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PISCEES - Quantification of Uncertainty in Sea-Level Rise from Next-Generation Ice Sheet Models

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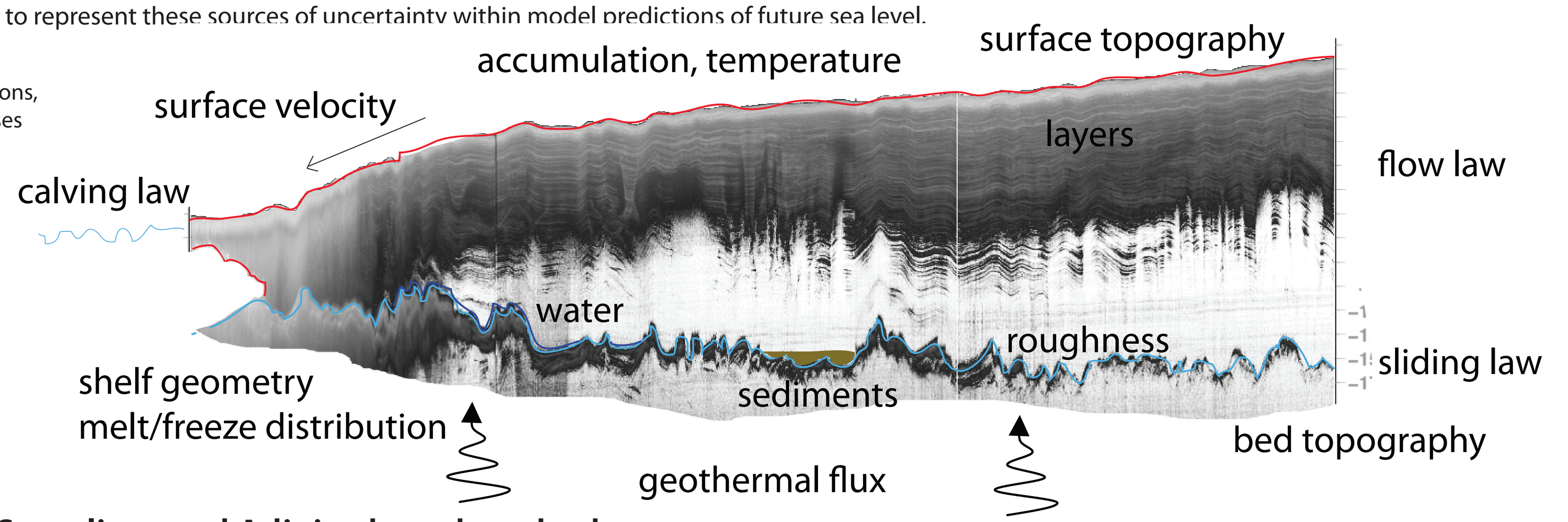
(1) Background and Motivation

The Initialization Problem

In order to conduct experiments about how the Antarctic and Greenland ice sheets could respond to changes in their environment, one needs to initialize the ice flow model so that it realistically simulates how modern ice sheets maintain their current states of mass balance. However, observations of Greenland and Antarctic ice sheets do not completely constrain boundary conditions and other processes, particularly at the rock-ice (basal) and ocean-ice (lateral) interfaces. This lack of adequate observations and understanding means there can be multiple ways to construct and initialize an ice flow model and maintain consistency with observations. A primary goal for PISCEES efforts in uncertainty quantification is to represent these sources of uncertainty within model predictions of future sea level.

Figure 1.1: Schematic of observations, boundary conditions, and processes affecting ice sheet initialization.



