Strongly Coupled Field Theories at the Energy Frontier

Ethan T. Neil (Fermilab)
for the USQCD Collaboration

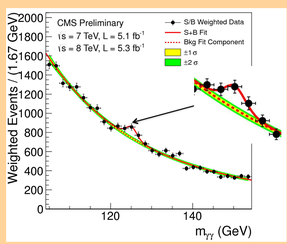
Abstract

The Large Hadron Collider is rapidly pushing back the frontiers of high-energy physics, most recently with the discovery of a new particle consistent with the Higgs boson. An interesting extension of the Standard Model is provided by "composite Higgs" models, in which the Higgs boson exists as a bound state of more fundamental particles. Because such models involve new strong forces, numerical lattice simulations are crucial for understanding their dynamics. I describe how the libraries of the QCD-API will be generalized in SciDAC-3 to the gauge groups and types of matter found in new strongly coupled theories.

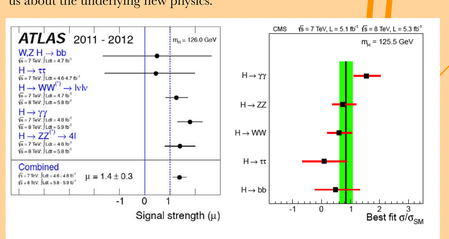
The Higgs boson

The Standard Model of particle physics is a unified framework which contains all visible matter in the universe, and describes all of its interactions (except for gravity.) The predictions of the Standard Model have been verified to fantastic accuracy. For the most precisely-measured quantities, theory and experiment agree at the level of 1 part per billion. Until this year, the Standard Model contained one particle which was theorized but never observed; the *Higgs boson*, whose field gives mass to all other particles.

On July 4, 2012, CERN announced the discovery of a new particle consistent with the long-sought Higgs boson. The two flagship experiments at the Large Hadron Collider, ATLAS and CMS, each announced evidence for the new particle with a confidence level of "5 sigma", or about 1 chance in 3,500,000 that the results are a statistical fluctuation. The Higgs boson is too unstable to be seen directly, but its signature is most easily found through decays to pairs of photons, as shown in the below plot from CMS. Decay products of the new particle stand out as a "bump" on top of a smooth spectrum of background radiation.

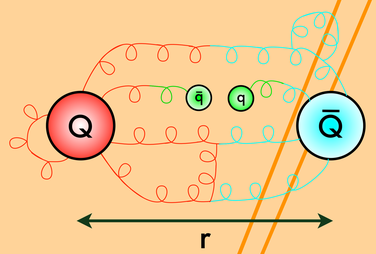


The role of the Higgs field in generating particle masses places strict requirements on how it must couple to all of the other particles. As a result, the various decay rates of the Higgs boson into different particles should be related by known factors, and together provide a strong confirmation that the new particle is indeed the Higgs. Experimental results for these decay rates, normalized to the theoretical predictions, are shown below. Although the results are broadly consistent with the Standard Model Higgs boson, there are hints of deviations from the expected rates in certain decays. As the experimental results become more precise, we must consider the possibility of a non-standard Higgs sector, and what its properties might tell us about the underlying new physics.



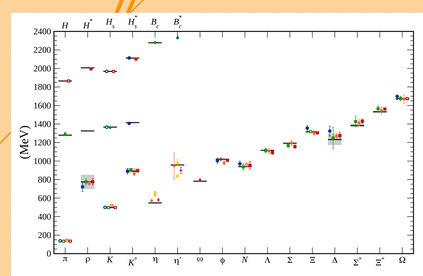
Composites and strong forces

One of the leading possibilities for new physics that could change the properties of the Higgs sector is the "composite Higgs" scenario. In this class of models, the Higgs boson is not fundamental, but instead exists as a bound state of some new particles. The strong forces binding the Higgs together would be described by a theory similar to "quantum chromodynamics" (**QCD**), the theory of the strong nuclear force. As opposed to the theory of electromagnetism, where there is a single unit of charge, the charges of QCD come in three "colors": red, green, and blue.



QCD is a strongly-coupled theory, in the sense that for many processes, a perturbative expansion in terms of single-particle exchanges fails to converge. Interactions between colored particles ("quarks") result from a sea of many-particle exchanges, as in the sketch above. Gluons, the force carriers of QCD, interact with all colored particles, including other gluons. The properties of these gluon exchanges lead to a condition known as "asymptotic freedom", which dictates that the force between pairs of colored particles actually *increases* as they are separated. This effect is opposed by the spontaneous creation of pairs of quarks, which "screen" the force between the real fermions, partially canceling the effects of gluon exchanges.

