White Paper for *Frontiers of Plasma Science Panel*

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Title:	Fundamental Variational Theory of Plasma Waves
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Abstract: *Strategic guidance for plasma physics cannot be limited to specifying problems to solve but should also reflect on the field's methodology and its potential impact on long-term future developments. Such reflection would be particularly timely in studies of plasma waves. One of today's frontiers in this area is to systematically develop a comprehensive theory of plasma waves based on fundamental variational principles. The research is low-risk, because the work is mostly analytic and thus would not require new research tools or capabilities. Potentially, the research is also high-gain, for it can help (i) improve robustness of wave simulations, (ii) identify new basic effects, as it already happened in the past, and, (iii) overall, help modernize the language of plasma physics. A fundamental wave theory will also help better understand waves in general, bridge plasma physics with other disciplines, and strengthen the status of plasma theory as a fundamental science.*

Fundamental Variational Theory of Plasma Waves

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Introduction. – Strategic guidance for plasma physics cannot be limited to specifying problems to solve but should also reflect on the field's methodology and its potential impact on long-term future developments. Such reflection is particularly important for studying waves, which are vital to understanding and taming the plasma state of matter. This White Paper will argue that, notwithstanding the long history of wave studies, plasma wave theory is still full of challenges and surprises.

Although wave phenomenology has advanced over decades immensely, its foundation, as it is commonly understood, has not progressed beyond the original Maxwell-Vlasov theory. This may be holding back further developments. A modern, yet so far largely academic, field-theoretical formulation [1] represents a beautiful and promising alternative. It is now timely to transform this alternative into a practical comprehensive variational theory of waves as fundamental dynamic objects. Developing such a theory is one of the key frontiers in basic plasma theory today. Apart from being an intellectually stimulating enterprise that can modernize the language of plasma physics in general, the development of a comprehensive fundamental wave theory also has obvious utilitarian aspects. Specifically, it can help (i) improve robustness of wave simulations and (ii) identify new basic effects that are harder to see from existing theories. Below, we argue this in detail on the example of radiofrequency (RF) waves, which are particularly important in magnetic fusion energy applications like plasma heating and stabilization, current drive, and alpha channeling. Full-wave and ray-tracing RF simulations, as well as related topics, will be discussed.

Full-wave simulations. – Full-wave simulations are based on direct integration of Maxwell's equations for electromagnetic fields. Since *ab initio* simulations are typically too slow for common applications, various approximations are applied to model the plasma response. Performing such approximations within differential equations typically leads to the loss of action and energy conservation, which is a problem especially at describing mode conversion. This is because such integrals are specific to waves as dynamic objects, so they are not manifest in equations aimed to describe electromagnetic fields in general. To preserve the conservation laws (absent dissipation, which can be added later), Maxwell's equations must be replaced with a variational formulation of the wave dynamics. This has several advantages.

First of all, if approximations are made within the action integral, even the approximate differential equations that it yields remain conservative. Second, a variational formulation also allows, potentially, to simulate RF dynamics using advanced symplectic methods. Even without serving applied purposes directly, such RF algorithms with hard-wired conservation laws could be useful for independent and decisive validation of other full-wave RF codes. Third, aside from numerical simulations, the variational formulation is the only unambiguous way to define the wave energy and momentum. This issue has been controversial [2], and resolving it unambiguously leads to important physical insights. (For example, Ref. [3] explained recently, for the first time in simple terms, why resonant absorption of the wave momentum transverse to the dc magnetic field can spin up a tokamak plasma.) Formulating variational principles suitable for practical full-wave simulations beyond the geometrical-optics (GO) limit is a work that is yet to be done.

Ray tracing. – Ray-tracing simulations are commonly believed to be less prone to inaccuracies, since they are based on the firm foundation of GO. However, the standard GO [1, 2] ignores polarization effects. Those can contribute terms of the order of λ/L to the ray Hamiltonian, thus altering the ray trajectory.

(Here λ is the wavelength, and L is the inhomogeneity scale.) This area, known as extended GO, has been actively studied in optical physics [4] but is largely ignored within the plasma community. The variational formulation offers a first deductive way of accounting for polarization effects. It introduces the general concept of the "photon wave function" [5, 6] and hence treats rays precisely like particles. A vector wave becomes mathematically similar to a Dirac electron and thus has a Stern-Gerlach-like Hamiltonian causing "spin-orbital" coupling. This is precisely the added effect of the wave polarization that is missing in GO. The Hamiltonian is straightforward to find in general [7], but applying the theory to the many specific waves of practical interest remains to be done.

Other impacts. – As a spin-off, the same variational theory automatically solves another long-standing basic-physics problem, namely, how to formulate a consistent classical Lagrangian for a point particle with spin [7, 8]. The same theory also advances the existing understanding of ponderomotive forces and, due to treating rays and particles on the same footing, even extends the concept of ponderomotive forces to waves. The general approach was recently outlined in Ref. [9] in application to scalar interactions and is yet to be developed in general.

The variational wave theory also paves the way for first-principle studies of nonlinear wave effects. The wave Lagrangian can be constructed out of single-particle ponderomotive Lagrangians. The theory explicitly shows how to calculate the particle ponderomotive Lagrangians even for strongly nonlinear dynamics, such as that of particles resonantly trapped by a wave. Already preliminary studies in this area have yielded many new findings regarding the wave dispersion, dynamics, and stability without even introducing the Maxwell-Vlasov system [10]. In particular, a classic problem of finding the Langmuir wave nonlinear dispersion was solved recently such that, for the first time, it captures *both* fluid and kinetic nonlinearities for general plasma and expresses the result simply in terms of the linear dielectric function [11].

Moreover, since the variational theory can treat waves and particles on the same footing, this theory is also applicable in wave kinetics. In particular, once photons are assigned a ponderomotive energy, they can be understood as polarizable particles; then their contribution to the plasma dielectric tensor can be readily found. Among other applications, this quantitatively explains the modulational stability of photon gas in plasma. It is also hoped that further developments in this area will contribute to studies of plasma turbulence.

Conclusions. – In summary, one of the key frontiers in plasma physics today is to systematically develop a comprehensive theory of plasma waves based on fundamental variational principles (as opposed to *ad hoc* solutions of Maxwell's equations). Although a lot has been done already, developing such a theory remains a major research program and a matter of strategic investment. The research is low-risk, because the work is mostly analytic and thus would not require new research tools or capabilities. Potentially, the research is also high-gain, for it can help (i) improve robustness of wave simulations, (ii) identify new basic effects, as it already happened in the past, and, (iii) overall, help modernize the language of plasma physics. A fundamental theory will also help better understand waves in general and bridge plasma physics with other disciplines, thus facilitating and stimulating cross-fertilization of ideas. This will also strengthen the status of plasma theory as a fundamental science, the status that is often called into question because of the overwhelming preponderance of phenomenology over methodology in mainstream plasma research.

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