White Paper for *Frontiers of Plasma Science Panel*

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Indicate the primary area this white paper addresses by placing "P" in right column.
Indicate secondary area or areas by placing "S" in right column

	"P", "S"
• Plasma Atomic physics and the interface with chemistry and biology	
• Turbulence and transport	
• Interactions of plasmas and waves	
• Plasma self-organization	P
• Statistical mechanics of plasmas	

Indicate type of presentation desired at Town Hall Meeting.

	"X"
Oral	
Poster	X
Either Oral or Poster	
Will not attend	

Title:	Self-organization phenomena in tokamaks
Corresponding Author:	Stephen C. Jardin
• Institution:	Princeton Plasma Physics Laboratory
• email:	jardin@pppl.gov
Co-Authors:	Nate Ferraro (GA), Isabel Krebs (PPPL/IPP)

(Limit text to 3-pages including this form. Font Times Roman size 11.
1 page of references and 1 page of figures may also be included. *Submit in PDF format*.)

- ***Describe the research frontier* and importance of the scientific challenge.**

Typical high-performance modes of tokamak operation undergo "sawtooth" cycles, in which the peaking of the toroidal current density triggers a periodic core instability which redistributes the current density. However, certain modes of operation are known, such as the "hybrid" mode in DIII-D [1] and other tokamaks [2-7], which do not experience this cycle of instability. Empirically, it is observed that these modes maintain a non-axisymmetric equilibrium which somehow limits the peaking of the toroidal current density. The physical mechanism responsible for this has not previously been understood, but has been referred to as "flux-pumping," in which poloidal flux is anomalously redistributed and the toroidal current broadened in order to maintain the central safety factor greater than unity, $q_0 > 1$ [1]. Here we show that in simulations of inductively driven tokamak plasmas, a steady-state non-axisymmetric magnetic equilibrium may be obtained in which $q_0 > 1$ is maintained by a nonlinear dynamo action driven by a stationary marginal core interchange mode.

- ***Describe the approach to advancing the frontier and indicate if new research tools or capabilities are required.***

An inductively driven tokamak plasma is said to be in a "stationary state" if the magnetic field, temperatures, and densities (and hence pressure) are not changing in time. If this state is achieved, it is normally reached late in the discharge after the plasma current is constant in time and has fully

penetrated. Since the magnetic field is not changing in time, it follows from Maxwell's induction equation that the electric field is the gradient of a single valued potential plus a constant times the gradient of the toroidal angle:

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} = 0 \quad \Rightarrow \quad \mathbf{E} = -\nabla \Phi + \frac{V_L}{2\pi} \nabla \varphi \tag{1}$$

Here, $\mathbf{B}$ and $\mathbf{E}$ are the magnetic and electric fields, Φ is a single valued potential, (R, φ, Z) are the usual cylindrical coordinates, and V_L is a spatial constant that represents the voltage the long way around the torus, created by external induction coils. In the resistive MHD description of a plasma, the generalized Ohm's law, $\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J}$, combined with Eq. (1), gives the condition:

$$-\mathbf{V} \times \mathbf{B} + \eta \mathbf{J} = -\nabla \Phi + \left(V_L / 2\pi\right) \nabla \varphi \tag{2}$$

Here, η is the resistivity (taken to be a function of the plasma temperature *T*), $\mathbf{V}$ is the fluid velocity, and $\mathbf{J} = \mu_0^{-1} \nabla \times \mathbf{B}$ is the plasma current density in the low frequency (MHD) limit. If we take the inner product of Eq. (2) with the magnetic field vector $\mathbf{B}$, the velocity vector is eliminated and we obtain:

$$\eta \mathbf{J} \cdot \mathbf{B} = -\mathbf{B} \cdot \nabla \Phi + \left(V_L / 2\pi\right) \mathbf{B} \cdot \nabla \varphi \tag{3}$$

