All mode averaging and calculations of the muon anomalous magnetic moment, $(g-2)_\mu$

Tom Blum (UConn/RBRC) and Taku Izubuchi (BNL/RBRC) for the USQCD Collaboration

Collaborators

Work on g-2 done in collaboration with

HVP	HLbL
Christopher Aubin (Fordham U)	Saumitra Chowdhury (UConn)
Maarten Golterman (SFSU)	Masashi Hayakawa (Nagoya)
Santiago Peris (Barcelona)	Taku Izubuchi (BNL/RBRC)
	Christoph Lehner (BNL)
RBC/UKQCD Collaboration	Norikazu Yamada (KEK)
	Norman Christ (Columbia)
	Luchang Jin (Columbia)

The magnetic moment of the muon

Interaction of particle with static magnetic field

$$V(\vec{x}) = -\vec{\mu}\cdot\vec{B}_{\rm ext}(\vec{x})$$

The magnetic moment $\vec{\mu}$ is proportional to its spin ($c=\hbar=1$)

$$\vec{\mu} = g\left(\frac{e}{2m}\right)\vec{S}$$

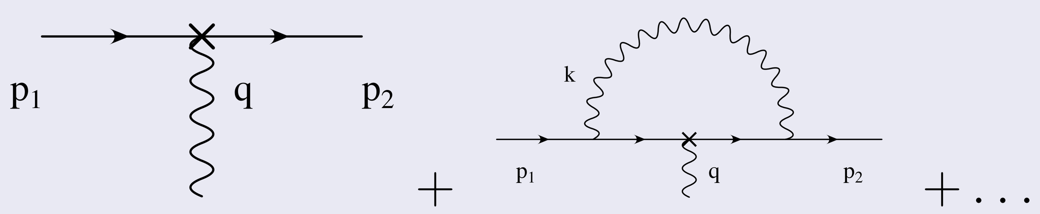
The Landé g-factor is predicted from the free Dirac eq. to be

