

### U.S. DOE SciDAC Partnership **Predictive Computing for Condensed Matter**

Bryan Clark (Physics)

So Hirata (Chemistry) Peter Abbamonte (Physics) Garnet Chan (Chemistry)<br>David Ceperley (Physics) Shiwei Zhang (Physics) Shinsei Ryu (Physics) Lucas Wagner (Physics)



# **Frontiers of predictive computing**



A DOE report on Computational Materials Science (2010):

*Characterization, and modeling of complex materials and chemical <i>characterization, and <b>modeling* of *and all and all a* processes will transform our ability to understand and design new **processes** *"We are at the threshold of a new era where the integrated synthesis, materials and chemistries with predictive power"*

 $Ani$ quest $^n$ Materials Genome Initiative for Global Competitiveness (2011): *"the development of advanced materials can be accelerated through advances in computational techniques"*

# **The team**

### So Hirata (Chemistry, UIUC) - Lead PI

Hirata is a theoretical/computational chemist and an expert in electron-correlation theories for molecules and solids. He is the primary author of the computer-generated, high-rank electroncorrelation modules in DOE's massively parallel NWCHEM suite of software, implemeting several of his original methods.

### Peter Abbamonte (Physics, UIUC)

Abbamonte, an experimental condensed-matter physicist, brings an invaluable experimental insight into the project. He is one of the originators of resonant soft x-ray scattering, with which he discovered a Wigner crystal in doped spin ladders and the charged stripes in copper-oxide superconductors.

#### Garnet K.-L. Chan (Chemistry, Princeton)

Chan is a recognized expert in strong correlation theories including density matrix renormalization group (DMRG), tensor networks, and density matrix functional theory. He has, in particular, established DMRG as a practical, powerful tool for strongly correlated molecular electronic structures.

#### Shinsei Ryu (Physics, UIUC)

Ryu specializes in mathematical theories of strong correlation and other condensed phase electronic structures. Ryu is a pioneer in the use of entanglement entropy in classifying topological phases of matter and has predicted the fractional topological insulator in two dimension.





Wagner is the principal author of the quantum Monte Carlo program, QWALK, with which he has performed predictively accurate calculations on strongly correlated systems.





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Ceperley is a theoretical/computational physicist and an authority of quantum Monte Carlo (OMC). He invented a number of QMC algorithms and is the author of massively parallel QMCPACK software.



#### Bryan Clark (Microsoft Station Q)

Clark has considerable experience in both conventional quantum Monte Carlo (QMC) and novel extensions such as QMC in the Hilbert space. He has developed a large-scale parallel algorithm of QMC in  $PIMC++$ .



#### Shiwei Zhang (Physics, W&M)

Zhang specializes in computational condensed matter physics and materials science. Zhang is a pioneer in the use of quantum Monte Carlo (QMC) in the Hilbert space for strong correlation. He is the inventor of the phaseless auxiliary field QMC.





### **Theories**

(Finite-*T*) MP, CC, RPA, etc.; QMC; DMRG, TN; FCIQMC, AFQMC, MC-MP2; DMFT

### **Software**

Algorithm redesign; massive parallelism; fast integrals

### **Applications – experimental verification**

Metals; (copper oxide) superconductors; graphene; molecular crystals; Peierls systems and Luttinger liquid; Mott-Hubbard systems; spin liquid



Abbamonte



**Schrödinger equation**  
\nfor water  
\n
$$
\frac{\partial}{\partial t}
$$
\n
$$
\left(-\frac{\hbar^2}{2m_e}\sum_{i=1}^{10}\nabla_i^2 - \frac{\hbar^2}{2m_f}\sum_{l=1}^{3}\nabla_l^2 - \sum_{l=1}^{3}\sum_{i=1}^{10}\frac{Z_ie^2}{4\pi\epsilon_0 r_{il}} + \sum_{i=1}^{9}\sum_{j=i+1}^{10}\frac{e^2}{4\pi\epsilon_0 r_{ij}} + \sum_{l=1}^{2}\sum_{j=l+1}^{3}\frac{Z_lZ_j e^2}{4\pi\epsilon_0 r_{lj}}\right)\Psi = E\Psi
$$
\n
$$
\frac{\partial^2}{\partial r_i^2} + \frac{2}{r_i}\frac{\partial}{\partial r_i} + \frac{1}{r_i^2}\left(\frac{1}{\sin^2\theta_i}\frac{\partial^2}{\partial \phi_i^2} + \frac{1}{\sin\theta_i}\frac{\partial}{\partial \theta_i^2}\sin\theta_i\frac{\partial}{\partial \theta_i^2}\right)
$$
Singularities