Solution Strategies: Sampling- and Adjoint-based methods

We will use non-intrusive (sampling-based) and intrusive (adjoint-based) methods to identify an optimal ice flow model initial state (velocities, temperatures, and thicknesses) and boundary conditions that minimize the distance to observations. These same methods can be used to seek the set of possible solutions around this optimal solution, given uncertainties in the data constraints, the model, and ice flow parameters. The impacts of these uncertainties on predictions of sea level can then be assessed through forward integrations with appropriate estimates of environmental forcing. The two approaches each have their strengths that address different challenges to this task.

$$\mathrm{cost}(\mathbf{m},\Omega) \rightarrow \text{log-likelihood}(\mathbf{m},\Omega) \rightarrow \left(\Psi(\mathbf{U},\mathbf{T},\mathbf{H}\mid\mathbf{m},\boldsymbol{\Omega})-\mathbf{Obs}\right)^{T}\mathbf{C}^{-1}\left(\Psi(\mathbf{U},\mathbf{T},\mathbf{H}\mid\mathbf{m},\boldsymbol{\Omega})-\mathbf{Obs}\right)$$

$\mathbf{U}$: velocity

$\mathbf{T}$: temperature

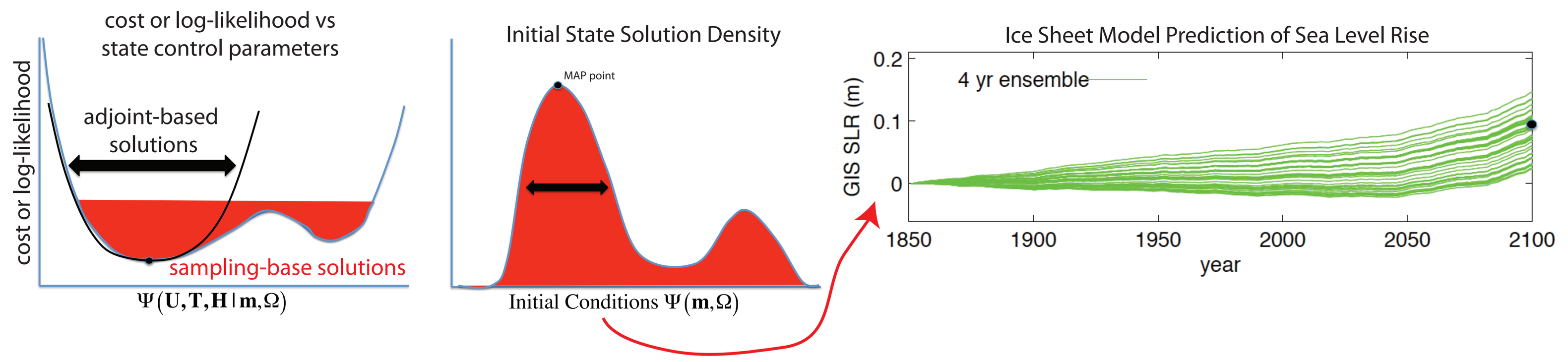
$\mathbf{H}$: thickness

$\mathbf{m}$: parameters

Ω : boundry conditions

$\mathbf{C}^{-1}$: inverse of covariance of errors

$\Psi(\mathbf{U},\mathbf{T},\mathbf{H}\mid\mathbf{m},\boldsymbol{\Omega}) \rightarrow$ initial state



Challenges

1. Sampling high dimensional parameter space uncertainties (e.g. 2d maps of basal traction coefficients, boundary conditions).
2. Data may be incompatible with model physics.
3. We assume steady state, although unlikely to be true.
4. Forcing from climate model contains long-term average errors (or "biases") within both the atmosphere and ocean models.

QUEST help with sampling-based solutions

1. Use DAKOTA to develop cheaply sampled surrogate ice sheet models, to represent behavior of models at arbitary points in parameter space
2. Parameterize high dimensional unknowns with a few parameters. Not clear how to do this for some boundary conditions such as topography and basal traction. An example for surface mass balance is given in Section 2.
4. Include descrepancy term or scaling factor in log-likelihood to account for biases.
5. How to account for the lack of treatment of transients in solutions?

