Optimal Deployment and Protection of High-speed Rail

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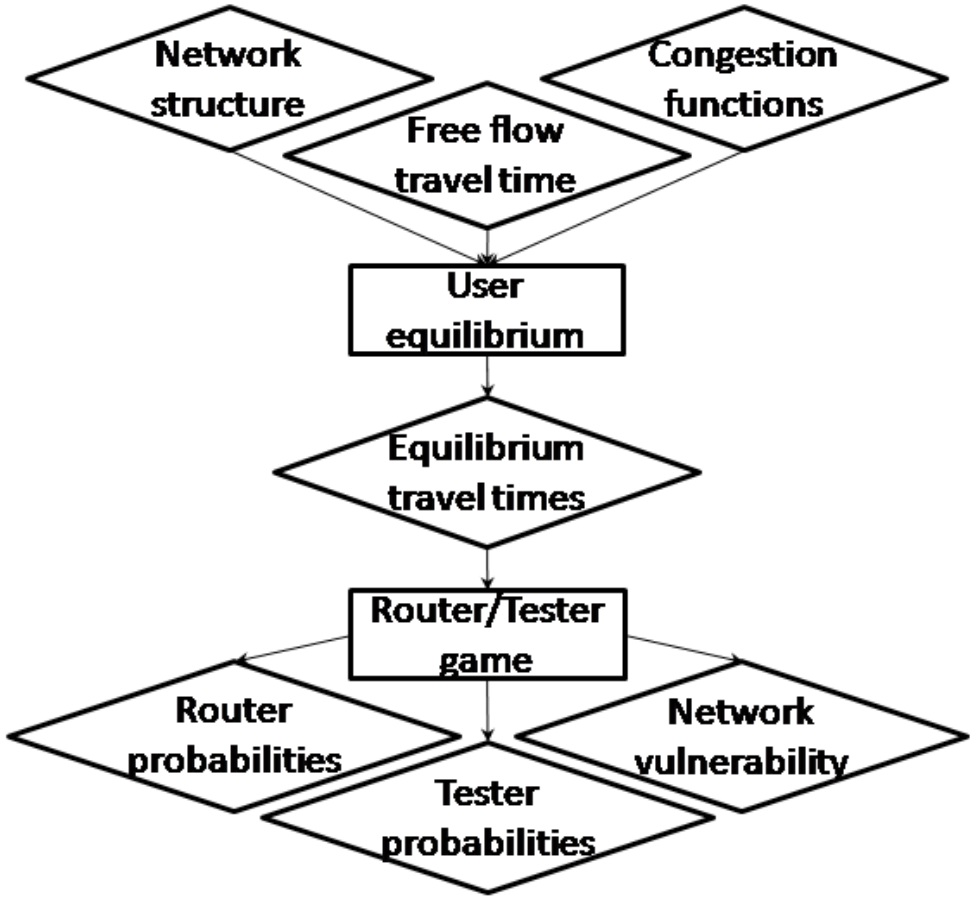
Motivation

- Billions of dollars to be invested in high-speed rail (HSR).
- Two primary concerns:
  - Supplement existing infrastructure (DOT).
  - Protect traveling public (DHS & TSA).
- Approach that satisfies all stakeholders must:
  - Include capacity planning.
  - Quantify network vulnerability and protection.

Overview

- Capacity planning:
  - Multi-modal transportation network (road, rail, aviation, etc…):
    - Network topology.
    - Intercity travel demand.
    - Highway congestion model.
  - Identify best combination of HSR lines to relieve congestion and minimize travel time.
- Network vulnerability:
  - Identify critical links.
  - Prevent attacks and mitigate damage by deploying technologies proportionate to risk.

Overview (contd..)



Problem definition

- User-equilibrium (UE):
  - Travellers from point A to B rationally distribute themselves so that their travel times are equal regardless of path taken [Wardrop 1952].
  - Traditionally solved with quadratic programming algorithm [Frank-Wolfe 1956].
  - Introduction of HSR complicates standard solution:
    - Slows convergence of underlying optimization problem.
    - Exploring new ways of modeling and solving problem.

