A Many to Many Game Theory Approach to Measuring Transportation Network Vulnerability

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Introduction

- Vulnerability of a transportation network strongly correlated with network's ability to withstand shocks and disruptions.
- High-volume edges with limited alternative paths represent obvious system vulnerabilities.
- May not be the whole story…
- Game theoretic approach offers approach that is theoretically and conceptually agreeable

Background Review (1)

Source	Methodology	Objectives/Result
Bell et al. (2002, 2003, 2008),	Game theoretic network vulnerability approaches that envision games between router and demon	Develop mixed strategy games for vulnerability assessment
Bar-Noy et al. (1995)	Use different approaches to the tester problem	Shown that the tester's problem is NP-hard
Murray-Tuite and Mahmassani (2004)	A non-zero-sum game envisioned between an evil entity and a traffic management agency	To identify vulnerable edges in a transportation network
Murray-Tuite (2008)	Incorporation of two types of substitution (method and target) into a methodology	Determine a transportation system's risk profile
Ukkusuri and Yushimito (2009)	A heuristic method incorporating a user equilibrium assignment procedure	To assess the importance of highway transportation networks

Background Review (2)

Source	Methodology	Objectives/Result
Latora and Marchiori (2005)	Method based on the difference in shortest paths between baseline and disrupted networks	To identify critical edges
Scott et al (2006)	a system-wide approach though the method was not optimization- or game-based	Identify critical links and evaluate network performance
Laporte et al (2010)	Corresponding mixed enlarged game yield robust network designs	designing an railway transit network that optimizes a certain utility function when failures occur

Context: Mixed strategy adversarial game

- We adopt the game-base approach to: Establish a vulnerability metric for all O-D pairs that is computationally efficient for large-scale application.

- Two-Player Game
  - Assumes perfect information
  - Non-cooperative

Role:	Router	Tester
Goal	Seeking shortest paths for all travelers	Disable Link(s) that most harm of network
Tool	Route assignment strategy	Link failure strategies

Objective Function

The objective of the game between router and tester can be represented as a mini-max formulation

$$min_{\gamma} max_{\rho} V(\gamma, \rho) = \sum_{e \in E} \gamma_e^n \rho_e^n C_e^{F,n}$$

Subject to

$$\sum_{e \in E} \rho_e = 1, \rho \geq 0; \quad \sum_{k_p \in K_p} h_{k_p} = 1, h > 0; \quad \sum_{k_p \in K_p} h_{k_p} a_{e,k_p} = \gamma_e$$

h_{k_p}	path choice probability between O-D pair, *p*
a_{e,k_p}	parameter that takes value 1 if $e \in k_p$ and 0 otherwise
d_p	travel demand between O-D pair, *p*
C_e^F	cost of edge *e* in failure scenario *F*
x_e^n	edge use probability differential of edge *e* at iteration *n*
ρ_e^n	probability the tester disables edge *e* at iteration *n*
γ_e^n	probability the router chooses edge *e* at iteration *n*

Solution Approach

Step 0: Initialize network

Step 1: Update edge costs under Tester's failure scenario,

Step 2: Calculate *s*-expected edge costs,

Router

Step 3: Identify shortest path(s) for all *O-D pairs*

Step 4: Calculate Router's edge use probability differential

Step 5: Update Router's edge use probability using MSA

Tester

Step 6: Calculate Tester's edge failure probability,

Step 7: Calculate Network Vulnerability

Step 8: If no change, stop. Otherwise, go to Step 1

Calculate edge failure probability

To solve the tester's strategy problem, a weighted entropy function was been proposed to calculate edge failure probability (Bell, 2008). The modified objective function is:

$$min_{\gamma} max_{\rho} V^n(\gamma, \rho) = \sum_{e \in E} \gamma_e^n \rho_e^n C_e^{F,n} - \left(\frac{1}{\theta}\right) \sum_{e \in E} \rho_e^n ln\, \rho_e^n$$