QUEST help with adjoint-based solutions

1. Adjoints are invaluable for finding the MAP point, or optimal fit between the model and observations (e.g., see Section 3).
2. Derivative-based UQ seeks to exploit sensitivity (1st-derivative, or adjoint) and geometric (2nd-derivative, curvature or Hessian) information. How best to do this is one of our research goals, and we are exploring how best to make use of DAKOTA tools.
3. Adjoint-based UQ research is being conducted in the following three areas:
   i. inverse propagation: Combine parameter uncertainties and observation uncertainties to estimate a quality of the fit of the model to observational data.
   ii. derivative-enhanced sampling: Hessian eigenvectors are a close-to-optimal basis for sampling dominant uncertainties and reducing the space that describes what matters to science. This may help to address the "curse of dimensionality" and improve sampling efficiency.
   iii. forward propagation: From a defined quantity of interest, adjoint and Hessian information links parameter, model, and data uncertainties to prediction uncertainties.

(2) Sampling-Based Methods

Initial Goals

1. Incorporate DAKOTA libraries into code base and implement test problems. (below)
2. Represent surface-mass balance boundary condition forcing errors and estimate their impacts on predictions of sea-level rise using forcing from the Community Earth System Model. (below)
3. Develop initialization strategies that reflect Bayesian approaches to incorporating known uncertainties. (ongoing)

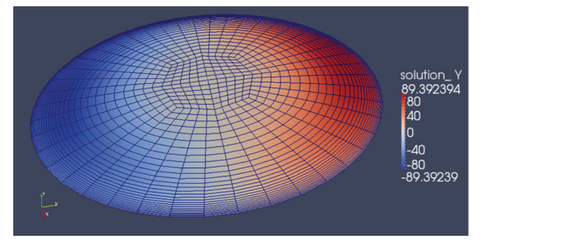


Figure 2.1: Ice sheet "Dome Problem" geometry with velocity in one dimension (coloring).

Synthetic Application #1: Use DAKOTA for Uncertainty Propagation

Here we demonstrate the capacity to use DAKOTA to perform a forward propagation of uncertainties in a single parameter sliding law for a simple ice "dome" test problem. The dome problem has a single basal sliding coefficent with a mean value of 1 and a sigma of 0.2 (KPa yr/m). DAKOTA managed 1000 samples run in Library mode (1 run of code = 1 setup cost). The result provides a PDF on model outputs (here, velocity) given uncertain parameter inputs.

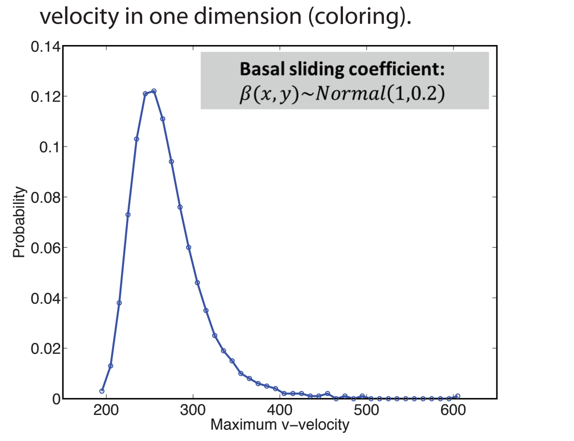


Figure 2.2: Forward propagation of uncertainty in the "Dome Problem" given uncertainty in basal sliding coefficient.

Synthetic Application #2: Use DAKOTA and QUESO for Bayesian Calibration

Here we demonstrate using the DAKOTA framework and the QUESO tool in collaboration with QUEST to conduct a Bayesian samping-based approach to initialize a dome ice sheet. Given observed velocities (here, synthetic model generated "data"), we estimate a joint probability to select 4 parameters in a polynomial sliding law. Sampling-based approaches have to work in lower dimensions, although in many cases one may find a low-dimensional way to parameterize high-dimensional unknowns.

