PLASMA FIELD THEORY

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Many collaborators and students over the past 35+ years. Recently: Tassi, Chandre, Vittot, Lingam, D'Avignon, Yoshida, Kawazura, Abdelhamid, Pegoraro, Andreussi, Brizard, Burby, Qin, ...

I apologize for giving talk in meeting I organized!

Overview

- Soap Box
- Theory Grand Challenge/Frontier

A First Tale

Kaluza-Klein (1919) 5D theory of $\{\text{Gravity}\} \cup \{\text{E\&M}\} \rightarrow$

Pauli (1953) 6D theory of Gravity (unpublished) $\rightarrow$

Yang-Mills (1954) nonabelian gauge theory $\rightarrow$

Symmetry Breaking (1960) $\rightarrow$ Standard Model

Moral

Things may not turn out like you think they will.

Yet, good things can happen.

A Second Tale

Dirac (1931,1948) symmetrize E&M by adding monopoles $\rightarrow$

Prolonged Search (1970s onward) $\rightarrow$ 0

Moral

Pure theory can be a waste. But ... ?

Fundamental

Fundamental vs. Applied

Plasma theory $= \{\text{Newton}\} \cup \{\text{Maxwell}\}$

Moral

Pure theory can be a waste. <u>But</u> ... ?

Magnetic monopoles in spin ice

C. Castelnovo[1], R. Moessner[1,2] & S. L. Sondhi[3]

Electrically charged particles, such as the electron, are ubiquitous. In contrast, no elementary particles with a net magnetic charge have ever been observed, despite intensive and prolonged searches (see ref. 1 for example). We pursue an alternative strategy, namely that of realizing them not as elementary but rather as emergent particles—that is, as manifestations of the correlations present in a strongly interacting many-body system. The most prominent examples of emergent quasiparticles are the ones with fractional electric charge *e*/3 in quantum Hall physics[2]. Here we propose that magnetic monopoles emerge in a class of exotic magnets known collectively as spin ice[3–5]: the dipole moment of the underlying electronic degrees of freedom fractionalises into monopoles. This would account for a mysterious phase transition observed experimentally in spin ice in a magnetic field[6,7], which is a liquid–gas transition of the magnetic monopoles. These monopoles can also be detected by other means, for example, in an experiment modelled after the Stanford magnetic monopole search[8].

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Moral

Models are models and
speculative theory can lead to good things too.

Theory for Theory's Sake

- Germinates ideas
- Provides vocabulary

Opinion: plasma physics needs more TTS.

Theory Grand Challenge/Frontier:

Topological Plasma Physics

MHD

Alfvén (1942) $\rightarrow$ Newcomb* (1962) $\rightarrow$ Hamiltonian theory (1980)

frozen flux $\neq$ frozen-in field lines

Lie dragged 2-form

$*$ Examined MHD as a field theory. Others in the 1960s looked at other plasma models: Sturrock, Low, Suramlishvili, ...

MHD Properties

Flux Conservation (Alfv́en):

$$\Phi_S(t) = \int_S d^2x \cdot \boldsymbol{B} \qquad \leftarrow \qquad S \text{ moves with } \boldsymbol{V}$$

Magnetic Helicity Conservation (Woltjer):

$$C_B = \int_D d^3x\, \boldsymbol{A} \cdot \boldsymbol{B} \qquad \leftarrow \qquad \boldsymbol{B} = \nabla \times \boldsymbol{A}$$

Cross Helicity Conservation (Woltjer):

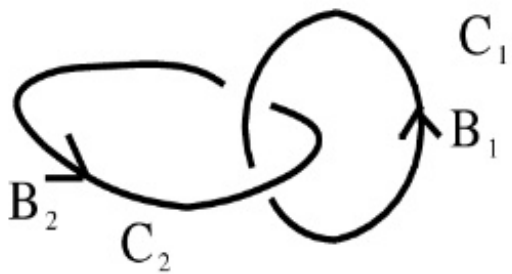
$$C_V = \int_D d^3x\, \boldsymbol{V} \cdot \boldsymbol{B} \qquad \leftarrow \qquad D \text{ is fixed volume}$$

Helicity Consequence

Magnetic helicity C_B is a "topological" quantity that measures, e.g., linking

$$C_B = \int_D d^3x\, \boldsymbol{A} \cdot \boldsymbol{B} = 2n\Phi_1\Phi_2\,, \qquad \Phi_{S_i} = \int_{S_i} d^2x \cdot \boldsymbol{B}\,,$$

where n is the linking number.