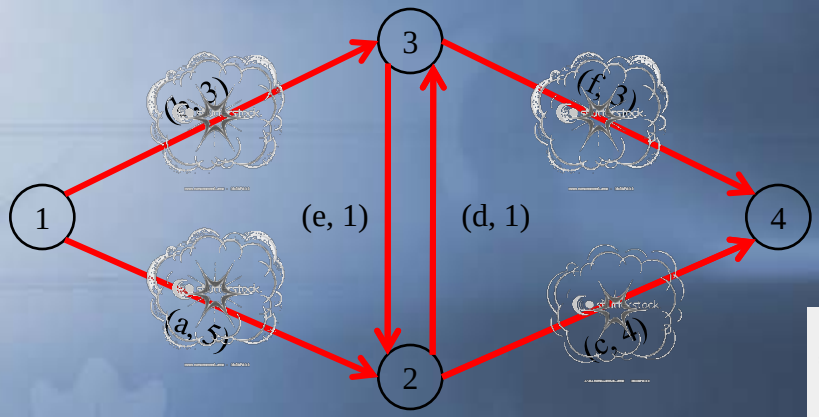
The explicit solution is:

$$\rho_e^{n+1} = \frac{exp\left(\theta \gamma_e^n C_e^{F,n}\right)}{\sum_{s \epsilon E} exp\left(\theta \gamma_s^n C_s^{F,n}\right)}$$

The parameter θ been called the aggressiveness of the tester, could also be interpreted as the confidence of their strategy.

Sample Demonstration

Figure below depicts the 4 nodes and 6 links sample network, with the edge identifier and edge cost noted in parenthesis above each edge.

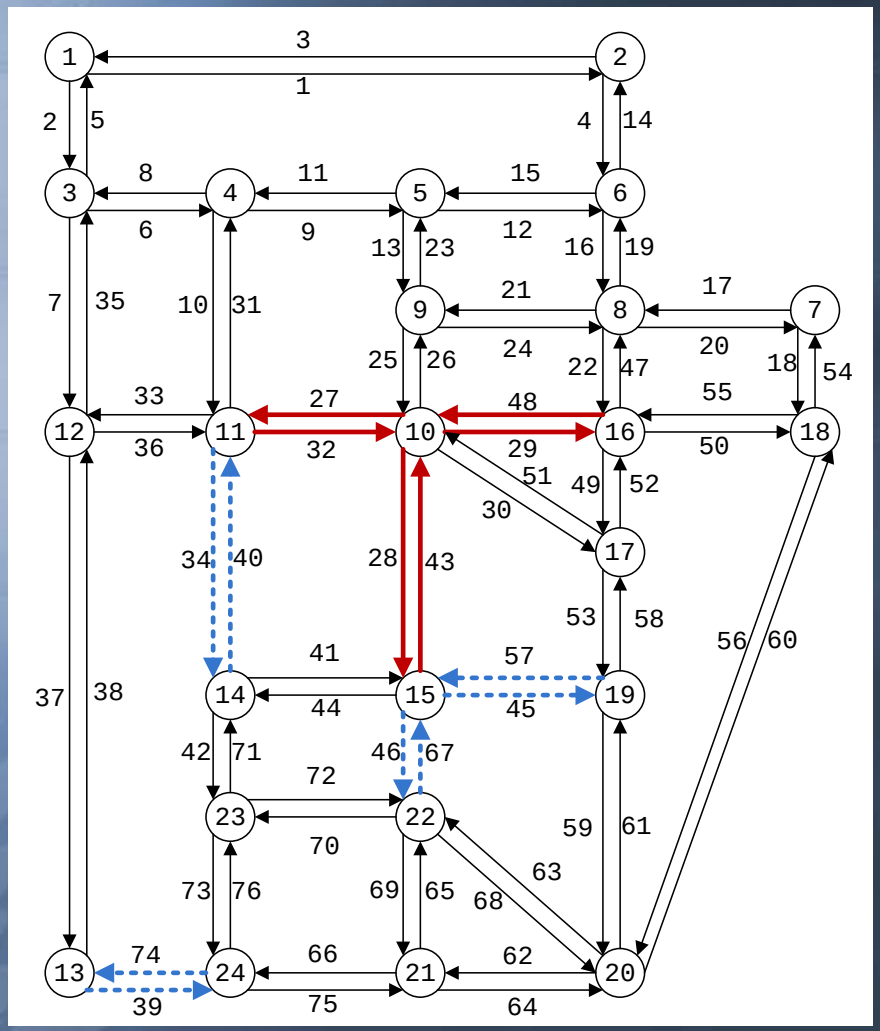


O-D Pair	Demand
(1, 2)	1
(1, 3)	1
(1, 4)	1
(2, 3)	1
(2, 4)	2
(3, 4)	1

	1st Iteration		2nd Iteration	
O-D Pair	SP	Cost	SP	Cost
(1, 2)	b-e	4	a	5
(1, 3)	b	3	a-d	6
(1, 4)	b-f	6	a-c	9
(2, 3)	d	1	d	1
(2, 4)	d-f, c	4	c	4
(3, 4)	f	3	e-c	5