The theory of QCD has been thoroughly tested and verified by experiment. Because many theoretical computations in QCD are inaccessible through perturbative expansion, non-perturbative numerical techniques (primarily, "lattice QCD") have been essential in the study of the strong nuclear force and to compare with experimental results. Below, a set of composite-state masses in QCD is shown, comparing predictions from lattice simulations (colored points) to experimental results (black lines). Using a very small subset of these states to fix the input parameters for the theory (shown as open symbols below), the predictions for the remaining states are seen to be in excellent agreement. This is an important confirmation of both the theory of QCD itself and of the numerical techniques used in order to obtain these results.



If a similar strongly-coupled theory does lie beneath the newly-observed Higgs, then lattice simulations will be similarly crucial in fully understanding its properties. As shown above, we would expect to again find a rich set of experimental predictions from a modest number of input parameters.

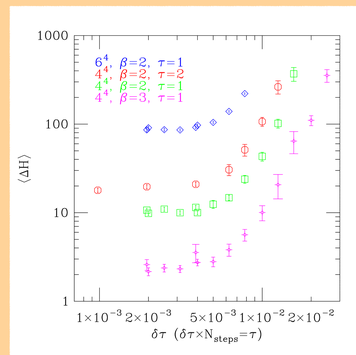
Moving beyond QCD

The mathematical description of a strongly-coupled force admits a much wider class of parameters than just the particular set which describe QCD. The number of colors and number of quarks could easily be different in a new, strongly-coupled sector, and changing either of these can have drastic effects on the dynamics which must be studied numerically.

As a first example, we can consider three-color theories with additional species of quarks. If too many quarks are added, their screening effects can completely negate the confining effect of the gluons, and there will be no strongly-bound states (and thus no candidates for a composite Higgs.) The exact number of quarks needed to trigger this effect can only be investigated non-perturbatively, through numerical studies. Furthermore, novel physical effects are generically predicted in confining theories with enough quarks to be near to the transition point; these effects can also be searched for and quantified in lattice simulations.

From a software perspective, relatively little modification to the existing code is required in order to work with a three-color theory including arbitrary numbers of quarks. Including duplicate copies of the quark contributions as already written into the Monte Carlo evolution is the main requirement. Within many evolution codes, this inclusion is as simple as changing a single parameter.

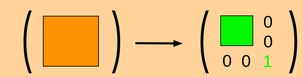
However, carrying out a complete lattice simulation requires a careful tuning of the input and evolution parameters. Intuition gained from years of experience with this tuning procedure in simulations of QCD does not generally apply to different theories with new dynamics. While pushing into unexplored regions of parameter space, algorithmic issues and bugs in the existing code can be revealed.



An example is shown in the plot above, using the MILC application code. In this simulation, a Markov Chain Monte Carlo step is proposed based on a numerical integration of the current state of the lattice. In the plot, change in the observable H, which is preserved by an exact integration, is plotted against the integration step size. The expected scaling (quadratic in the step size) is violated at very small step sizes. The bug leading to this effect was present even for QCD simulations, but it was discovered by simulations of a many-fermion theory. The large cancellation between gluon and quark forces in such a theory forces the use of much smaller integration step sizes, where this effect was revealed.

Changing colors in SciDAC-3

A more substantial departure from QCD is to change the number of colors in the theory. The case of two-color theories is particularly interesting, with theoretical arguments pointing to a unique spectrum of relatively light bound states compared to theories with more colors. In a strongly-coupled theory with N colors, the gluons can be represented by an NxN symmetric, complex matrix with zero trace. With three colors, such a matrix is described by 8 real numbers, while the two-color matrix requires only 3. The lower computational cost for dealing with these smaller matrices is an extra motivation for working with two-color theories.

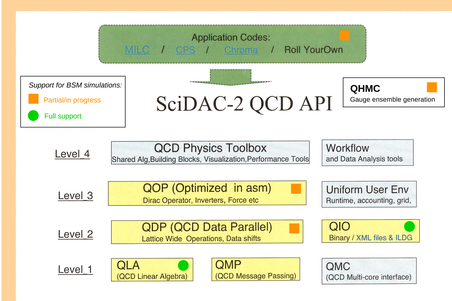


Changing the number of colors in the simulation codes is a much more difficult task, with many subroutines optimized or even hard-coded to work with 3x3 matrices only. The fastest way to implement two-color simulations is by "hacking" the three-color code to use a 2x2 submatrix, as shown above. However, this approach is far from ideal, discarding a large fraction of the performance gain expected from working with smaller matrices.