Basal sliding coefficient:
$\beta(x,y) = \beta_0 + \beta_1 x + \beta_2 y + \beta_3 r$

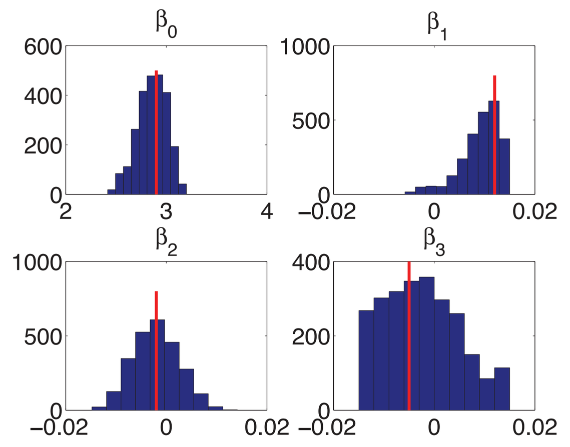


Figure 2.3: "Dome Problem" Bayesian calibration of four parameters within an idealized distribution of basal sliding coefficients.

Representing Boundary Condition Uncertainties

We focus here on representing uncertainties and biases in surface mass balance (SMB = accumulation minus ablation). While there are sophisticated observational and modeling efforts aimed at estimating SMB, our science experiments will generally rely on SMB as simulated in a global climate model, which can have sizable differences from more data-driven estimates. Our approach[1] is to use the scatter that is present in short-term average SMB that will contain the imprint of individual weather events, rather than long-term climatological means, in order to represent the amplitude and spatial correlations that exist in modeled SMB biases. Because we hold these shorter-term averages constant, excessive accumulation or ablation can be amplified as ice sheet growth or decay, in response to these anomalies, feeds back onto itself and amplifies the errors. We have learned that the types of errors that exist in simulated SMB can create uncertainty of more than 100% in Greenland's future contribution to sea level.

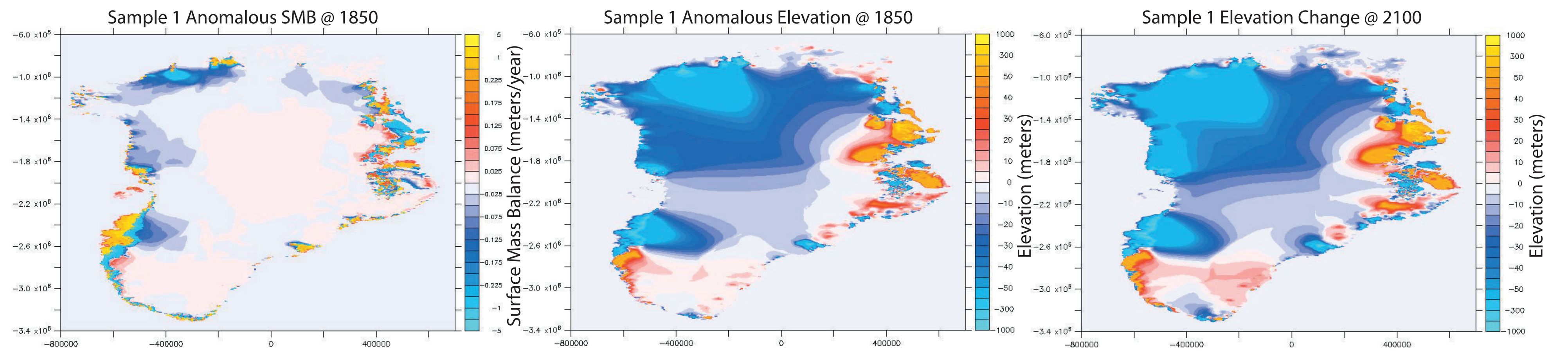


Figure 2.4: (left) Surface Mass Balance (SMB) bias in one ensemble member taken from a 4-year mean of a 300 year long preindustrial control simulation. (center) SMB is applied to an ice flow model for 9000 years to equilibrate to the new SMB forcing. The excessive ablation occuring on the east and northeast sections result in an ice sheet that is more than 1 km below the average ensemble elevation. (right) With imposed changes to predicted climate out to 2100, the pattern of elevation change is similar to the anomalies that exist at 1850, meaning that all feedbacks amplify the original anomalies.