Conditions arising from the indistinguishability of electrons

$$
\Psi(\mathbf{r}_{e1}, \mathbf{r}_{e2}, \mathbf{r}_{e3}, \mathbf{r}_{e4}, \mathbf{r}_{e5}, \mathbf{r}_{e6}, \mathbf{r}_{e7}, \mathbf{r}_{e8}, \mathbf{r}_{e9}, \mathbf{r}_{e10}, \mathbf{r}_{H1}, \mathbf{r}_{H2}, \mathbf{r}_{O}) =
$$
\n
$$
-\Psi(\mathbf{r}_{e2}, \mathbf{r}_{e1}, \mathbf{r}_{e3}, \mathbf{r}_{e4}, \mathbf{r}_{e5}, \mathbf{r}_{e6}, \mathbf{r}_{e7}, \mathbf{r}_{e8}, \mathbf{r}_{e9}, \mathbf{r}_{e10}, \mathbf{r}_{H1}, \mathbf{r}_{H2}, \mathbf{r}_{O}) =
$$
\n
$$
\Psi(\mathbf{r}_{e3}, \mathbf{r}_{e1}, \mathbf{r}_{e2}, \mathbf{r}_{e4}, \mathbf{r}_{e5}, \mathbf{r}_{e6}, \mathbf{r}_{e7}, \mathbf{r}_{e8}, \mathbf{r}_{e9}, \mathbf{r}_{e10}, \mathbf{r}_{H1}, \mathbf{r}_{H2}, \mathbf{r}_{O}) =
$$
\n
$$
-\Psi(\mathbf{r}_{e3}, \mathbf{r}_{e2}, \mathbf{r}_{e1}, \mathbf{r}_{e4}, \mathbf{r}_{e5}, \mathbf{r}_{e6}, \mathbf{r}_{e7}, \mathbf{r}_{e8}, \mathbf{r}_{e9}, \mathbf{r}_{e10}, \mathbf{r}_{H1}, \mathbf{r}_{H2}, \mathbf{r}_{O}) = \dots
$$

3,628,800 terms!

Many-body

# **Systematic many-body methods**



### **MP2**



# **Systematic many-body methods**





# **Quantum Monte Carlo**

 $\hat{H}\Psi = E\Psi$ 

• Gives the *best* wave function and energy in  $E = \int$  $^{\dagger} \Psi d$ r the variational sense, which is usually nearly exact. **•** Easily and efficiently parallelized. **function** • Nearly zero memory cost. **Requirement as statistical errors, which decay slowly as <b>grable** *O*(*N*−1/2) with the number of MC steps (*N*). • Calculations of structures and properties (including energy bands) and applications to Requ\reme**excited states difficult.**n *g* behaves like the integra<mark>/nd *f*</del></mark>  $H\Psi/\Psi = E$ 

# **Contents**

- Research highlights of 2012-2013
	- QMC for copper oxide superconductors
	- Auxiliary field QMC for band gaps
	- Full CI QMC for excited states
- Monte Carlo MP2 and MP3
- MP2 for molecular crystals: solid-solid phase transition in  $CO<sub>2</sub>$