$$g = 2$$

for elementary spin-1/2 fermion

The magnetic moment of the muon

In interacting quantum (field) theory g gets corrections



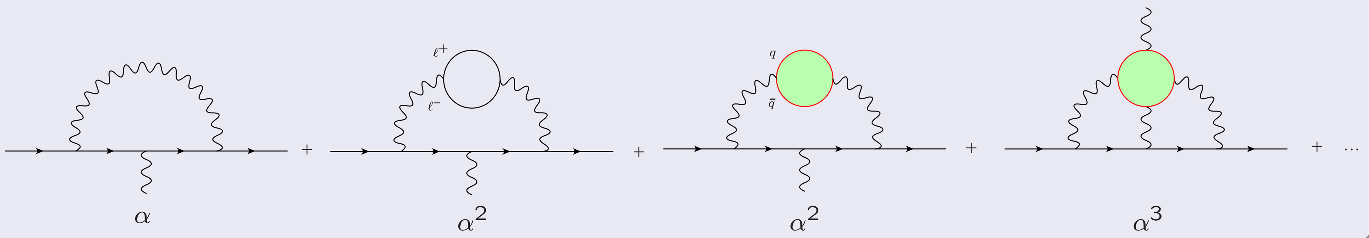
$$\frac{g-2}{2} \equiv a_\mu$$

the anomalous magnetic moment, or anomaly

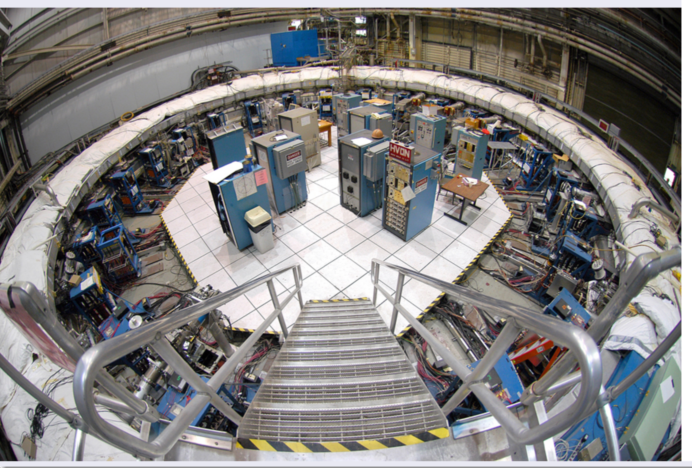
The magnetic moment of the muon

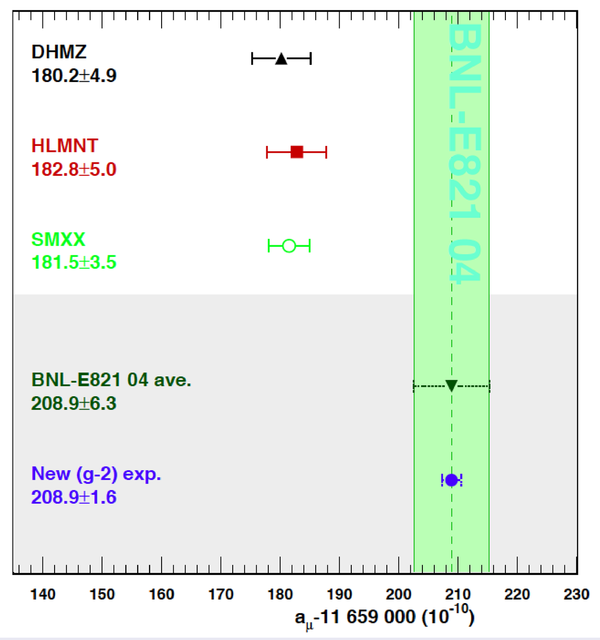
Compute corrections in pert. theory in QED coupling constant

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137} + \ldots$$



E821 at BNL $a_\mu^{\rm exp} = 116\,592\,089(63)\times 10^{-11}$ (0.54 ppm)





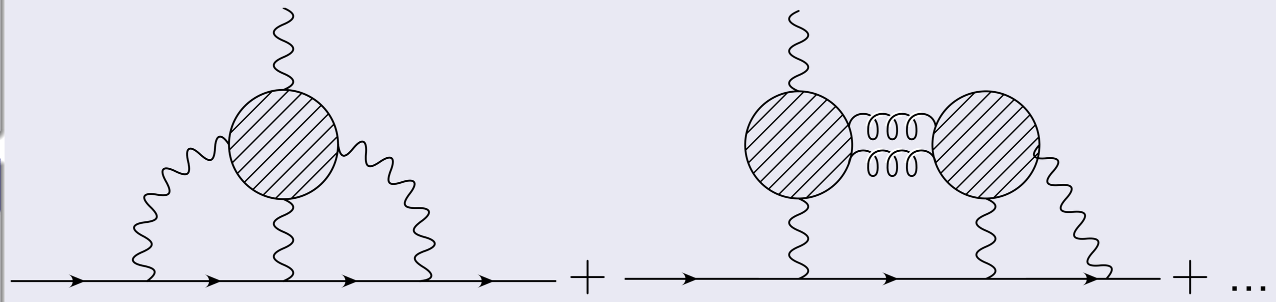
Fermilab E989, J-PARC E34, Goal: 0.14 ppm

New experiments + new theory = (?) new physics

Muon anomaly a_μ provides (the most) important test of the SM

- a_μ(Expt)-a_μ(SM) $= 287(63)(51)\ (\times 10^{-11})$, or $\sim 3.6\sigma$
  to $249(87)\ (\times 10^{-11})$, or $\sim 2.9\sigma$
- Big discrepancy! (New Physics $\sim 2\times$ Electroweak)
- Theory must improve too. Hadronic (QCD) contributions dominate theory error
- Lattice QCD calculations crucial

The hadronic light-by-light amplitude



- Model estimates: about $(10-12)\times 10^{-10}$ with a 25-40% uncertainty (difficult to quantify)
- Lattice calculation: model independent, approximations (non-zero a, finite V, ...) systematically improvable
- Compute directly on lattice, using QCD and QED
- Dominated by quark propagators, inverse of large, sparse matrix. Use conjugate gradient (CG).

All Mode Averaging (AMA)

[Blum, Izubuchi, Shintani PRD88 (2013) 9, 094503, arXiv:1208.4349]

$$\mathcal{O} = \mathcal{O}^{\rm (appx)} + \mathcal{O}^{\rm (rest)}$$

Original → unbiased improved

Lattice Symmetry

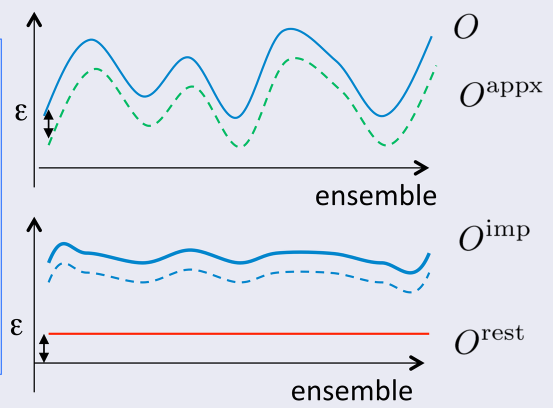
$$\mathcal{O}^{\rm (imp)} = \mathcal{O}^{\rm (rest)} + \frac{1}{N_G}\sum_{g\in G}\mathcal{O}^{{\rm (appx)},g}$$

Expensive : infrequently measured

Cheap : frequently measured

- $O^{(appx)}$: cheap & not to be too accurate
- N_G average suppresses the bulk part of noise cheaply