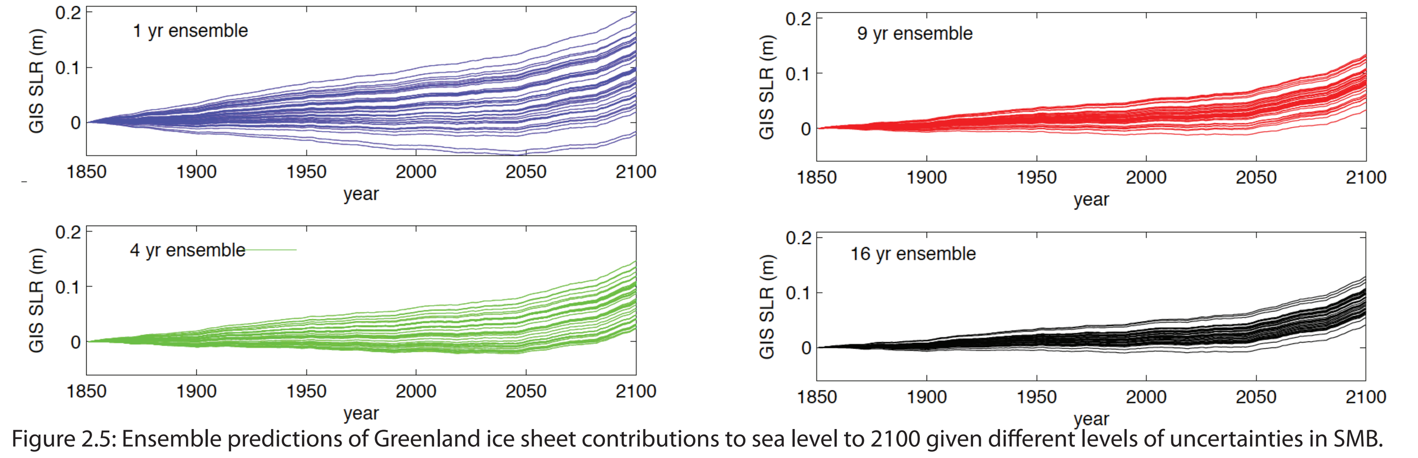


Figure 2.5: Ensemble predictions of Greenland ice sheet contributions to sea level to 2100 given different levels of uncertainties in SMB.

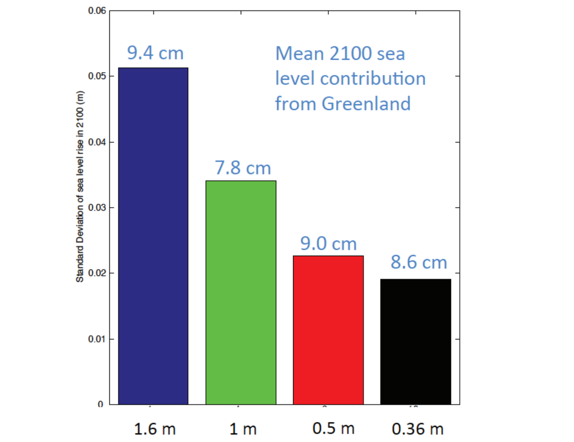


Figure 2.6: Standard deviation in uncertainties in sea-level rise at 2100.