*Research Highlights of 2012-2013*

### **QMC for copper oxide superconductors**

**Wagner and Abbamonte, to be published (2013)**



**Wagner** 









*For more details, please visit the poster by Lucas Wagner*

*Research Highlights of 2012-2013*

# **Auxiliary field QMC for band gaps**

**Zhang** *et al.***, to be published (2013)**







Zhang



*Research Highlights of 2012-2013*

# **Full CI QMC for excited states**

**Booth and Chan,** *JCP Comm* **(2012)**



**Chan** 

$$
\exp[-\beta(H-E)]
$$

Standard propagator: **max at ground-state**

$$
\exp[-\beta(H-\omega)^2]
$$

Gaussian propagator: **max at specified omega**



**Willow, Kim and Hirata,** *JCP* **(2012),** *JCP* **(2013), under review by** *JCTC* **(2013)**



**Willow, Kim and Hirata,** *JCP* **(2012)** 

**Very long** *O***(***n***4) summation of products of 2 x 6-dimensional integrals**

$$
E^{(2)} = \sum_{i,j}^{\infty} \sum_{a,b}^{\text{virt.}} \frac{\langle ab|ij\rangle\langle ij|ab\rangle}{\langle i,j\rangle_{a,b}^{\text{eff.}}} \\
E\times \text{plicit two-electron integrals}
$$
\n
$$
E^{(2)} = \sum_{i,j}^{\infty} \sum_{a,b}^{\text{virt.}} \frac{\int \varphi_i(\mathbf{r}_1)\varphi_j(\mathbf{r}_2)\frac{1}{r_{12}}\varphi_a(\mathbf{r}_1)\varphi_b(\mathbf{r}_2)d\mathbf{r}_1d\mathbf{r}_2\int \varphi_i(\mathbf{r}_3)\varphi_j(\mathbf{r}_4)\frac{1}{r_{34}}\varphi_a(\mathbf{r}_3)\varphi_b(\mathbf{r}_4)d\mathbf{r}_3d\mathbf{r}_4}{\varepsilon_i + \varepsilon_j - \varepsilon_a - \varepsilon_b}
$$
\nLaplace transformation of the denominator

\n
$$
E^{(2)} = -\sum_{i,j}^{\infty} \sum_{a,b}^{\text{virt.}} \int \varphi_i(\mathbf{r}_1)\varphi_j(\mathbf{r}_2)\frac{1}{r_{12}}\varphi_a(\mathbf{r}_1)\varphi_b(\mathbf{r}_2)d\mathbf{r}_1d\mathbf{r}_2\int \varphi_i(\mathbf{r}_3)\varphi_j(\mathbf{r}_4)\frac{1}{r_{34}}\varphi_a(\mathbf{r}_3)\varphi_b(\mathbf{r}_4)d\mathbf{r}_3d\mathbf{r}_4\int_0^\infty e^{\left(\varepsilon_i+\varepsilon_j-\varepsilon_a-\varepsilon_b\right)\tau}d\tau
$$
\nChange of orders of summations and integrations

\n
$$
E^{(2)} = -\int \cdots \int \int_0^\infty \frac{\sum_{i}^{\infty} \varphi_i(\mathbf{r}_i)\varphi_i(\mathbf{r}_3)e^{\varepsilon_i \sum_{j}^{\infty} \varphi_j(\mathbf{r}_2)\varphi_j(\mathbf{r}_4)e^{\varepsilon_j \sum_{a}^{\infty} \varphi_a(\mathbf{r}_1)\varphi_a(\mathbf{r}_3)e^{-\varepsilon_a \sum_{b}^{\infty} \varphi_b(\mathbf{r}_2)\varphi_b(\mathbf{r}_4)e^{-\varepsilon_b \sum_{b}^{\infty} \varphi_b(\mathbf{r}_4)e^{-\varepsilon_b \sum_{
$$

**Willow, Kim and Hirata,** *JCP* **(2012)** 

$$
E = \int f(x) dx = \int \frac{f(x)}{g(x)} g(x) dx = \sum_{x \in g} \frac{f(x)}{g(x)} \cdot \int g(x) dx
$$
  
\nEquation function Metropolis  
\nRequirement 1: analytically integrable  $\int g(x) dx = N$   
\nRequirement 2: cancellation of singularities  $f(x)/g(x)$   
\n
$$
E^{(2)} = -\int \cdots \int_{0}^{\infty} \frac{G^{-}(r_{1}, r_{3}, r)G^{-}(r_{2}, r_{4}, r)G^{+}(r_{1}, r_{3}, r)G^{+}(r_{2}, r_{4}, r)}{(r_{1}, r_{3}, r_{3})} dr_{1} \cdots dr_{4} dr
$$
\nSingularities  
\n
$$
g(r_{1}, r_{2}, r_{3}, r_{4}) = \frac{\rho(r_{1})\rho(r_{2})}{r_{12}} \frac{\rho(r_{3})\rho(r_{4})}{r_{34}}
$$
\n
$$
\int g(r_{1}, r_{2}, r_{3}, r_{4}) dr_{1} dr_{2} dr_{3} dr_{4} = 4E_{\text{Coulomb}}^{2}
$$

