

# U.S. DOE SciDAC Partnership Predictive Computing for Condensed Matter

Garnet Chan (Chemistry) Bryan Clark (Physics) So Hirata (Chemistry) Peter Abbamonte (Physics) David Ceperley (Physics) Shinsei Ryu (Physics) Lucas Wagner (Physics)

Shiwei Zhang (Physics)



### **Frontiers of predictive computing**



A DOE report on Computational Materials Science (2010):

"We are at the threshold of a new era where the integrated synthesis, characterization, and <u>modeling of complex materials</u> and chemical processes <u>will transform our ability to understand and design new</u> <u>materials and chemistries</u> with predictive power"

Materials Genome Initiative for Global Competitiveness (2011): "the development of advanced materials can be accelerated through advances in <u>computational techniques</u>"

### The team

#### So Hirata (Chemistry, UIUC) - Lead PI

Hirata is a theoretical/computational chemist and an expert in electron-correlation theories for molecules and solids. He is the primary author of the computer-generated, high-rank electroncorrelation modules in DOEs massively parallel NWCHEM suite of software, implemeting several of his original methods.

#### Peter Abbamonte (Physics, UIUC)

Abbamonte, an experimental condensed-matter physicist, brings an invaluable experimental insight into the project. He is one of the originators of resonant soft x-ray scattering, with which he discovered a Wigner crystal in doped spin ladders and the charged stripes in copper-oxide superconductors.

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Chan is a recognized expert in strong correlation theories including density matrix renormalization group (DMRG), tensor networks, and density matrix functional theory. He has, in particular, established DMRG as a practical, powerful tool for strongly correlated molecular electronic structures.

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Ryu specializes in mathematical theories of strong correlation and other condensed phase electronic structures. Ryu is a pioneer in the use of entanglement entropy in classifying topological phases of matter and has predicted the fractional topological insulator in two dimension.



Wagner is the principal author of the quantum Monte Carlo program, QWALK, with which he has performed predictively accurate calculations on strongly correlated systems.





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Ceperley is a theoretical/computational physicist and an authority of quantum Monte Carlo (QMC). He invented a number of QMC algorithms and is the author of massively parallel QMCPACK software.



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Zhang specializes in computational condensed matter physics and materials science. Zhang is a pioneer in the use of quantum Monte Carlo (QMC) in the Hilbert space for strong correlation. He is the inventor of the phaseless auxiliary field QMC.





#### Theories

(Finite-7) MP, CC, RPA, etc.; QMC; DMRG, TN; FCIQMC, AFQMC, MC-MP2; DMFT

#### Software

Algorithm redesign; massive parallelism; fast integrals

#### **Applications – experimental verification**

Metals; (copper oxide) superconductors; graphene; molecular crystals; Peierls systems and Luttinger liquid; Mott-Hubbard systems; spin liquid



Abbamonte



$$\begin{array}{c} \textbf{Schrödinger equation}\\ \textbf{for water}\\ \hline \textbf{10-particle 30-dimensional partial differential equation}\\ \hline \begin{pmatrix} \hat{h}^2\\ 2m_e \sum_{i=1}^{10} \nabla_i^2 - \frac{\hbar^2}{2m_I} \sum_{I=1}^{3} \nabla_I^2 - \sum_{I=1}^{3} \sum_{i=1}^{10} \frac{Z_I e^2}{4\pi \varepsilon_0 r_{iI}} + \sum_{i=1}^{9} \sum_{j=i+1}^{10} \frac{e^2}{4\pi \varepsilon_0 r_{ij}} + \sum_{I=1}^{2} \sum_{j=I+1}^{3} \frac{Z_I Z_J e^2}{4\pi \varepsilon_0 r_{iJ}} \right) \Psi = E \Psi\\ \hline \begin{pmatrix} \partial^2\\ \partial r_i^2 + \frac{2}{r_i} \frac{\partial}{\partial r_i} + \frac{1}{r_i^2} \left( \frac{1}{\sin^2 \theta_i} \frac{\partial^2}{\partial \phi_i^2} + \frac{1}{\sin \theta_i} \frac{\partial}{\partial \theta_i^2} \sin \theta_i \frac{\partial}{\partial \theta_i^2} \right) \end{array}$$

Conditions arising from the indistinguishability of electrons

$$\Psi(\mathbf{r}_{e1}, \mathbf{r}_{e2}, \mathbf{r}_{e3}, \mathbf{r}_{e4}, \mathbf{r}_{e5}, \mathbf{r}_{e6}, \mathbf{r}_{e7}, \mathbf{r}_{e8}, \mathbf{r}_{e9}, \mathbf{r}_{e10}, \mathbf{r}_{H1}, \mathbf{r}_{H2}, \mathbf{r}_{O}) =$$