(3) Progress Using Adjoint-Based Methods

Problem 1: Coupled Ice Sheet and Climate Model Initialization

A number of methods for optimizing uncertain ice sheet model parameters have been proposed and applied recently (e.g 2,3). One example[2] is shown in Fig. 3.1, in which modeled ice speeds (right) are compared to those from a target field (left). Despite the good match between the two, the flux divergence implied by the model velocity field, shown in Fig. 3.2 (right), is noisy and unphysical. This presents a significant problem for coupling between ice sheet models and climate models: in steady-state, the surface mass balance (snow accumulation minus melting), provided by the climate model, should balance the flux divergence. A sample surface mass balance field from a climate model is shown in Fig. 3.2 (left). The difference between the two will lead to a "shock" to the ice sheet model when it is coupled to the climate model and forced with realistic surface mass balance. Evidence for such a shock can be seen in Fig. 3.3; the ice sheet volume initially grows in size. Even after 50 yrs the unphysical initial transient has not damped out. Non-physical initial transients of this type must be removed (or at least minimized) for coupled ice-sheet and climate model projections to be of use for sea-level rise projection.

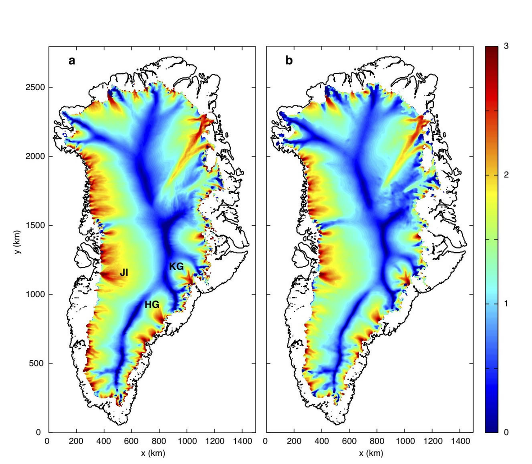


Figure 3.1: Depth-averaged ice speed for Greenland based on observations (left) and from a tuned ice sheet model[2] (CISM) (Ice speeds are on a log10 scale).

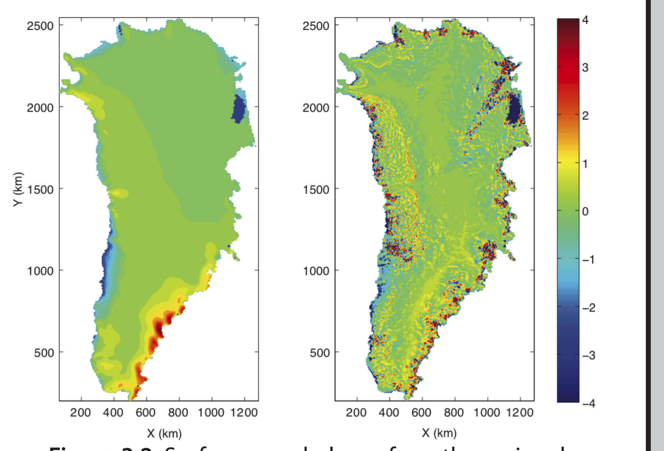


Figure 3.2: Surface mass balance from the regional climate model RACMO[4] (left) vs. the flux divergence calculated from the velocity field shown in the right panel of Fig. 3.1. Colorbar is in units of m / yr.

A New Approach

Under DOE PISCEES, we are developing new methods for adjoint-based optimization[6]. The goal is to derive optimal ice sheet model initial conditions for coupled, forward model simultions, which avoid the type of initalization "shock" shown in Figure 3.3.

Goal: Find an initial condition such that the ice sheet is at quasi-thermomechanical equalibrium with a given initial geometry (from observations) and surface mass balance (from a climate model) and the mismatch between modeled and observed velocities is minimized. Model parameters that are optimized include the 2d (x,y) fields describing the frictional basal sliding coefficient, β, and the observed ice thickness, H, which has a specified range of uncertainty.

$$\frac{\partial H}{\partial t} = \underbrace{-\mathrm{div}\,(\mathbf{U}H)}_{\text{flux divergence}} + \overbrace{\tau_s}^{\text{surface mass balance}}, \qquad \mathbf{U} = \frac{1}{H}\int_z \mathbf{u}\,dz. \qquad \text{At equilibrium: } \mathrm{div}\,(\mathbf{U}H) = \tau_s$$