18

**Willow, Kim and Hirata,** *JCP* **(2012)** 



# **Monte Carlo MP2 for self-energies**

**Willow, Kim and Hirata,** *JCP* **(2013)** 



**Dyson self-energies (quasiparticle energies) for multiple states obtainable from a single MC run**

## **Real-space Green's function**

**Willow, Kim and Hirata, to be published (2013)** 



**Willow, Kim and Hirata, to be published (2013)** 





## **Redundant-walker MC-MP2**

**Willow, Hermes, Kim and Hirata, under review by** *JCTC* **(2013)** 



# **Massively parallel MC-MP2**

**Willow, Hermes, Kim and Hirata, under review by** *JCTC* **(2013)** 

 $C_{60}$  cc-pVDZ on 320 processors of Blue Waters



# **MP2 for molecular crystals**





## **Embedded-fragment approach**

**Hirata** *et al***.,** *MP* **(2005); Kamiya, Hirata, and Valiev,** *JCP* **(2008)**

*N***-body (***N* **> 2) Coulomb in point-charge or dipole approximation**

 $E = \sum_{i=1}^{n} E'_{i} + \sum_{i$ 

**Pair energy in the presence of selfconsistent atomic charges or dipoles**

**1 and 2-body Coulomb Exchange Correlation**

## **Molecular crystals**

### **Hirata,** *JCP* **(2008)**

 $\sum_{(0)} + \sum \Big\{ E_{i(0)j(0)} - E_{i(0)} - E_{j(0)} \Big\} + \frac{1}{2} \sum (1 - \delta_{m0}) \sum \Big\{ E_{i(0)j(m)} - E_{i(0)} - E_{j(m)} \Big\}$ ,  $\frac{1}{2} \sum_{i=1}^{+L} (1$ 2 *L*  $\mu(i0)^{-1} \sum_i \left\{ \frac{\mu_i(0)}{j(0)} \cdot \frac{\mu_i(0)}{\mu_i(0)} \right\} \left( \frac{1}{2} \sum_i \left( \frac{\mu_i(0)}{m} \right) \sum_i \left( \frac{\mu_i(0)}{m} \right) \cdot \frac{\mu_i(0)}{\mu_i(0)} \cdot \frac{\mu_i(0)}$ *i i j m L i j*  $E = \sum E_{i(0)} + \sum \{E_{i(0)}\}_{i(0)} - E_{i(0)} - E_{i(0)} \} + \frac{1}{2} \sum (1 - \delta_{m0}) \sum \{E_{i(0)}\}_{i(m)} - E_{i(0)} - E_{i(0)}$ +  $\langle j \rangle$   $\sim m$  =  $\sim$  $=\sum E_{i(0)}+\sum\bigl\{E_{i(0)j(0)}-E_{i(0)}-E_{j(0)}\bigr\}+\frac{1}{2}\sum\bigl(1-\delta_{m0}\bigr)\sum\bigl\{E_{i(0)j(m)}-E_{i(0)}-E_{j(0)}\bigr\}$  $\frac{(0)}{100} + \sum_{i=1}^{\infty} \left\{ \frac{GL_{i}(0)j(0)}{2m} - \frac{GL_{i}(0)}{2m} - \frac{GL_{i}(0)}{2m} \right\} + \frac{1}{2} \sum_{i=1}^{\infty} \left( 1 - \delta_{m0} \right) \sum_{i=1}^{\infty} \left\{ \frac{CL_{i}(0)j(m)}{2m} - \frac{CL_{i}(0)}{2m} - \frac{CL_{i}(0)}{2m} \right\}$ ,  $\frac{1}{2} \sum_{i=1}^{+L} (1$ 2 *L*  $\mu(i)$   $\sum_{i} \left[ \sum_{i} U_{ij}(0)j(0) - U_{ij}(0) - U_{ij}(0) \right]$   $\sum_{i} \left[ \sum_{i} (1 - S_i) \sum_{i} \sum_{i} U_{ij}(0)j(m) - U_{ij}(0) - U_{ij}(0) \right]$ *m i i i*<sub>k</sub>*j*  $\left(\begin{array}{ccc} \mathcal{U}\mathcal{X} & \mathcal{U}\mathcal{X} & \mathcal{U}\mathcal{X} \end{array}\right)$   $\mathcal{L}_{m=-L}$  *i*, *j*  $E = \sum_i \partial E_{i(0)}$   $\sum_i \left[ \partial E_{i(0)}_{i(0)} - \partial E_{i(0)} \right]$   $\partial E_{i(0)}$   $\Big|$   $\Big|$   $\Big|$   $\sum_{i=1}^{+L}$   $\Big|$   $\Big|$   $\Big|$   $\sum_i \left[ \partial E_{i(0)}_{i(m)} - \partial E_{i(0)} \right]$   $\Big|$   $\partial E_{i(0)}$  $x \leftarrow i \partial x \leftarrow i \xi$   $\partial x$   $\partial x$  $\delta$ +  $\langle \xi, \xi \rangle$   $\langle \xi, \xi \rangle$  $\partial E \nabla \partial E_{i(0)}$   $\nabla \left[ \partial E_{i(0)} i(0)} \partial E_{i(0)} \partial E_{i(0)} \right]$  1  $\stackrel{+L}{\nabla}$  (1 c )  $\nabla \left[ \partial E_{i(0)} i(m) \partial E_{i(0)} \partial E_{i(0)} \right]$  $\frac{\partial E}{\partial x} \approx \sum_{i} \frac{\partial E_{i(0)}}{\partial x} + \sum_{i < j} \left\{ \frac{\partial E_{i(0)j(0)}}{\partial x} - \frac{\partial E_{i(0)}}{\partial x} - \frac{\partial E_{j(0)}}{\partial x} \right\} + \frac{1}{2} \sum_{m=-L} (1 - \delta_m) \sum_{i,j} \left\{ \frac{\partial E_{i(0)j(m)}}{\partial x} - \frac{\partial E_{i(0)}}{\partial x} - \frac{\partial E_{j(m)}}{\partial x} \right\}$  $(0) j(m)$   $\longrightarrow$   $\overline{L}$   $( m)$   $\Big\downarrow \quad \overline{L}$   $\overline{L}$   $\overline{L}$  $(m)$   $\qquad \qquad \mathfrak{Z}_{\mathbf{z}}(m)$ , 1 2 *L*  $i(0) j(m)$   $\mathcal{L}$   $j(m)$  $j(m)$   $\qquad \qquad \Delta J(m)$ *m* = - L *i*, *j*  $E \left[ \left. \begin{array}{c} +L \ L \end{array} \right] \sum \sum_{\mu} \left| \partial E_{i(0)j(m)} \right| \partial E_{j(m)} \left| \right| \right] \partial E$ *m*  $a \quad 2 \sum_{m=-L}^{\infty} \sum_{i,j} \sum_{\gamma}^{\gamma} \left[ \partial z_{\gamma}^{j(m)} \quad \partial z_{\gamma}^{j(m)} \right] \quad \partial a$ + =−  $\partial E \equiv 1 + L \sum_{i \in (0, j(m))} \partial E_{i(0, j(m))} \quad \partial E_{j(m)} \equiv \partial E_{j(m)}$  $\frac{\partial E}{\partial a} \approx \frac{1}{2} \sum_{m=-L} \sum_{i,j} \sum_{\gamma} m \left\{ \frac{\partial E_{i(0)j(m)}}{\partial z_{\gamma}^{j(m)}} - \frac{\partial E_{j(m)}}{\partial z_{\gamma}^{j(m)}} \right\} + \frac{\partial E}{\partial a}$ **Energy per unit cell Energy gradients** Linear scaling regardless of gradients algorithms (analytical vs. numerical) Long-range electrostatic correction essential for cell gradients

