The magnetic moment of the muon

\[ \vec{\mu} + \text{JPsi Mcore-hr} \]

at BNL

Work on \( g-2 \) done in collaboration with

Collaborators

Christopher Aubin (Fordham U)
Saumitra Chowdhury (UConn)
Maarten Golterman (FSU)
Massashi Hayakawa (Nagoya)
Santiago Peris (Barcelona)
Taku Izubuchi (BNL/RCBC)
Christoph Lehner (BNL)

RBC/UKQCD Collaboration
Norikazu Yamada (KEK)
Norman Christ (Columbia)
Luchang Jin (Columbia)

The magnetic moment of the muon

\[ V(\vec{\mu}) = -\vec{\mu} \cdot B \cdot \vec{\mu} \]

The magnetic moment \( \vec{\mu} \) is proportional to its spin \( (c = h = 1) \)

\[ \vec{\mu} = g \left( \frac{e}{2m} \right) S \]

The Landé \( g \)-factor is predicted from the free Dirac eq. to be

\[ g = 2 \]

for elementary spin-1/2 fermion

The magnetic moment of the muon

In interacting quantum (field) theory \( g \) gets corrections

\[ g - \frac{2}{3} = \alpha \]

the anomalous magnetic moment, or anomaly

The magnetic moment of the muon

Compute corrections in pert. theory in QED coupling constant

\[ \alpha = \frac{e^2}{4\pi} = \frac{1}{137} + \ldots \]

The magnetic moment of the muon

New experiments + new theory = (? ) new physics

Muon anomaly \( \alpha_g \) provides the most important test of the SM

\[ \alpha_g(\text{Expt}) - \alpha_g(\text{SM}) = 287(63)[51] \times (10^{-11}) \]

or \( \sim 3.6 \sigma \)

\[ 249(87) \times (10^{-11}) \]

or \( \sim 2.9 \sigma \)

Big discrepancy! (New Physics \( \sim 2 \times \) Electroweak)

Theory must improve too! Hadronic (QCD) contributions dominate theory error

Lattice QCD calculations crucial

The hadronic light-by-light amplitude

Model estimates: \( (10 - 12) \times 10^{-10} \) with a 25-40% uncertainty (difficult to quantify)

Lattice calculation: model independent, approximations (non-zero \( \alpha \), finite \( V \), ...) systematically improvable

Compute directly on lattice, using QCD and QED

Dominated by quark propagators, inverse of large, sparse matrix. Use conjugate gradient (CG).

All Mode Averaging (AMA)

\[ f(x) \]

accuracy control:

- low mode part: \# of eig-mode
- mid-high mode: degree of poly

Speedup and memory requirements for AMA

HLLBL calculation: RBC/UKQCD 2+1f DWF ensemble

\[ m_\pi = 329 \text{ MeV} \]

\[ (24^3 \times 64 / 2 \times 64) \times (2 \times 3 \times 4) \times 4 \times 400 \approx 272 \text{ GB} \]

Approximation: 400 low modes, 10^6 stop res. (exact: 10^-11)

Error completely dominated by approximation, \( N_g = 6^2 = 36 \) propagators, 4 exact propagators

No direct test, but similar nucleon calculation was \( \approx 10^5 \) less expensive

HLBL contribution from lattice QCD+QED using AMA

Stat. errors only, lowest non-trivial momentum

Several source/sink separations for muon

Checked in pure QED

\( m_\pi = 329 \text{ MeV} \)

Statistical error: \( \approx 2 \times \) QCD error

Significant excited state contamination

Model value/error is "Glasgow Consensus"

USQCD projects using AMA

FNAL Clusters (400 J-Psi Mcore-hours allocated in total)

\begin{tabular}{|c|c|}
  \hline
  PI & J-Psi Mcore-hr \\
  \hline
  Aubin & 32 \\
  Ishikawa & 26 \\
  Izubuchi & 11 \\
  Soni & 47 \\
  Syritsin & 8 \\
  Witzel & 134 \\
  \hline
\end{tabular}

ANL BG/ Q

Many groups now using AMA: BMWc, Ke-computer, Mainz, RBC-UKQCD, ...

Summary

- Muon anomalous magnetic moment is measured precisely in exp. (BNL E821)

- In potential disagreement with Standard Model

- Going to be measured even more precisely (FNAL E989, J-PARC E34) in ~ 2 years

- Theory error dominated by QCD corrections

- Lattice QCD calculations provide systematically improvable results with controlled errors

- Combination may lead to discovery of NEW PHYSICS