Development and A Priori Testing of Sparse Adaptive Pseudospectral Polynomial Chaos Expansions

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Abstract

A crucial aspect in the application of uncertainty quantification methods to complex models concerns the ability to sample quantities of interest efficiently. This poster presentation discusses the development and implementation of a sparse, adaptive sampling strategy to efficiently and reliably construct suitable polynomial chaos surrogates. The approach relies on a pseudospectral construction that accommodates arbitrary admissible sparse grids. An a priori analysis of the implementation and performance of the algorithm is conducted, based on existing databases of ocean global circulation simulations. The tests demonstrate that in the present setting order-of-magnitude savings are obtained for isotropic sparse sampling.

Model Test Case: Hybrid Coordinate Ocean Model

Four Uncertain Parameters

Parameter | Distribution
---|---
1 critical Richardson number | $U_l(0.25, 0.7)$
2 background viscosity ($m^2/s$) | $U_l(10^{-4}, 10^{-3})$
3 background diffusivity ($m^2/s$) | $U_l(10^{-5}, 10^{-4})$
4 stochastic wind drag coefficient | $U_l(0.2, 1.0)$

Gulf of Mexico during Hurricane Ivan (September 2004)

Polynomial Chaos Expansions

Fourier-like expansion over orthogonal basis function $\Psi_k$

$U(\xi, t) = \sum_{k=1}^{\infty} U_k(\xi, t)\Psi_k(\xi)$

Coefficients $U_k$ given by $U_k = \frac{(U, \Psi_k)_{\omega}}{(\Psi_k, \Psi_k)_{\omega}}$ where $(\cdot, \cdot)_{\omega}$ represents scalar product performed with numerical quadrature.

Full Database Solution: Sensitivity Analysis

Database of 385 realizations for up to fifth order polynomials, using Smolyak quadrature, and equal resolution in all dimensions. $T_i$ is the total sensitivity index due to the $i^{th}$ random variable.

Integrated SST Sensitivity Analysis

• 3rd and 4th dimensions dominate sensitivity
• Can accuracy be reduced in first two dimensions?
• Should accuracy be increased in the latter two dimensions?

Dimensionally Adaptive Representation

- Enrich along 3rd and 4th dimensions (128 additional realizations)
- Generate Latin-Hypercube sample with 256 realizations for verification

<table>
<thead>
<tr>
<th>Accuracy</th>
<th>P</th>
<th>N</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5.5.5.5)</td>
<td>126</td>
<td>385</td>
<td>Original</td>
</tr>
<tr>
<td>(5.5.7.7)</td>
<td>168</td>
<td>513</td>
<td>Enriched in 3 &amp; 4</td>
</tr>
<tr>
<td>(2.2.5.5)</td>
<td>36</td>
<td>73</td>
<td>Reduced in 1 &amp; 2</td>
</tr>
<tr>
<td>(2.2.7.7)</td>
<td>59</td>
<td>169</td>
<td>Enriched and Reduced</td>
</tr>
</tbody>
</table>

P: Number of Polynomials
N: Number of Realizations

Integrated SST (degrees C)

Integrated SST PDF at T=60hr comparison

69 Realizations
513 Realizations

Smolyak Pseudospectral Projection

Instead of a direct Smolyak quadrature, build final projection as a Smolyak sum of internal-aliasing-free projections

Example:

is a weighted superposition of

and all smaller projections

Remarks:

- In addition to avoiding internal aliasing, the Smolyak pseudospectral projection permits higher-order polynomials (especially monomial terms) with the same realization stencil
- Inherent in construction is an error indicator for adaptive refinement

Adaptive Refinement

- Start with a base representation
- Choose multi-index with highest indicator
- Enrich index with forward neighbors
  - Require the index remains admissible for proper telescoping
  - Require associated realizations be in database (for a priori testing)

Adaptive Driver Results

Conclusions:

- Reach error threshold with 69 realizations compared to 513
- Probability Density Function estimations are nearly identical
- Adaptive refinements leads to order-of-magnitude savings with no prior knowledge of full solution

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