**Energy Hessian**  $(1-\delta_{m0})$  $2 \mathbf{E}$   $\partial^2 \mathbf{F}$   $\left( \partial^2 \mathbf{F}$   $\partial^2 \mathbf{F}$   $\partial^2 \mathbf{F}$   $1 \mathbf{L}$   $\left( \partial^2 \mathbf{F}$   $\partial^2 \mathbf{F}$   $\partial^2 \mathbf{F}$  $(0)$  (0)  $\sum_{i=1}^{\infty}$  (0)  $\sum_{i=1}^{\infty}$  (0)  $(i)$  (0)  $\sum_{i=1}^{\infty}$  (0)  $(i)$  (1)  $\sum_{i=1}^{\infty}$  (1)  $\sum_{i=1}^{\infty}$  (0)  $(i)$  (m)  $\sum_{i=1}^{\infty}$  (0)  $\sum_{i=1}^{\infty}$  (n) 0 ,  $\frac{1}{2} \sum_{i=1}^{+L} (1$ 2 *L*  $\mu(i)$   $\sum_{i}$   $\sum_{i}$   $\sum_{i}$   $\mu(i)$   $\mu(j)$   $\sum_{i}$   $\sum_{i}$   $\mu(j)$   $\sum_{i}$   $\sum_{i}$   $\sum_{i}$   $\sum_{i}$   $\sum_{i}$   $\sum_{i}$   $\sum_{i}$   $\mu(i)$   $\sum_{i}$   $\mu(i)$   $\sum_{i}$   $\mu(i)$   $\sum_{i}$   $\mu(i)$   $\sum_{i}$   $\mu(i)$   $\sum_{i}$   $\mu(i)$   $\sum_{i}$   $\mu(i)$  *m i i i i i j*  $\left\{\n \begin{array}{ccc}\n \text{CAVy} & \text{CAVy} \\
 \text{CAVx} & \text{CAVy}\n \end{array}\n \right\}\n \rightarrow_{m=-L} \quad i,j$  $E = \sum_{i=0}^{\infty} \frac{\partial^2 E_{i(0)}}{\partial \ln \sum_{i=0}^{\infty} E_{i(0)i(0)}} \frac{\partial^2 E_{i(0)}}{\partial \ln \sum_{i=0}^{\infty} E_{i(0)}} \left[ \frac{1}{2} \sum_{i=0}^{+L} \frac{1}{2} \sum_{i=0}^{+L} \frac{\partial^2 E_{i(0)i(m)}}{\partial \ln \sum_{i=0}^{\infty} E_{i(0)}} \right] \frac{\partial^2 E_{i(0)}}{\partial \ln \sum_{i=0}^{\infty} E_{i(0)}}$ *x* $\partial$ y  $\leftarrow$   $\partial$ *x* $\partial$ y  $\leftarrow$   $\partial$ *x* $\partial$ y  $\partial$ *x* $\partial$ y  $\partial$ *x* $\partial$ y  $\partial$ *x* $\partial$ *y*  $\delta$ +  $\langle z_j |$   $U\lambda UY$   $U\lambda UY$   $U\lambda UY$   $\Delta_{m=-}$  $\partial^2 E \nabla \partial^2 E_{i(0)}$   $\nabla \left[ \partial^2 E_{i(0)} i(0)} \partial^2 E_{i(0)} \partial^2 E_{i(0)} \right]$   $1 \nabla f_{i(0)}$   $\partial^2 E_{i(0)} i(m)$   $\partial^2 E_{i(0)}$   $\partial^2 E_{i(0)}$  $\frac{\partial E}{\partial x \partial y} \approx \sum_i \frac{\partial^2 E_{i(0)}}{\partial x \partial y} + \sum_{i < j} \left\{ \frac{\partial^2 E_{i(0)j(0)}}{\partial x \partial y} - \frac{\partial^2 E_{j(0)}}{\partial x \partial y} \right\} + \frac{1}{2} \sum_{m=-L} (1 - \delta_{m0}) \sum_{i,j} \left\{ \frac{\partial^2 E_{i(0)j(m)}}{\partial x \partial y} - \frac{\partial^2 E_{j(m)}}{\partial x \partial y} \right\}$ 