The need for an efficient, flexible code base to support lattice simulations in non-QCD theories, particularly theories with various numbers of colors, is becoming urgent in light of the rapidly developing experimental situation. There are already some scattered, private codes which have been built for the study of composite Higgs theories. However, there is a clear advantage to developing common and optimized codes which are available to the entire community. The SciDAC-2 QCD API, a set of low-level libraries which underpin most modern lattice simulation application packages, provides such a foundation for simulations of QCD. Thus, the best way to proceed is through modification of the existing API to extend support to more general theories. This will be one of the main efforts under SciDAC-3.

Currently, the lowest-level libraries of the QCD API, such as QIO (input/output support) and QLA (linear algebra), already support the use and efficient manipulation of general NxN matrices. Other components like QMP (message passing) are not sensitive to the implementation of color matrices, and so already provide support for more general simulations.

Under SciDAC-3, the implementation of this same level of support for general theories will be extended to the higher-level libraries. In addition, a new application package, QHMC, is under development. Unlike the other API libraries, which are implemented in C or C++, QHMC is implemented in the Lua language. This allows for greater flexibility in use and scripting of the application, without compromising speed, since the numerically intensive computations are carried out using the low-level libraries. QHMC will support simulations of QCD, but is also designed from the beginning with the study of more general theories in mind.



Studying the Higgs

For any numerical studies of theories different from QCD, regardless of the choice of colors or quark number, a common consideration must now be the study of the bound state which can be identified as a composite Higgs boson. The corresponding state in QCD simulations is known as the sigma meson, and it is known to be very difficult to study numerically.

The first step in the study of a particular bound state is the determination of its mass. The probability for a particle to "propagate" between two points is exponentially suppressed by its mass, so that the mass can be determined using a numerical evaluation of this probability. In lattice simulations, this is expressed in terms of the "propagator" M^{-1}, a very large matrix which gives this probability between any two points on the lattice:

$$\mathrm{Tr}\left[M^{-1}(0,t)\Gamma M^{-1}(t,0)\Gamma\right] \sim e^{-m_H t}$$

Even for lattices of modest size, the matrix M is very large, so it is impractical to compute its inverse exactly. A common technique is to use stochastic or "noisy" estimators in order to evaluate the trace shown above. The first step is to draw N_r random vectors containing some sort of noise (Gaussian, Z_2, etc.) which averages to unity:

$$\sum_r < \eta_r^\dagger \eta_r >= 1$$

Next, the system

$$M\psi_r = \eta_r$$

is solved using standard methods, e.g. conjugate-gradient. Dotting the noise vector into the solution of the above system then gives a stochastic estimate for any component of the inverse matrix:

$$M^{-1}_{ij} \approx \sum_r \psi_r^i \eta_r^j$$

Unfortunately, the same properties which make the sigma state viable as a composite Higgs boson lead to difficulty in studying it using standard methods. The ability of a Higgs to decay democratically to many types of particles, which is linked to the generation of particle masses, leads to significant amounts of noise in the numerical study of its propagator, with contributions from a large number of diverse intermediate states. The control of additional variance introduced by stochastic estimation is therefore crucial in the study of a candidate composite Higgs.

A technique known as "dilution" offers a way to make stochastic estimates more precise. A set of N_D noise vectors, each with support on only a subset of the vector space, are drawn. Although this increases the cost of the technique, now requiring N_D solutions of a system of equations, there will be a corresponding reduction in the variance of the trace estimate, as noise in off-diagonal components of the matrix is explicitly zeroed out. Dilution is used already in the study of QCD, but software built on top of the SciDAC libraries explicitly directed at the study of composite Higgs states in general theories is currently under development.

Summary

The discovery of a Higgs-like particle at the LHC represents a breakthrough for physics at the energy frontier, and the precise study of the new particle's properties in the coming years will provide stringent tests of the Standard Model, and may well reveal the presence of new physics. For strongly-coupled theories which underpin models in which the Higgs is a composite state, there is a large parameter space available, and numerical studies are needed to fully understand the dynamics of such theories and make contact with experiment. Lattice simulations, supported by SciDAC-3 through the modification of QCD libraries to deal with more general strongly-coupled theories, may be essential in understanding the Higgs sector as it is revealed by experiment.