Problem definition (contd..)

- UE formulation:

$$\min \sum_{a \in A} \int_0^{v_a} t_a(\omega) d\omega$$

$$s.t. \quad v_a = \sum_{r \in R} \sum_{s \in S} \sum_{k \in K_{rs}} f_k^{rs} \delta_{a,k}^{rs} \quad \forall a \in A$$

$$\sum_{k \in K_{rs}} f_k^{rs} = q_{rs} \quad \forall r \in R, \forall s \in S$$

$$f_k^{rs} \geq 0 \quad \forall k \in K_{rs}, \forall r \in R, \forall s \in S$$

- Minimize sum of travel times ($t_a(v_a)$) on links (A).
- Volumes (v_a) equal to sum of flows (f_k^{rs}) on link (a).
- Sum of k flows from r to s (f_k^{rs}) equal to demand from r to s (q_{rs}).
- Link flows (f_k^{rs}) positive.

Congestion function

- Bureau of Public Roads [BPR 1964] model:

$$t(v) = t_0\left(1 + B\left(\frac{v}{c}\right)^{\alpha}\right)$$

  – *t(v)* - travel time on link with volume *v*.
  – t_0 - free flow travel time (FFTT).
  – *B* - congestion coefficient (*0.15*).
  – *c* – link capacity.
  – α - rate of congestion (*4.0*).
- Possible that *v>c*.

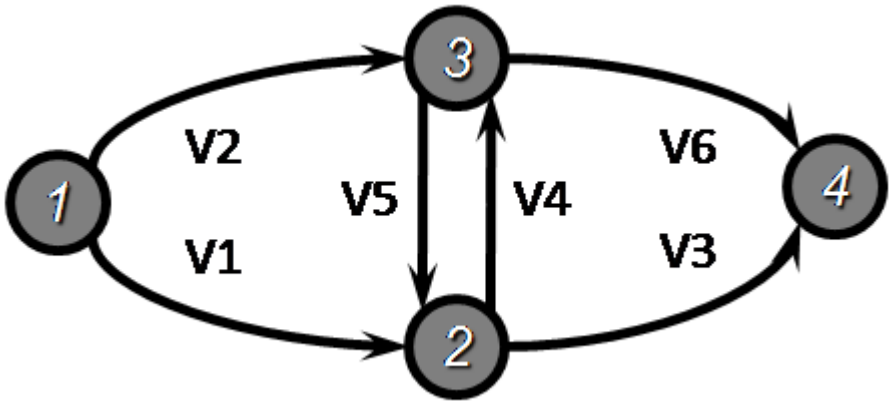
Problem solution technique

- **<u>INPUTS</u>**: Network topology, demand matrix, link congestion parameters.

1. Randomly allocate increments of demand.
2. While not equilibrium:
   - Find flows with minimum travel time.
   - Shift fraction of traffic to minimum travel time flow.

- **<u>OUTPUTS</u>**: Link and path volumes, travel times.

Case Study



$$Q = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Link #	V1	V2	V3	V4	V5	V6
FFTT	5	3	4	1	1	3
Capacity	2	1	2	1	2	1

- Baseline solution of UE: *29.70*.

Case Study (contd..)

- Sensitivity of UE to HSR:
  - HSR link compliments automobile link.
  - FFTT=*1/2* of parallel automobile link
  - HSR Capacity=*1*.
  - Not permitted *v>c*.

Link #	V1	V2	V3	V4	V5	V6
Travel time	**24.69**	25.39	25.37	28.68	29.70	25.75
Rank	**1**	3	2	5	6	4

Conclusions

- Large investment in HSR requires upfront deployment and protection studies.
- Must quantify capacity improvements and vulnerability.
- Developing efficient algorithm for finding user-equilibrium of bi-modal transportation network.

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