*x* and *y* need not be in-phase coordinates: phonon dispersions can be obtained

### **Finite pressures and temperatures**

**Li, Sode, Voth and Hirata, under review by** *Nature Comm* **(2013)**

**Gibbs free energy per unit cell**

 $G = E + E_{zR} + P(abc) - TS$ 

For an orthorhombic unit cell

Partition function  

$$
Z = \prod_{i} \prod_{k_i} \frac{\exp(-\omega_{ik_i}/2k_B T)}{1 - \exp(-\omega_{ik_i}/k_B T)}
$$

**Zero-point energy, entropy, and heat capacity**

$$
E_{\text{ZP}} = k_{\text{B}} T^2 \frac{\partial \ln Z}{\partial T} \qquad \qquad S = -\frac{\partial (k_{\text{B}} T \ln Z)}{\partial T} \qquad \qquad C_{\text{V}} = \frac{\partial (E + E_{\text{ZP}})}{\partial T}
$$









# **Acknowledgements**

### *Dr. James Davenport, Dr. Mark Pederson, and Dr. Ceren Susut*



SciDAC **Scientific Discovery** through **Advanced Computing** 

#### Peter Abbamonte (Physics, UIUC)

Abbamonte, an experimental condensed-matter physicist, brings an invaluable experimental insight into the project. He is one of the originators of resonant soft x-ray scattering, with which he discovered a Wigner crystal in doped spin ladders and the charged stripes in copper-oxide superconductors.

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<sup>33</sup> *For more information, please visit http://www.predictive-scidac.org*