If magnetic surfaces exist everywhere, we can perform a surface average of Eq. (3) to obtain the well-known condition for tokamak stationary states:

$$\left\langle \mathbf{J} \cdot \mathbf{B} \right\rangle / \left\langle \mathbf{B} \cdot \nabla \varphi \right\rangle = V_L / 2\pi \eta \tag{4}$$

Here, $\langle \ \rangle$ is the standard surface average operator [8] that annihilates $\mathbf{B} \cdot \nabla$. The plasma resistivity is a strong function of the plasma temperature: $\eta \sim \eta_0 T^{-3/2}$ (Spitzer). From Eq. (4) we see that where the temperature is largest (typically in the center of the discharge due to central heating and geometrical effects) the surface averaged parallel current density will also be largest. This large current will lead to a large Ohmic heating term in the temperature equation which tends to increase the central temperature even more, thus requiring the plasma current to peak even more in order to satisfy Eq. (4). This tendency for the parallel current and temperature to peak in an inductively driven tokamak has been known for almost 60 years.

The conventional explanation [9] for why this thermal instability doesn't continue to peak the current to very large values is the "sawtooth" cycle. We discuss here a different mechanism for preventing the current and temperature profiles from peaking in a truly stationary state tokamak. For certain global parameters, regardless of the initial state, the plasma profiles will evolve into a self-organized state with the central safety factor q_0 approaching unity from above and constant in a central volume. We illustrate such a calculation in Figures (1)-(2). The solid curves in Figure 1 show the steady state pressure and safety factor profiles for a 3D simulation (red), and for an identical 2D (axisymmetric) calculation (black) with the same transport model and parameters. It is seen that the toroidally averaged 3D and 2D profiles differ in the center. Figure 2 is a Poincaré plot of the final 3D configuration showing some island structure. The central shear-free region has $q \sim 1$ and constant. It is omitted from the Poincaré plot since flux surfaces in the conventional sense do not exist there.

Such a large shear free region with *q* just above unity is known to be linearly unstable to interchange modes driven by any non-zero pressure gradient [11-14]. Unlike an unstable configuration with a *q*=1 resonant surface in the plasma, the unstable linear eigenfunction for a ultra-low shear configuration with *q* just above 1 throughout a volume is distributed out to the region where the shear begins (about the *q*=1.01

surface) and the instability drives a strong (1,1) helical flow. The unstable eigenfunction found in [13] is given in terms of the stream function U by:

$$U = U_0 r\left[1-\left(r/r_1\right)^2\right]\sin(\theta-\varphi) \tag{5}$$

Here the minor radius is $r=\left|R-R_0\right|$ and r_1 is the minor radius where the shear becomes non-zero. The steady-state condition, Eq. (3), which resulted from the component of Eq. (2) parallel to the magnetic field, does not explicitly contain the velocity $\mathbf{V}$. However, we will see that the driven flow creates a (1,1) component of the potential Φ and of the magnetic field $\mathbf{B}$ that combine to create a (0,0) spatially varying voltage that prevents the current from peaking in the center region and hence maintains q slightly above unity and shear-free in the central region. The flow field also nonlinearly acts to partially flatten the temperature and hence resistivity profile in the center, but this effect is normally secondary to the generation of the spatially varying voltage. This dynamo mechanism was discovered with the M3DC1 toroidal 3D MHD code[15], and could possibly explain non-sawtoothing discharges with $q_0 \cong 1$ such as hybrid modes in DIII-D[1], ASDEX-U[2], JT-60U[3]and JET[4-5] , and long-lived modes in NSTX[6] and MAST[7]. Stationary non-sawtoothing behavior has been observed in other 3D tokamak MHD simulations [16-18] but an explanation of how these configurations maintain themselves over resistive timescales has not appeared.

