Reduced collisional dynamics in strongly magnetized plasmas

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I. RESEARCH FRONTIER

The existence of widely separated time scales observed in a strongly-magnetized nonuniform plasma offers exciting theoretical and numerical opportunities in the study of magnetically-confined charged particles. In a strong magnetic field, for example, the shortest orbital time scale involves the fast gyromotion of a charged particle about a magnetic-field line. When the time scales of interest are much longer than the gyromotion time scale, there are significant theoretical and computational advantages in adopting a reduced dynamical representation of magnetically-confined charged-particle dynamics, whereby the fast gyromotion time scale has been asymptotically decoupled from the slow reduced dynamics. Among the computational advantages of this reduced dynamical description are the adoption of longer time steps in kinetic particle simulations and the opportunity to consider realistic magnetic geometries.

Significant advances have been achieved through the derivation of reduced dynamical equations by Lie-transform methods resulting from the asymptotic elimination of fast time scales. Examples of such reduced dynamical equations include the guiding-center equations for charged particle motion [1, 2] and the nonlinear gyrokinetic Vlasov-Maxwell equations for the self-consistent kinetic treatment of particles and fields [3]. In general, dynamical reduction is carried out by transformations on dynamical variables, which not only reveals the existence of adiabatic invariants and other approximate conservation laws but also allows the development and use of theoretical models that focus on physical mechanisms affecting the long-time confinement of charged particles in magnetized plasmas [4]. We note that transformations generated by Lie-transform methods are not restricted to Hamiltonian systems and can also be applied to dissipative equations.

The formulation of collisional plasma kinetic theory in terms of the adiabatic invariants of guiding-center and bounce-center motion of charged particles allows a compact representation of transport processes in strongly magnetized inhomogeneous plasmas [5]. The combination of the compact representation of general collision operators with the reduced plasma gyrokinetic and gyrobounce-kinetic descriptions provides several theoretical and computational advantages in the investigation of transport processes in laboratory and space magnetized plasmas over long dissipative time scales [6]. The reduced magnetohydrodynamic descriptions of strongly magnetized laboratory, space, and astrophysical plasmas provide a formulation of global nonlinear plasma dynamics in which the fast time scale associated with compressional Alfvén waves is asymptotically eliminated. Lie-transform methods can not only provide a systematic procedure for the dynamical reduction of dissipationless and dissipative fluid models, but can also systematically introduce ponderomotive effects on large spatial-scale or long time-scale dynamics associated with the asymptotic elimination of small spatial-scale or short time-scale dynamics.

II. ADVANCES BEYOND THE FRONTIER

The Fokker-Planck collision operator [7] represents an essential paradigm for the introduction of collisional effects in a kinetic plasma theory [6]. While the standard Fokker-Planck collision operator does not represent the effect of a single collision event (instead it represents the effects of multiple small-angle collisions), it is nevertheless useful to consider Fokker-Planck "collisions" as transitions between Hamiltonian orbits. Since Hamiltonian orbits do not cross in particle phase space (and hence the initial conditions associated with a single Hamiltonian orbit are never lost), these collisional transitions allow for loss of information about initial conditions to occur (and thus entropy to increase). Consequently, the Fokker-Planck formalism can be used to describe irreversible transport processes in nonuniform magnetized plasmas.

In strongly magnetized plasmas, the characteristic collisional time scale is much longer than the typical gyromotion time scale, leading to the small dimensionless parameter $\epsilon_\nu \equiv \nu/\Omega = \rho/\lambda \ll 1$ defined in terms of the collision frequency ν (or the collisional mean free path λ) and the gyrofrequency Ω (or the thermal gyroradius ρ). If we now introduce the small parameter $\epsilon_B \equiv \rho/L_B \ll 1$ associated with the guiding-center dynamical reduction [1, 2], where L_B denotes the characteristic magnetic nonuniformity length scale, we can also introduce the useful dimensionless mean-free-path parameter $\Lambda_B \equiv \epsilon_B/\epsilon_\nu = \lambda/L_B$. This new parameter enables us to differentiate between three different collisional regimes. In the collisional regime $\Lambda_B \ll 1$, collisions are so frequent that particles never sample the nonuniformity of the magnetic field and, hence, for all practical purposes, collisions are viewed as taking place in a uniform magnetic field (i.e., $\lambda \ll L_B \to \infty$). In the neoclassical regime $\Lambda_B \simeq 1$, collisions are infrequent enough that particles do sample

the magnetic-field nonuniformity (and topology) and magnetic-drift effects manifest themselves directly in transport coefficients (e.g., finite-banana-width effects). Lastly, in the collisionless regime $\Lambda_B \gg 1$, collisions are rare so that particles mostly follow Hamiltonian orbits over long time scales before they experience random transitions. In this regime, it is necessary to treat the magnetic field as being nonuniform since $L_B \ll \lambda$.

A guiding-center Fokker-Planck collision was derived by Lie-transform methods in 2004 [8] for arbitrary values of the mean-free-path parameter Λ_B. This guiding-center collision operator was also derived explicitly to describe the collisions of energetic test-particles with a near-Maxwellian field-particle background plasma, which was then implemented in several Fokker-Planck numerical codes, most recently in a five-dimensional Monte Carlo scheme [9].

The exact energy-momentum conservation laws of a guiding-center Fokker-Planck collision operator, expressed in terms of the Landau form, were recently [10] investigated in the limits of finite ϵ_B and finite truncation order in ϵ_B. These exact conservation laws play fundamental roles in understanding the numerical accuracy of particle simulations. They have so far been applied only in linearized δf codes in which the background magnetic-field nonuniformity has been ignored (i.e., in the collisional regime $\Lambda_B \ll 1$).

III. IMPACT OF RESEARCH IN PLASMA SCIENCE

Given the importance of the collisionless gyrokinetic formalism in the analytical and numerical investigations of magnetized plasmas, the inclusion of self-consistent collisional effects in the neoclassical and collisionless regimes ($\Lambda_B \geq 1$) within the gyrokinetic formalism is of crucial importance in our ability to understand the complex nonlinear dynamics of magnetized fusion plasmas over long time scales.

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