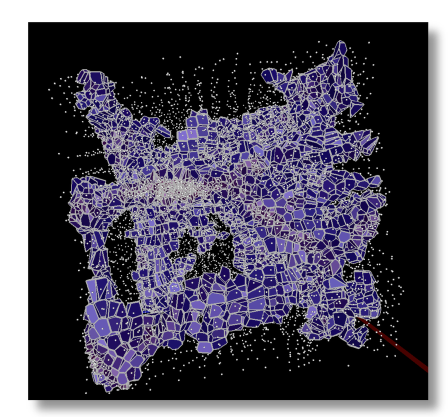




Above: Density contrast distribution of evolving Voronoi cells at three time steps statistically are consistent with the formation of large-scale cosmological structures.

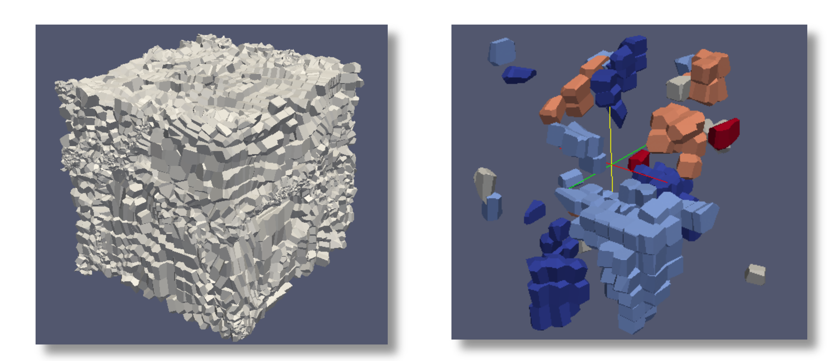


Above: Cosmology tools plugin in ParaView promotes interactive feature exploration.



Above: Thresholding cell volume reveals large-scale structures such as halos, voids, filaments, and walls.

Right: Connected components of Voronoi cells that have been filtered on cell volume are further characterized.



SDAV
Scalable Data Management, Analysis, and Visualization

Tom Peterka (ANL), Dmitriy Morozov (LBNL), Juliana Kwan, Adrian Pope, Hal Finkel, Katrin Heitmann, Salman Habib (ANL), George Zagaris, Berk Geveci (Kitware), Wei-keng Liao, Alok Choudhary (NU)