 $-\Psi(\mathbf{r}_{e2},\mathbf{r}_{e1},\mathbf{r}_{e3},\mathbf{r}_{e4},\mathbf{r}_{e5},\mathbf{r}_{e6},\mathbf{r}_{e7},\mathbf{r}_{e8},\mathbf{r}_{e9},\mathbf{r}_{e10},\mathbf{r}_{H1},\mathbf{r}_{H2},\mathbf{r}_{O}) =$ 

 $\Psi(\mathbf{r}_{e3}, \mathbf{r}_{e1}, \mathbf{r}_{e2}, \mathbf{r}_{e4}, \mathbf{r}_{e5}, \mathbf{r}_{e6}, \mathbf{r}_{e7}, \mathbf{r}_{e8}, \mathbf{r}_{e9}, \mathbf{r}_{e10}, \mathbf{r}_{H1}, \mathbf{r}_{H2}, \mathbf{r}_{O}) =$ 

Many-body

$$-\Psi(\mathbf{r}_{e3},\mathbf{r}_{e2},\mathbf{r}_{e1},\mathbf{r}_{e4},\mathbf{r}_{e5},\mathbf{r}_{e6},\mathbf{r}_{e7},\mathbf{r}_{e8},\mathbf{r}_{e9},\mathbf{r}_{e10},\mathbf{r}_{H1},\mathbf{r}_{H2},\mathbf{r}_{O}) = \dots$$

(3,628,800) terms!

## Systematic many-body methods



#### MP2



## Systematic many-body methods



Exact solution of the Schrödinger equation



## **Quantum Monte Carlo**



### Contents

- Research highlights of 2012-2013
  - QMC for copper oxide superconductors
  - Auxiliary field QMC for band gaps
  - Full CI QMC for excited states
- Monte Carlo MP2 and MP3
- MP2 for molecular crystals: solid-solid phase transition in CO<sub>2</sub>

Research Highlights of 2012-2013

#### **QMC** for copper oxide superconductors

Wagner and Abbamonte, to be published (2013)



Wagner



Magnetic Method J/eV moment DFT (LDA) 0.82 0.3 DFT (GGA) 0.5 0.6 QMC 0.15 0.62 Expt 0.13 0.6





For more details, please visit the poster by Lucas Wagner

Research Highlights of 2012-2013

### **Auxiliary field QMC for band gaps**

Zhang et al., to be published (2013)







Zhang

Method	Band gap / eV	Method	Band gap / eV
DFT (LDA)	0.67	DFT (LDA)	1.0
GW	4.8	GW	2.4, 2.6, 2.8
QMC	3.97	QMC	3.26
Expt	3.7, 3.87, 4.0, 4.3	Expt	3.3 – 3.57

Research Highlights of 2012-2013

### **Full CI QMC for excited states**

#### Booth and Chan, JCP Comm (2012)



Chan

$$\exp[-\beta(H-E)]$$

Standard propagator: **max at ground-state** 

$$\exp[-\beta(H-\omega)^2]$$

Gaussian propagator: max at specified omega



Willow, Kim and Hirata, JCP (2012), JCP (2013), under review by JCTC (2013)



Willow, Kim and Hirata, JCP (2012)