→ $O^{(imp)}$ has smaller error



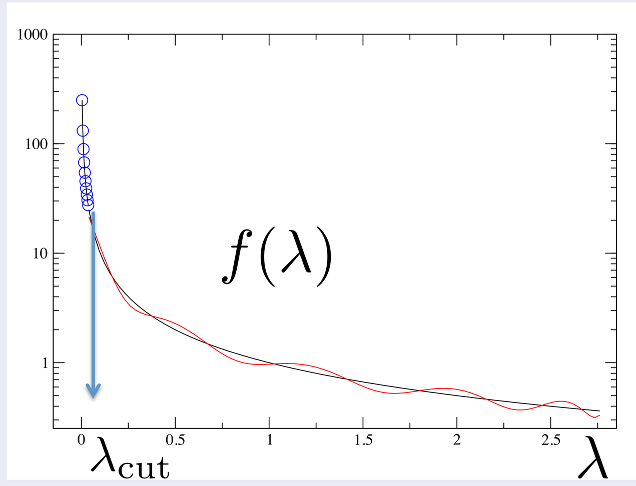
All Mode Averaging (AMA)

- Quark propagator is approximated by Sloppy Conjugate Gradient,

i.e. a crude polynomial approximation of inverse Dirac Matrix

$$\mathcal{O}^{\rm (appx)} = \mathcal{O}[S_l],$$
$$S_l = \sum_\lambda v_\lambda f(\lambda) v_\lambda^\dagger,$$
$$f(\lambda) = \begin{cases}\frac{1}{\lambda}, & |\lambda|<\lambda_{\rm cut}\\ P_n(\lambda) & |\lambda|>\lambda_{\rm cut}\end{cases}$$
$$P_n(\lambda) \approx \frac{1}{\lambda}$$

If quark mass is heavy, e.g. ~ strange, low mode isolation may be unnecessary

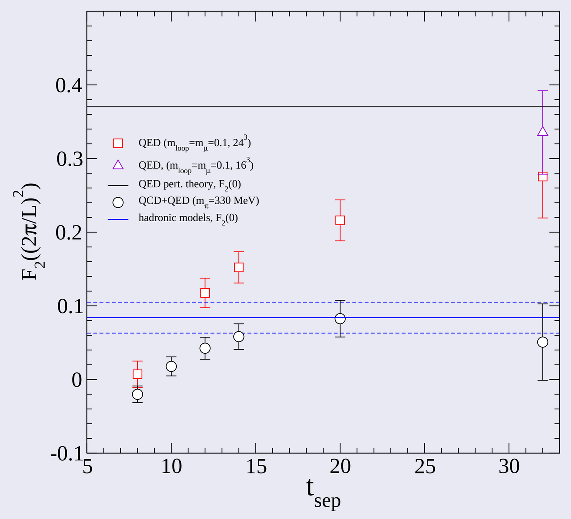


accuracy control :
- low mode part : # of eig-mode
- mid-high mode : degree of poly.

Speedup and memory requirements for AMA

- HLbL calculation: RBC/UKQCD 2+1f DWF ensemble
  - $m_\pi = 329$ MeV
  - $(24^3\times 64/2\times 16)*(2\times3\times4)*4*400 \approx 272$ GB
  - Approximation: 400 low modes, 10^{-4} stop res. (exact: 10^{-10})
  - Error completely dominated by approximation, $N_g = 6^3 = 216$ propagators, 4 exact propagators
  - No direct test, but similar nucleon calculation was $16\times$ less expensive

HLbL contribution from lattice QCD+QED using AMA



- Stat. errors only, lowest non-trivial momentum
- Several source/sink separations for muon
- Checked in pure QED
- $m_\pi = 329$ MeV

Significant excited state contamination

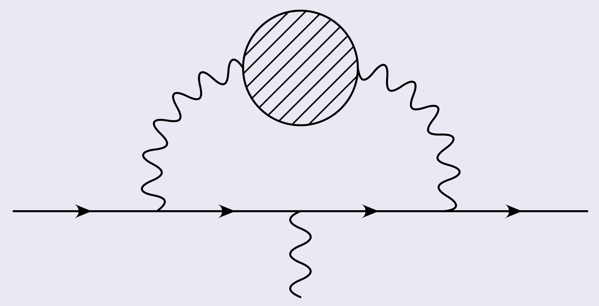
Model value/error is "Glasgow Consensus"