Basal boundary condition: $(\sigma\mathbf{n} + \beta\mathbf{u})_{\parallel} = \mathbf{0} \quad on \quad \Gamma_\beta$

$\boldsymbol{U}$: Model depth-averaged velocity
H : Ice thickness
β : Basal sliding coefficient
τ_s : Surface mass balance

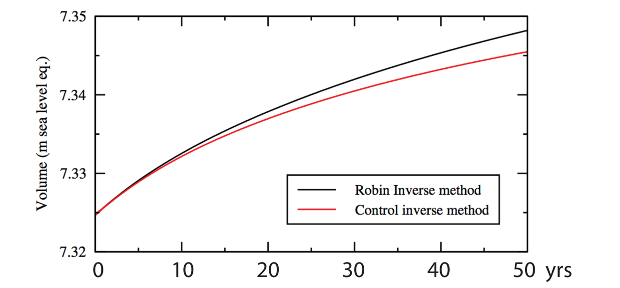


Figure 3.3: Drift in ice sheet mass balance (in units of m of cumulative sea-level rise) after initialization using two different control methods[5].

Optimization Problem: Find β and H that minimize the cost functional $\mathcal{J}$:

$$\begin{aligned}\mathcal{J}(\beta,H) = \; & \tfrac{1}{2}\alpha_d \int_\Gamma |\mathrm{div}(\boldsymbol{U}H) - \tau_s|^2\, ds + && \text{- flux div. vs. surf. mass bal. mismatch -}\\ & \tfrac{1}{2}\alpha_v \int_{\Gamma_{top}} |\mathbf{u} - \mathbf{u}^{obs}|^2\, ds + && \text{- model vs. observed velocity mismatch -}\\ & \tfrac{1}{2}\alpha_H \int_\Gamma |H - H^{obs}|^2\, ds + && \text{- model vs. observed thickness mismatch -}\\ & \mathcal{R}(\beta) + \mathcal{R}(H) && \text{- regularization terms -}\end{aligned}$$

The cost functional $\mathcal{J}$ is minimized under the constraint that velocities obey a 3d, 1st-order approximation to the Stokes flow equations with constant ice temperatures. Tikhonov regularization is applied to both β and H. Optimization, which uses the *Moocho* package from *Trilinos*, applies Sequential Quadratic Programming and LBFGS for approximating the reduced Hessian, in order to simultaneously reduce the residual of the constraint (the ice dynamics model) and the cost functional. First derviatives for the constraint and cost functional are provided by the *LifeV* finite element library[7].

Results: Fig 3.4 shows the results for an idealized test case where β is initially specified as a doubly-periodic "egg crate" pattern. Noise is then added to the forward modeled velocities and the calculated flux divergence and treated as synthetic "data" for the optimization problem, which recovers the initial β field, albeit with some error. Fig. 3.5 shows the method applied to a realistic, coarse-resolution (20 km) Greenland ice sheet problem, using observed ice sheet geometry and velocity fields.