Very long  $O(n^4)$  summation of products of 2 x 6-dimensional integrals

$$E^{(2)} = \sum_{i,j}^{\infty} \sum_{a,b}^{\text{vir.}} \frac{\langle ab | ij \rangle \langle ij | ab \rangle}{\varepsilon_i + \varepsilon_j - \varepsilon_a - \varepsilon_b}$$
Explicit two-electron integrals
$$E^{(2)} = \sum_{i,j}^{\infty} \sum_{a,b}^{\text{vir.}} \frac{\int \varphi_i(\mathbf{r}_1)\varphi_j(\mathbf{r}_2) \frac{1}{r_{12}} \varphi_a(\mathbf{r}_1)\varphi_b(\mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2 \int \varphi_i(\mathbf{r}_3)\varphi_j(\mathbf{r}_4) \frac{1}{r_{34}} \varphi_a(\mathbf{r}_3)\varphi_b(\mathbf{r}_4) d\mathbf{r}_3 d\mathbf{r}_4}{\varepsilon_i + \varepsilon_j - \varepsilon_a - \varepsilon_b}$$
Laplace transformation of the denominator
$$E^{(2)} = -\sum_{i,j}^{\infty} \sum_{a,b}^{\text{vir.}} \int \varphi_i(\mathbf{r}_1)\varphi_j(\mathbf{r}_2) \frac{1}{r_{12}} \varphi_a(\mathbf{r}_1)\varphi_b(\mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2 \int \varphi_i(\mathbf{r}_3)\varphi_j(\mathbf{r}_4) \frac{1}{r_{34}} \varphi_a(\mathbf{r}_3)\varphi_b(\mathbf{r}_4) d\mathbf{r}_3 d\mathbf{r}_4 \int_0^{\infty} e^{(\varepsilon_i + \varepsilon_j - \varepsilon_a - \varepsilon_b)\mathbf{r}} d\mathbf{r}$$
Change of orders of summations and integrations
$$E^{(2)} = -\int \cdots \int \int_0^{\infty} \frac{\sum_i \varphi_i(\mathbf{r}_1)\varphi_i(\mathbf{r}_3)e^{\varepsilon_i r} \sum_j \varphi_j(\mathbf{r}_2)\varphi_j(\mathbf{r}_4)e^{\varepsilon_i r} \sum_a \varphi_a(\mathbf{r}_1)\varphi_a(\mathbf{r}_3)e^{-\varepsilon_i r} \sum_b \varphi_b(\mathbf{r}_2)\varphi_b(\mathbf{r}_4)e^{-\varepsilon_b r}} d\mathbf{r}_1 \cdots d\mathbf{r}_4 d\mathbf{r}$$
Single 13-dimensional integral evaluated by Monte Carlo
$$E^{(2)} = -\int \cdots \int \int_0^{\infty} \frac{G^-(\mathbf{r}_1, \mathbf{r}_3, \tau)G^-(\mathbf{r}_2, \mathbf{r}_4, \tau)G^+(\mathbf{r}_1, \mathbf{r}_3, \tau)G^+(\mathbf{r}_2, \mathbf{r}_4, \tau)}{r_{12}r_{34}} d\mathbf{r}_1 \cdots d\mathbf{r}_4 d\tau$$

Willow, Kim and Hirata, JCP (2012)

$$E = \int f(x) dx = \int \frac{f(x)}{g(x)} g(x) dx = \sum_{x \in \mathbb{R}} \frac{f(x)}{g(x)} \cdot \int g(x) dx$$
  
Requirement 1: analytically integrable  $\int g(x) dx = N$   
Requirement 2: cancellation of singularities  $f(x) / g(x)$   

$$E^{(2)} = -\int \cdots \int \int_{0}^{\infty} \frac{G^{-}(\mathbf{r}_{1}, \mathbf{r}_{3}, \tau) G^{-}(\mathbf{r}_{2}, \mathbf{r}_{4}, \tau) G^{+}(\mathbf{r}_{1}, \mathbf{r}_{3}, \tau) G^{+}(\mathbf{r}_{2}, \mathbf{r}_{4}, \tau)}{r_{12} f_{12}} d\mathbf{r}_{1} \cdots d\mathbf{r}_{4} d\tau$$
  

$$g(\mathbf{r}_{1}, \mathbf{r}_{2}, \mathbf{r}_{3}, \mathbf{r}_{4}) = \frac{\rho(\mathbf{r}_{1}) \rho(\mathbf{r}_{2})}{r_{12}} \frac{\rho(\mathbf{r}_{3}) \rho(\mathbf{r}_{4})}{r_{34}}$$
  

$$\int g(\mathbf{r}_{1}, \mathbf{r}_{2}, \mathbf{r}_{3}, \mathbf{r}_{4}) d\mathbf{r}_{1} d\mathbf{r}_{2} d\mathbf{r}_{3} d\mathbf{r}_{4} = 4E^{2}_{\text{Coulomb}}$$

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Willow, Kim and Hirata, JCP (2012)



### **Monte Carlo MP2 for self-energies**

Willow, Kim and Hirata, JCP (2013)



Dyson self-energies (quasiparticle energies) for multiple states obtainable from a single MC run

#### **Real-space Green's function**

Willow, Kim and Hirata, to be published (2013)



Willow, Kim and Hirata, to be published (2013)





### **Redundant-walker MC-MP2**

Willow, Hermes, Kim and Hirata, under review by JCTC (2013)



### **Massively parallel MC-MP2**

Willow, Hermes, Kim and Hirata, under review by JCTC (2013)

C<sub>60</sub> cc-pVDZ on 320 processors of Blue Waters



### **MP2 for molecular crystals**





### **Embedded-fragment approach**

Hirata et al., MP (2005); Kamiya, Hirata, and Valiev, JCP (2008)

*N*-body (N > 2) Coulomb in point-charge or dipole approximation

Pair energy in the presence of self-1 and 2-body consistent atomic Coulomb charges or dipoles Exchange Correlation  $E = \sum_{i=1}^{n} E'_{i} + \sum_{i < i}^{n} \left( E'_{ij} - E'_{i} - E'_{j} \right) + \Box$ 