To illustrate the self-organized voltage clamping mechanism, we present results from 5 long time simulations. The runs were largely identical with central resistivity in MKS units $\eta_0 = 10^{-6}(\mu_0/\tau_A)m^2$, where $\tau_A = a\sqrt{\mu_0\rho}/B$, and plasma beta: $\beta \equiv 2\mu_0\langle p\rangle/B^2 = 2\%$. The four fully 3D simulations differed in that a multiplier was applied to both the isotropic thermal conductivity and the heating sources so that they would balance and keep $\beta = 2\%$. The 4 values used were $\kappa_0 = C\times\eta_0/\mu_0$ with $C = 18,36,72,144$. A parallel thermal conductivity of $\kappa_{\parallel} = (10/\tau_A)\,m^2$ was used in all simulations. Each simulation was also run in a 2D (axisymmetric) mode in which all toroidal derivatives were set to zero, for comparison. The simulations were run to long times: $T = 10^5\ \tau_A$ such that everything was stationary. A typical stationary self-organized state for the 3D C=144 case is shown in Figures (1) -(3). The inset in Figure (3) shows contours of the M3DC1 poloidal velocity stream function U interior to the q=1.01 surface, and the line plot shows a comparison of the computed stream function and the linear unstable eigenfunction as given by Eq. (5) with $U_0 = 1.43\times10^{-4}\ m/\tau_A$ and $r_1 = 0.3m$.

Consider now Eq. (3). In Figure (4) we show the difference of the mid-plane value of the toroidally averaged voltage drop along the magnetic field $(-\mathbf{B}\cdot\nabla\Phi)$ that is present in the four 3D runs from that in the corresponding 2D runs. This fundamentally 3D dynamo voltage is obtained by plotting the difference of the toroidally averaged first term on the right side of Eq. (4) for each of the four 3D runs from that in its corresponding 2D run:

$$\frac{B}{2\pi R}V_{dyn} = \int_0^{2\pi}\left[-\mathbf{B}\cdot\nabla\Phi\right]_{3D}\frac{d\varphi}{2\pi} - \left[-\mathbf{B}\cdot\nabla\Phi\right]_{2D} \tag{6}$$

Here, V_{dyn} is the axisymmetric voltage that is present in the 3D runs that is not present in the 2D run and is therefore due to the 3D dynamo effect driven by the interchange instability. This voltage is exactly that needed to reduce the central current density so as to keep q in the center just above 1 as can be seen in the q-profiles for the four runs in Figure (5). The required dynamo voltages for each of the four runs are different because of the differing temperature and hence resistivity flattening for the four runs as shown in Figure (6) The runs with the lowest thermal conductivity and energy source terms are most

affected (flattened) by the interchange instability and so need the least dynamo voltage to maintain q=1 in the center. This clearly illustrates that there is a self-organized "feedback" mechanism at play.

- ***Describe the impact of this research on plasma science and related disciplines and any potential for societal benefit.***

This hybrid mode of operation may be desirable for ITER because it is stationary and does not have sawteeth transients. While normally benign, sawteeth oscillations can sometimes excite or couple to other modes such as neoclassical tearing modes, or edge localized modes. These modes can degrade confinement, produce large transient heat loads on the divertor, or, in the worst case, lead to a plasma disruption.

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Figures

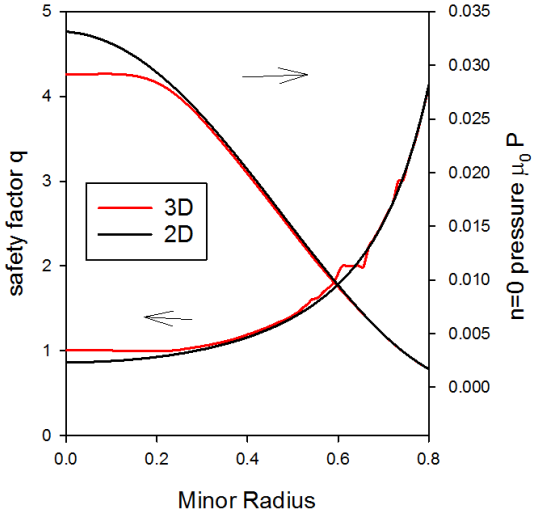

Figure 1: Comparison of q-profile and toroidally averaged pressure for stationary state 3D run and 2D run with exactly the same transport coefficients.

