1 Motivation

We study the joint problem of detection and identification of a sudden and unobservable change in the statistical pattern of an information sequence to one of several alternative patterns. This problem has applications in various fields including sensor management, bio-surveillance, threat detection and identification. We propose a practical online change detection and identification strategy that is asymptotically optimal under the fixed-error and the Bayesian formulations.

Suppose we observe an information sequence, and it is crucial to detect a change in the threat level as early as possible and identify its cause as accurately as possible. The infected threat is initially low, but at some unknown and unobservable time, the threat level jumps suddenly to one of pre-described categories. To maintain the public safety, it is important to detect this change as early as possible, and at the same time, assess the nature of the change as accurately as possible in order to take the most appropriate countermeasures. This boils down to solving optimally the tradeoff between the detection delay cost and the false alarm and misdiagnosis costs to the society.

2 Problem description

A system is subject to be failed at some uncertain and unobservable time \( \theta \) by some unknown \( \theta \).

Cause \( \theta \) can be any one of \( M = \{1, 2, \ldots, M\} \) with \( P(\theta = \mu) = p_{\mu} \).

The decision time \( \tau \) is non-sequential and deterministic; \( P(\tau = 0) = p_{\tau} \) \( P(\tau > 0) = (1 - p_{\tau}) p_{\mu} \).

We want to:
- detect the decision time \( \tau \) as early as possible, and
- identify its cause \( \mu \) as accurately as possible.

Based on a sequence of observations, \( \mathbf{X} = \{X_n\} \), such as

\[
X_0, X_1, \ldots, X_0, X_0, X_0, X_0, X_0, \ldots
\]

condionally independent given \( \theta = \mu \).

We want to determine a decision rule \( (\tau, d) \), that declares at time \( \tau \) that the deflection has been triggered by cause \( \mu \) if and only if the trackoff among the decision delay cost \( R_{\tau}(\delta, d) = c(\tau - \theta) \) or \( \delta(\tau - \theta) \), and the decision loss cost \( \mathbf{C} = \{C_1, C_2, \ldots, C_N\} \). We seek \( \min_{\tau, d} \{R_{\tau}(\delta, d)\} \) such that for every \( \tau, \mu \),

\[
\mathbf{X} = \{X_n\}, \quad \mathbb{E}[\mathbf{X}] = \{\mathbf{E}[X_n]\}, \quad \mathbb{E}[\mathbf{X}^2] = \{\mathbb{E}[X_n^2]\},
\]

We define decision rule \( (\tau, d, \mathbf{X}) \) as the following:

\[
\tau = \inf \left\{ \mu > 0 \mid R_{\tau}(\delta, d) > 1 \right\} \quad \text{for some } \mu \in M \quad \text{and} \quad d = \arg \min_{\delta} R_{\tau}(\delta, d).
\]

4 Results

4.1 Asymptotic of each loss as \( \theta \rightarrow 0 \)

In order to choose the values of \( \theta \), the asymptotic optimality is crucial to know how the expected delay loss and the terminal decision loss behave as \( \theta \rightarrow 0 \). We can in fact obtain their approximations when \( \theta \) is small. Let \( \phi_{\tau, d} \) denote, respectively, the conditional probabilities given \( \theta = \tau \) and given \( \theta = 0 \) and \( \tau = d \). Define

\[
L_{\tau}(j) = \mathbb{E}[\phi_{\tau, d}(j)g(|q(j) - \delta|)], \quad j = 0
\]

in terms of the Kullback-Leibler divergence

\[
D_{KL}(j) = \int q(x) \log \left( \frac{q(x)}{p(x)} \right) dx, \quad j \neq 0.
\]

We compute the decision time loss:

\[
R_{\tau}(c) = -\log \mathbb{E}[\min_{\tau, d}(I_{\tau, d})] + a(4), \quad a(4) \leq a(0).
\]

The asymptotics of \( R_{\tau}(\delta, d) \) can be obtained by nonlinear renewal theory as in Reinhart and Verrallini [4]

\[
\mathbf{F} = \min_{\tau, d}\{R_{\tau}(\delta, d)\}
\]

Let \( \mathbf{F} = \min_{\tau, d}\{R_{\tau}(\delta, d)\} \) be the optimal decision time loss.

We define decision rule \( (\tau, d, \mathbf{X}) \) as the following:

\[
\tau = \inf \left\{ \mu > 0 \mid R_{\tau}(\delta, d) > 1 \right\} \quad \text{for some } \mu \in M \quad \text{and} \quad d = \arg \min_{\delta} R_{\tau}(\delta, d).
\]

We consider two formulations:

- the misdiagnosis cost

\[
\mathbf{F} = \min_{\tau, d}\{R_{\tau}(\delta, d)\}
\]

4.2 Asymptotic Optimality

Asymptotic optimality in the fixed-error formulation is obtained by setting \( A(\mu) = 1\) if the family \( \mathbf{X} = \{X_n\} \) is uniformly integrable.

\[
\mathbf{F} = \min_{\tau, d}\{R_{\tau}(\delta, d)\}
\]

\[
\mathbf{F} = \min_{\tau, d}\{R_{\tau}(\delta, d)\}
\]

The asymptotic optimality in the Bayesian formulation can be obtained by setting \( A(\mu) = 1\) if the family \( \mathbf{X} = \{X_n\} \) is uniformly integrable.