### **Molecular crystals**

#### Hirata, JCP (2008)

Energy per unit cell  $E = \sum_{i} E_{i(0)} + \sum_{i < j} \left\{ E_{i(0) j(0)} - E_{i(0)} - E_{j(0)} \right\} + \frac{1}{2} \sum_{m=-L}^{+L} \left( 1 - \delta_{m0} \right) \sum_{i,j} \left\{ E_{i(0) j(m)} - E_{i(0)} - E_{j(m)} \right\}$ Energy gradients  $\frac{\partial E}{\partial x} \approx \sum_{i} \frac{\partial E_{i(0)}}{\partial x} + \sum_{i < j} \left\{ \frac{\partial E_{i(0) j(0)}}{\partial x} - \frac{\partial E_{i(0)}}{\partial x} - \frac{\partial E_{j(0)}}{\partial x} \right\} + \frac{1}{2} \sum_{m=-L}^{+L} \left( 1 - \delta_{m0} \right) \sum_{i,j} \left\{ \frac{\partial E_{i(0) j(m)}}{\partial x} - \frac{\partial E_{i(0)}}{\partial x} - \frac{\partial E_{j(m)}}{\partial x} \right\}$ 

$$\frac{\partial E}{\partial a} \approx \frac{1}{2} \sum_{m=-L}^{+L} \sum_{i,j} \sum_{\gamma} m \left\{ \frac{\partial E_{i(0)j(m)}}{\partial z_{\gamma}^{j(m)}} - \frac{\partial E_{j(m)}}{\partial z_{\gamma}^{j(m)}} \right\} + \frac{\partial E_{LR}}{\partial a}$$

Linear scaling regardless of gradients algorithms (analytical vs. numerical) Long-range electrostatic correction essential for cell gradients

 $\frac{\partial^{2} E}{\partial x \partial y} \approx \sum_{i} \frac{\partial^{2} E_{i(0)}}{\partial x \partial y} + \sum_{i < j} \left\{ \frac{\partial^{2} E_{i(0) j(0)}}{\partial x \partial y} - \frac{\partial^{2} E_{i(0)}}{\partial x \partial y} - \frac{\partial^{2} E_{j(0)}}{\partial x \partial y} \right\} + \frac{1}{2} \sum_{m=-L}^{+L} (1 - \delta_{m0}) \sum_{i,j} \left\{ \frac{\partial^{2} E_{i(0) j(m)}}{\partial x \partial y} - \frac{\partial^{2} E_{j(m)}}{\partial x \partial y} - \frac{\partial^{2} E_{j(m)}}{\partial x \partial y} \right\}$ 

x and y need not be in-phase coordinates: phonon dispersions can be obtained

#### **Finite pressures and temperatures**

Li, Sode, Voth and Hirata, under review by Nature Comm (2013)

Gibbs free energy per unit cell

 $G = E + E_{zp} + P(abc) - TS$ 

For an orthorhombic unit cell

Partition function  

$$Z = \prod_{i} \prod_{\mathbf{k}_{i}} \frac{\exp\left(-\omega_{i\mathbf{k}_{i}}/2k_{\mathrm{B}}T\right)}{1 - \exp\left(-\omega_{i\mathbf{k}_{i}}/k_{\mathrm{B}}T\right)}$$

Zero-point energy, entropy, and heat capacity

$$E_{\rm ZP} = k_{\rm B} T^2 \frac{\partial \ln Z}{\partial T}$$
  $S = -\frac{\partial \left(k_{\rm B} T \ln Z\right)}{\partial T}$   $C_{\rm V} = \frac{\partial \left(E + E_{\rm ZP}\right)}{\partial T}$ 







![](_page_31_Figure_2.jpeg)

### **Acknowledgements**

#### Dr. James Davenport, Dr. Mark Pederson, and Dr. Ceren Susut

![](_page_32_Picture_2.jpeg)

Scientific Discovery through Advanced Computing

#### Peter Abbamonte (Physics, UIUC)

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![](_page_32_Picture_10.jpeg)

#### Lucas Wagner (Physics, UIUC) - Co-lead PI Wagner is the principal author of the quantum

Wagner is the principal author of the quantum Monte Carlo program, QWALK, with which he has performed predictively accurate calculations on strongly correlated systems.

![](_page_32_Picture_13.jpeg)

![](_page_32_Picture_14.jpeg)

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![](_page_32_Picture_21.jpeg)

![](_page_32_Picture_22.jpeg)

For more information, please visit http://www.predictive-scidac.org

![](_page_32_Picture_24.jpeg)