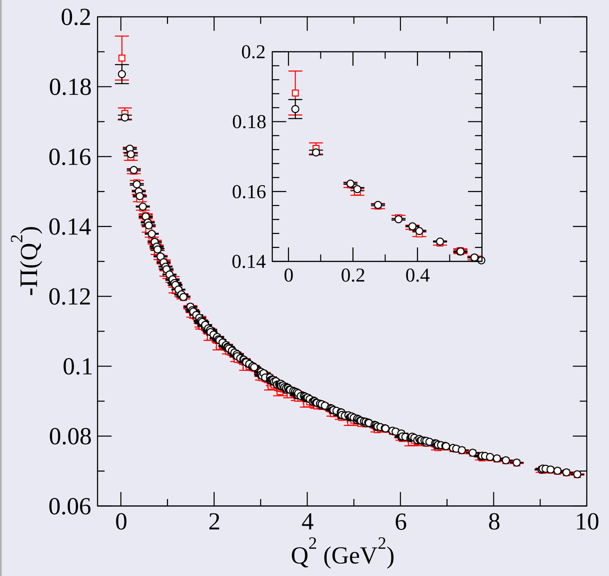
(arXiv:0901.0306 [hep-ph])

Speedup and memory requirements for AMA

- Next HLbL calculation: RBC/UKQCD 2+1f DWF ensemble
  - $m_\pi = 170$ MeV
  - $(32^3\times 64/2\times 16)*(2\times3\times4)*4*1000 \approx 1.6$ TB
  - Approximation: Möbius Dirac op, 1000 low modes
  - based on nucleon calculation, $\sim 110\times$ less expensive
- Next-to-Next HLbL calc: RBC/UKQCD 2+1f Möbius-DWF
  - $m_\pi = 140$ MeV
  - $(48^3\times 96/2\times 24)*(2\times3\times4)*4*1000 \approx 12$ TB
  - current nucleon run: 500 eigenvectors, 400 iters, $\approx$ 40-50$\times$ less expensive
  - Approx.: 1000 low modes, 200-400 CG iters (exact: 20k), $\approx 100\times$ less expensive (or better)
- Factor of 2 reduction in size possible using further tricks for the approx. (MADWF preconditioning or smaller L_s, ...)

LO hadronic vacuum polarization contribution



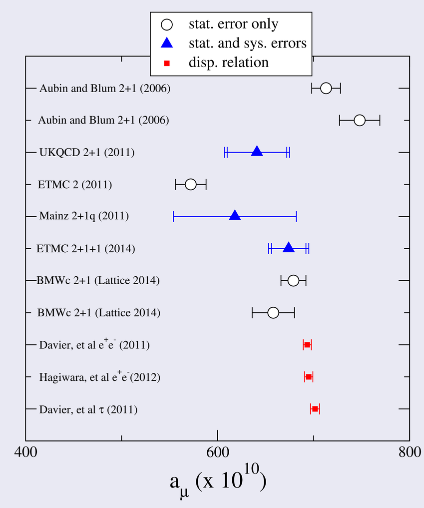


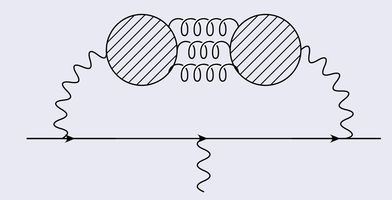
- 2+1f improved staggered fermions
- $m_\pi = 315$ MeV
- $48^3\times 96$ lattice size
- 1400 low modes, CG 1×10^{-4} stopping residual
- $\approx 12\times$ less expensive
- expect 20-50$\times$ better for lighter quarks
- more for DWF

Blum, Izubuchi, Shintani, Phys. Rev. D88 (2013)

hadronic vacuum polarization contribution status

- Statistical error: → AMA, smart subtractions
- Finite volume effect (small $q^2\sim m_\mu^2$): → Twisted Boundary Condition, Twist averaging (C. Lehner, T.Izubuchi 2014), Fit/ansatz (Pade, coordinate space moments,...)
- Physical mass ensemble: → RBC/UKQCD, BMWc, ETMC...
- Disconnected diagrams





USQCD projects using AMA

FNAL Clusters (400 JPsi Mcore-hours allocated in total)

PI	JPsi Mcore-hr
Aubin	32
Ishikawa	10
Izubuchi	26
Soni	11
Syritsin	47
Witzel	8
	134

ANL BG/Q

PI	JPsi Mcore-hr
Mawhinney	194

AMA increasingly effective as m_π decreases

AMA is diplomatic: Can combine with other algorithms: A2A, distillation, domain decomposition, hierarchical deflation, multi-grid, ...

Many groups now using AMA: BMWc, Kei-computer, Mainz, RBC-UKQCD, ...

Summary

- Muon anomalous magnetic moment is
  - measured precisely in exp. (BNL E821)
  - in potential disagreement with Standard Model
  - going to be measured even more precisely (FNAL E989, J-PARC E34) in ~ 2 years
- Theory error dominated by QCD corrections
- Lattice QCD calculations provide systematically improvable results with controlled errors
- Combination may lead to discovery of NEW PHYSICS