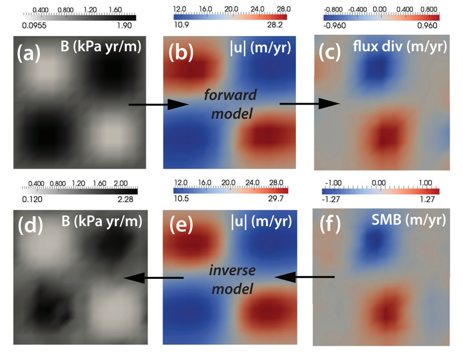
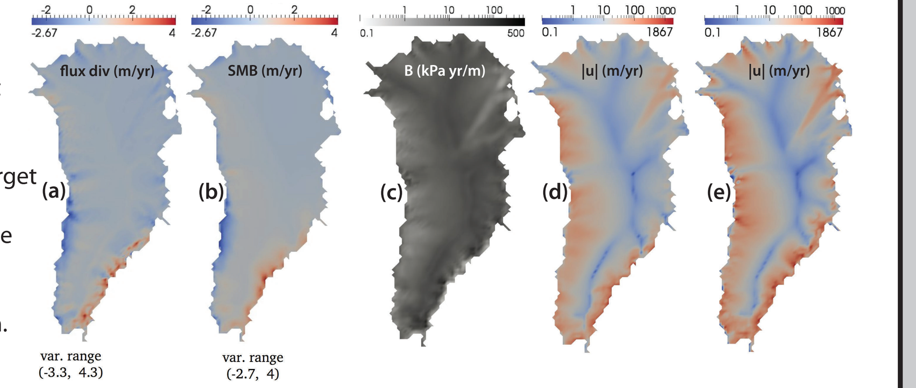


Figure 3.4: (left) top row: (b) Forward modeled velocity and (c) flux divergence for (a) specified sliding coefficient field. bottom row: (a) Recovered sliding coefficient field when using noisy input (e) velocities and (f) surface mass balance as optimization inputs.

Figure 3.5: (right) (a) Model flux divergence compared with (b) target surface mass balance for (c) basal sliding coefficients recovered using realistic geometry and velocities for Greenland (compare the model flux divergence here with that shown in Figure 3.2 above). The modeled and target velocity fields are shown in (d) and (e), respectively. Ice thickness uncertainty was held constant at 200 m.



Problem 2: Idealized Ice-Ocean Coupling Simulations

Adjoint-based approaches have also been used to perform time dependent (synthetic) data assimilation and sensitivity analysis for icealized, ice-ocean coupled simulations[8,9].

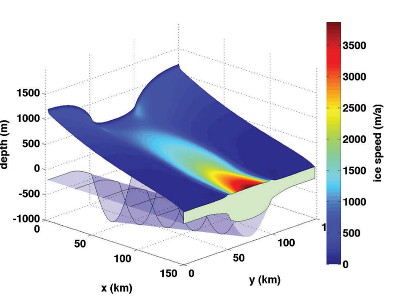


Figure 3.6: (left) Idealized ice-ocean coupled initial condition[8] showing ice sheet / shelf velocity (color) and geomtry. The upstream boundary (x=0) represents grounded ice coupled to an ice sheet. The ice begins to float by bouyancy near x=100 km.

Figure 3.7: (right) Adjoint-diagnosed sensitivity of volume loss for the coupled system to submarine melting rates[8]. Counterintuitively, sensitivity is the greatest at margins (y~55-60, 90-95) rather than at the deepest part of the grounding line (x~110).

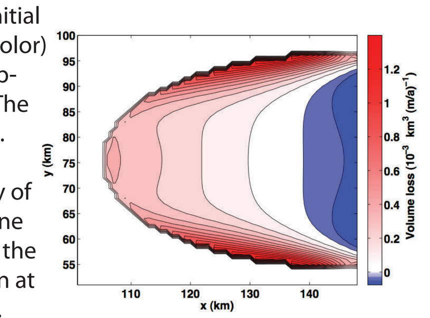
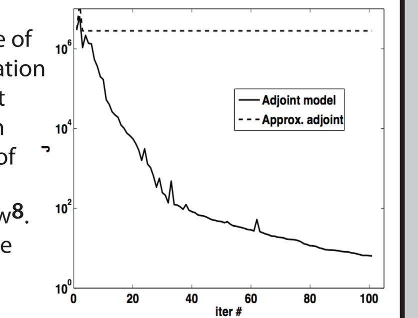


Figure 3.8: (right) Convergence of optimization with data assimilation when using a linearized adjoint (dashed line) vs. when using an adjoint that takes full account of the nonlinear nature of the momentum balance for ice flow[8]. In the former case, convergence is not always realized.



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