Similarly cross helicity is linking of $\boldsymbol{V}$ with $\boldsymbol{B}$ (Moffatt).

Religion: Beltrami states that minimize magnetic energy $\int d^3x\, |\boldsymbol{B}|^2/2$ at fixed C_B are stable and preferred:

$$\nabla \times \boldsymbol{B} = \lambda \boldsymbol{B}\,.$$

Actually merely an easy variational principle for equilibria. $\exists$ such variational principles for all MHD equilibria.

More MHD Helicity Etc.

- Taylor States, Helicity injection, toroidal current, continuous operation, etc. (Finn and Antonsen e.g.)

- W. Irvine (Chicago)
  - Generates and visualizes fluid helicity: $\int (\nabla \times \mathbf{v}) \cdot \mathbf{v}$
  - Examines knotted vortices and effect of viscosity.
  - For vortex bundle: Helicity = linking + writhe + twist
  - Helicity conserved upon reconnection but not linking.

Hamiltonian Field Theory (HFT)

Noncanonical Poisson bracket:

$$\frac{\partial \psi}{\partial t} = \mathcal{J}\frac{\delta H}{\delta \psi} = \{\psi, H\}\,,$$

Retains Poisson Bracket Properties:

antisymmetry $\longrightarrow$ $\{F, G\} = -\{G, F\}\,,$

Jacobi identity $\longrightarrow$ $\{\{F, G\}, H\} + \{\{G, H\}, F\} + \{\{H, F\}, G\} = 0$

Casimir Invariants

Helicities are Casimir Invariants:

$$\{F, C_{B,V}\}^{MHD} = 0 \qquad \forall \text{ functionals } F.$$

Casimirs are a consequence of Poisson bracket degeneracy.

Automatic if know Hamiltonian structure.

Extended MHD (XMHD) Scaled

Ohm's Law:

$$\boldsymbol{E} + \boldsymbol{V} \times \boldsymbol{B} = \frac{d_e^2}{\rho}\left(\frac{\partial \boldsymbol{J}}{\partial t} + \nabla \cdot \left(\boldsymbol{V}\boldsymbol{J} + \boldsymbol{J}\boldsymbol{V} - \frac{d_i}{\rho}\boldsymbol{J}\boldsymbol{J}\right)\right) + \frac{d_i}{\rho}\left(\boldsymbol{J} \times \boldsymbol{B} - \nabla p_e\right).$$

Momentum:

$$\rho\left(\frac{\partial \boldsymbol{V}}{\partial t} + (\boldsymbol{V} \cdot \nabla)\boldsymbol{V}\right) = -\nabla p + \boldsymbol{J} \times \boldsymbol{B} - d_e^2\, \boldsymbol{J} \cdot \nabla\left(\frac{\boldsymbol{J}}{\rho}\right).$$

Two parameters, $d_e = \frac{c}{\omega_{pe} L}$ measures electron inertia and $d_i = \frac{c}{\omega_{p_i} L}$ accounts for current carried by electrons mostly ... .

HMHD + IMHD = XMHD

Is XMHD a HFT?

Candidate Hamiltonian:

$$H[d_e; \boldsymbol{B}] = \int d^3x \left[\rho \frac{|\boldsymbol{V}|^2}{2} + \rho U(\rho) + \frac{|\boldsymbol{B}|^2}{2} + d_e^2 \frac{|\boldsymbol{J}|^2}{2\rho} \right]$$

$H[d_e; \boldsymbol{B}]$ is conserved. Pressure, $p = \rho^2 \partial U / \partial \rho$.

