Three Coordinate Systems as a Frontier of Plasma Physics

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Plasmas of interest in the laboratory and in nature characteristically have an embedded magnetic field. The behavior of the plasma and of the magnetic field has fundamental differences between systems that depend on two and versus three spatial coordinates. Both theory and experiments have focused on two-coordinate systems, which leaves systems that have a dependence on all three coordinates as a research frontier. In the examples given below, existing knowledge is sufficiently great to establish the importance of this frontier but too little is known for the implications to be assimilated. The physics of plasmas that depend on three coordinates was recently reviewed in Nuclear Fusion **55,** 025001 (2015).

I. Space and Astrophysical Plasmas

1. Magnetic Reconnection

It is known that the generic evolution of magnetic fields that depend on three--but not on two--coordinates gives an increasing σ in the separation $\delta = \delta_0 e^{\sigma(\ell)}$ of neighboring magnetic field lines. When σ becomes sufficiently large, $\sigma \sim 20$, it is known that magnetic field line connections must be broken over large spatial regions even though the current density need not be large. What is not known are the implications for energy transfer from magnetic fields to plasmas, the expected effects on measurements seen by spacecraft, or the predictions of phenomena such as space weather. Most reconnection theories and experiments are focused on two-coordinate systems. Simulations alone cannot explore this part of the three-coordinate frontier since the largest σ that can be numerically explored is the natural logarithm of the system to the grid size, at present ~10.

The equation for a magnetic field line $\vec{x}_0(\ell)$ is $d\vec{x}_0 / d\ell = \hat{b}(\vec{x}_0)$, where $\hat{b} \equiv \bar{B} / B$ and ℓ is the distance along the line. The limit as the separation $\delta = |\vec{\delta}| \to 0$, defines a neighboring line, which has the location $\vec{x}_0(\ell) + \vec{\delta}(\ell)$. $\vec{\delta}$ obeys the linear equation $d\vec{\delta} / d\ell = (\vec{\delta} \cdot \vec{\nabla})\hat{b}$. Solutions have the form $\delta = \delta_0 e^{\sigma(\ell)}$, where $\sigma(\ell)$ is the number of exponentiations. The number of exponentiations $\sigma(\ell)$ is not conserved in an ideal evolution, and typically increases with time when the dependence on all three spatial coordinates is non-trivial. A plasma can never act to enforce an ideal evolution over a spatial scale smaller than an electron inertial scale, c / ω_{pe}. The implication is that magnetic field lines can break and connect on the scale $e^{\sigma} c / \omega_{pe}$, so large-scale reconnection is triggered when σ becomes sufficiently large. In the solar corona, the ratio of the radius of the sun to c / ω_{pe} is equivalent to $\sigma \sim 23$.

2. Magnetic Dynamos

It is known that magnetic dynamos require three spatial coordinates, but how dynamos work in the low dissipation limit is not known and remains a research frontier.

II. Magnetic Fusion Plasmas

Much of the knowledge of three-coordinate plasma physics comes from the magnetic fusion program. Two reasons are: (a) experiments, and not just observations, can be made and (b) the

periodicity of toroidal fusion plasmas provides constraints that are not applicable to the open magnetic field lines in space and astrophysics. Uncertainty in the boundary conditions that should be applied has led to obscurity in the understanding of open magnetic fields. This has often led to the in-appropriate adoption of a periodicity constraint.

1. It is known that even axisymmetric plasmas have orders of magnitude differences in sensitivity to various non-axisymmetric external magnetic perturbations. The spectrum of external perturbations ordered by importance is not known. The implications for plasma control and error field mitigation are not assimilated.

2. It is known that the confinement of the trajectories of collisionless trapped particles is assured in systems that depend on only two spatial coordinates through p_φ invariance. It is also known that this confinement can be destroyed by small variations due to the third coordinate but can be maintained by careful design even when the dependence on the third coordinate is strong. A number of implications remain unexplored.

3. It is known that confined plasmas that depend on all three spatial coordinates, called stellarators, can avoid issues associated with (a) maintaining the plasma configuration, (b) with a catastrophic loss of plasma equilibrium called a disruption, and (c) with a plasma current sufficiently large, ~5MA, to be converted by an avalanche mechanism into a dangerous current of relativistic electrons. Although successful stellarator designs and experiments have been made, the trade-offs between different designs are not well understood.

4. It is known that the exponential decay of curl-free magnetic fields through space allows the magnetic field distributions produced by distant coils to be ordered by their difficulty of production—independent of the actual design of the coils. Greatly improved coil systems would presumably result by using the external magnetic field distributions that can be efficiently produced to control those fields to which the plasma is sensitive. This has not been done, though a poof-of-principle demonstration has been made.

5. It is known that a toroidal plasma with net toroidal magnetic flux must have an encircling external coil, but only one encircling coil is required. The other coils could be dipole-like or could be replaced in three-dimensional stellarator systems by superconducting wafers. Design efforts of this sort, which require coils with locations that depend on all three spatial coordinates, could ensure good maintenance access to the plasma chamber, a major problem in existing reactor designs.