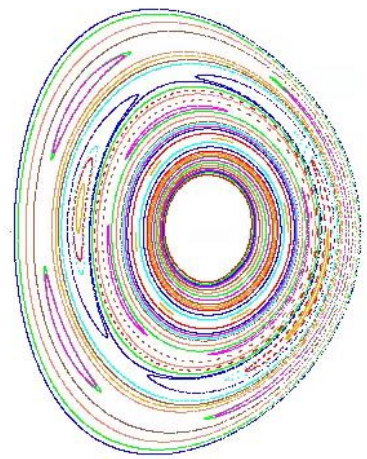

Figure 2: Poincare plot of final state with (2,1) and (3,1) islands present. Center volume has q=1 and no flux surfaces.

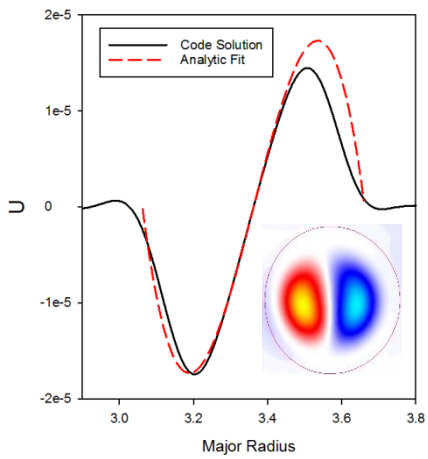

Figure 3: Inset shows contours of m3dc1 velocity stream function U interior to the q=1.01 surface. Curve compares midplane values with eigenfunction found in [13].

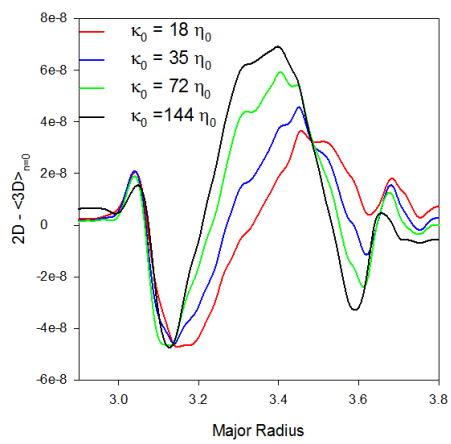

Figure 4: Effective mid-plane toroidal voltage increase in 4-different 3D stationary states over that in the equivalent 2D case. This voltage is due to dynamo action.

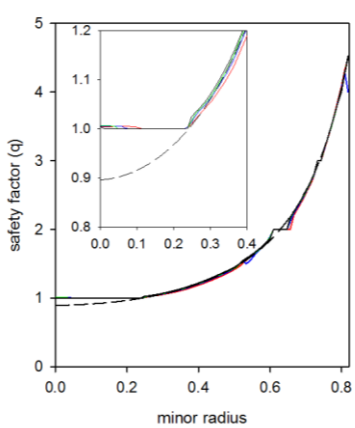

Figure 5: Final safety factor profile in the 4 3D runs (solid colors) and in the equivalent 2D runs (dashed).

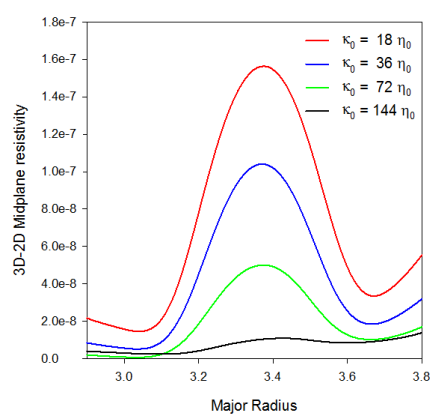

Figure 6: Difference in the final n=0 midplane resistivity profiles for each of the 4 cases from the equivalent 2D case.