What is the Poisson bracket? Casimirs? Helicities?

HMHD, IMHD, and XMHD are HFTs

Yoshida, Abdelhamid, Kawazura, pjm, Lingam, Miloshevich

Poisson Brackets:

$$\begin{aligned}\{F,G\}^{HMHD} &= \{F,G\}^{MHD} + \text{new term} \\ &= \{F,G\}^{MHD} + \{F,G\}^{Hall}\end{aligned}$$

$$\{F,G\}^{IMHD} = \{F,G\}^{MHD} + \text{complicated terms}$$

$$\{F,G\}^{XMHD} = \{F,G\}^{MHD} + \text{complicated terms}$$

Casimirs:

There exist generalized helicities. Whence?

Three Remarkable Identities I

New magnetic variable:

$$\boldsymbol{\mathcal{B}}_i = \mathbf{B} + d_i \nabla \times \mathbf{V} \,.$$

Identity I:

$$\{\tilde{F}, \tilde{G}\}^{HMHD}[d_i; \mathbf{B}] \equiv \{F, G\}^{HMHD}\,[-d_i; \boldsymbol{\mathcal{B}}_i]$$

HMHD Helicity/Casimirs:

$$\begin{aligned} C_B &= \int_D d^3x\, \boldsymbol{A} \cdot \boldsymbol{B} \\ C_2 &= \int_D d^3x\, \boldsymbol{\mathcal{A}}_i \cdot \boldsymbol{\mathcal{B}}_i = \int_D d^3x\, \left(\mathbf{A} + d_i \mathbf{V}\right) \cdot \left(\mathbf{B} + d_i \nabla \times \mathbf{V}\right) \,. \end{aligned}$$

Three Remarkable Identities II

Introduce 'inertial' magnetic field $\mathbf{B}^\star$ and $\boldsymbol{\mathcal{B}}_e$:

$$\begin{aligned} \mathbf{B}^\star &= \mathbf{B} + d_e^2 \nabla \times \left(\frac{\nabla \times \mathbf{B}}{\rho} \right) , \\ \boldsymbol{\mathcal{B}}_e &= \mathbf{B}^\star - d_e \nabla \times \mathbf{V} , \end{aligned}$$

<u>Identity II</u>:

$$\{\tilde{F}, \tilde{G}\}^{IMHD}[d_e; \mathbf{B}^\star] \equiv \{F, G\}^{HMHD}[2d_e; \boldsymbol{\mathcal{B}}_e] \ .$$

IMHD Helicities/Casimirs:

$$C^{\pm}_{IMHD} = \int_D d^3x \ (\mathbf{A}^\star \pm d_e \mathbf{V}) \cdot (\mathbf{B}^\star \pm d_e \nabla \times \mathbf{V}) \ ,$$

Three Remarkable Identities III

Last new variable:

$$\mathbf{B}_{\lambda_\pm} = \mathbf{B}^\star + \lambda_\pm^{-1} \nabla \times \mathbf{V} \,, \qquad \lambda_\pm = \frac{-d_i \pm \sqrt{d_i^2 + 4 d_e^2}}{2 d_e^2} \,.$$

Identity III:

$$\{\tilde{F}, \tilde{G}\}^{XMHD}[d_i, d_e; \mathbf{B}^\star] \equiv \{F, G\}^{HMHD} \left[d_i - 2\lambda_\pm^{-1}; \mathbf{B}_{\lambda_\pm} \right] \,,$$

XMHD Helicities/Casimirs:

$$C^\pm_{XMHD} = \int_D d^3r \; (\mathbf{V} + \lambda_\pm \mathbf{A}^\star) \cdot (\nabla \times \mathbf{V} + \lambda_\pm \mathbf{B}^\star) \,,$$

Questions

- What do these helicities mean for theory and/or experiment?
- To what extent do these helicities reflect resilient propreties?
- What about more realistic magnetic reconnection?
- Can one inject them?
- Etc.