		1st Iteration			2nd Iteration		
e	C_e^-	S_e^n	γ_e^n	ρ_e^{n+1}	S_e^n	γ_e^n	ρ_e^{n+1}
a	5	5	0	0.001	5.036	0.214	0.371
b	3	3	0.429	0.490	16.237	0.214	0.044
c	4	4	0.143	0.014	4.496	0.286	0.531
d	1	1	0.286	0.003	1.030	0.286	0.007
e	1	1	0.143	0.002	1.015	0.143	0.004
f	3	3	0.429	0.490	16.237	0.214	0.044

Sioux Falls network Application



The Sioux Falls network is the well-known and well-studied , it has 24 nodes, 76 edges and 552 O-D pairs.

High Critical

Median Critical

Low Critical

$\theta = 0.5$			$\theta = 1$			$\theta = 10$			Ukkusuri & Yushimito (*8*)	
Iter = 9114 RT= 63.5 s			Iter = 3442 RT = 23.0 s			Iter = 5049 RT= 40.3 s				
Edge #	Failure%	Use %	Edge #	Failure%	Use %	Edge #	Failure%	Use %	Edge #	Critical
4	31.51%	27.15%	4	34.08%	24.66%	4	34.78%	23.20%	39	80.34%
6	13.97%	29.87%	39	16.62%	29.03%	39	20.55%	28.87%	4	71.78%
12	13.96%	29.87%	6	15.69%	28.88%	6	17.66%	28.83%	**75**	67.87%
39	10.99%	28.67%	12	15.65%	28.88%	12	16.87%	28.82%	64	50.02%
20	4.93%	32.88%	36	5.23%	17.43%	36	8.32%	19.10%	20	46.23%
7	4.74%	24.46%	20	2.89%	32.87%	64	1.81%	18.84%	16	42.96%
2	2.98%	22.14%	2	2.87%	24.64%				1	30.43%
64	2.89%	14.66%	7	2.68%	24.47%				2	26.19%
75	2.62%	28.67%	64	1.95%	15.78%				60	20.17%
36	2.60%	14.31%	32	0.91%	17.42%				50	19.09%
32	1.27%	14.31%	75	0.91%	29.03%				7	17.61%
68	1.18%	14.01%	29	0.16%	17.43%				54	16.08%
16	0.95%	32.88%	68	0.11%	13.25%				37	12.65%
9	0.70%	29.87%	16	0.11%	32.87%				6	11.02%
29	0.62%	14.31%	9	0.05%	28.88%				**26**	9.02%
37	0.47%	17.24%	37	0.03%	17.25%				12	7.84%
60	0.35%	11.42%	60	0.02%	11.78%				**9**	7.79%
38	0.33%	14.83%	38	0.01%	14.47%				3	7.71%
50	0.20%	11.55%							68	4.63%
56	0.18%	8.20%							**76**	4.61%
18	0.18%	16.08%							**72**	4.48%
35	0.17%	7.73%							**30**	4.06%
65	0.14%	14.01%							**52**	3.99%
1	0.14%	4.51%							**32**	3.84%
47	0.07%	2.77%							**10**	3.30%
All other Failure % < 0.07%			All other Failure < 0.01%			All other Failure < 0.01%				

Conclusion/Future work

- **Conclusion**
  - Estimates the many-to-many Transportation Network.
  - Method adopts a game-theoretic framework and heuristics method.
  - Solves the problem rapidly and efficiently.

- **Future work**
  - Improve Tester/ Router strategies
  - Computational efficiency in large network
  - Multi-model transportation

Acknowledgment

- This material is based upon work supported by the U.S. Department of Homeland Security under Grant Award Number 2008-ST-061-TS002. The views and conclusions contained in this document are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the U.S. Department of Homeland Security.

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