A Markov Model of Particle Transport in Indoor Air

Rachael M. Jones, MPH
Doctoral Candidate
University of California, Berkeley

This research is funded by
U.S. EPA - Science To Achieve Results (STAR) Program and U.S. Department of Homeland Security University Programs
Grant # R83236201

Context

- Infection risks are poorly understood:
  - Risk=f(**exposure**, dose-response)
- Computational Fluid Dynamics (CFD) simulations are detailed, expensive, time-consuming and sensitive to boundary conditions
- Mathematical modeling techniques often treat aerosols as gases/vapors

Why model exposures?

- Describe exposures that cannot be monitored (i.e. low concentrations, highly toxic/pathogenic)
- Capture spatial and temporal variability
- Describe loss of pathogen viability
- **Predictive!**

Markov Chain: Definition

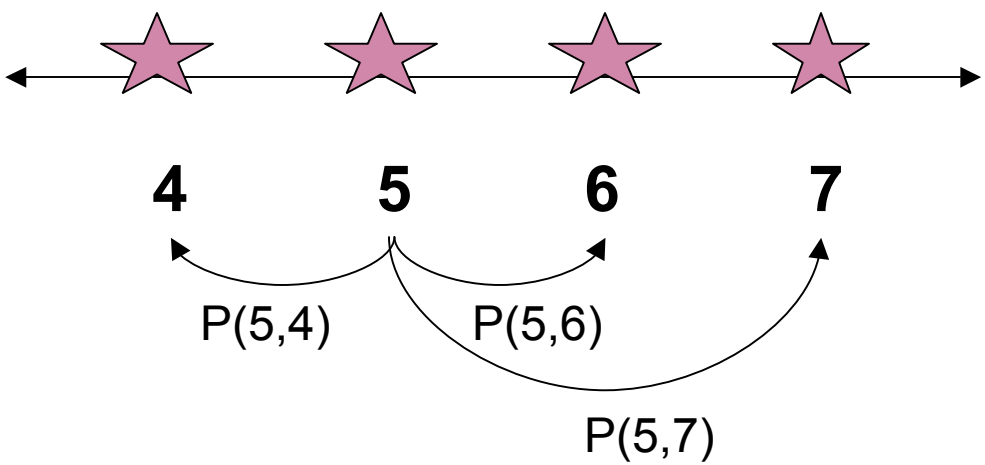
- A statistical process that retains **no memory**: Only the current state influences where the process goes next
- Has a countable number of states

  $S=\{s_o, s_1, s_2, \ldots s_n\}$
- Has a countable time index

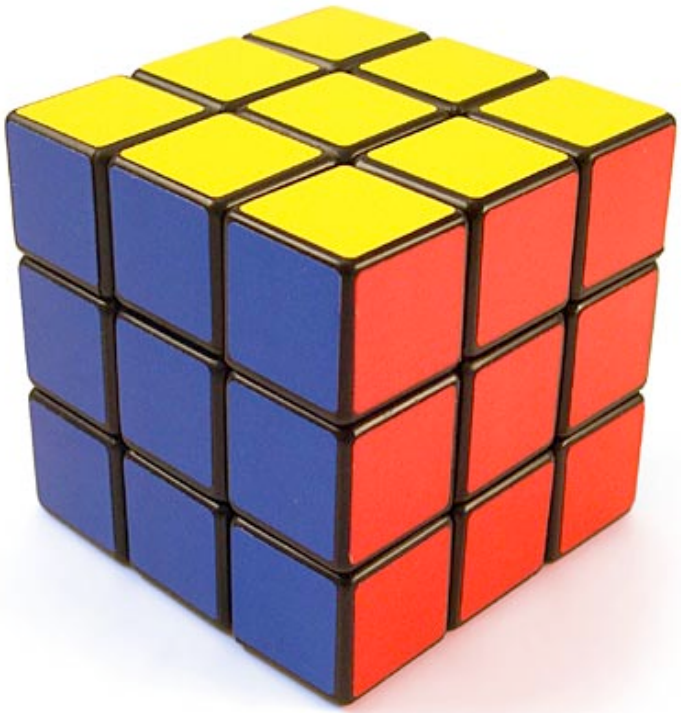
  $T=\{0, 1, 2, \ldots\}$

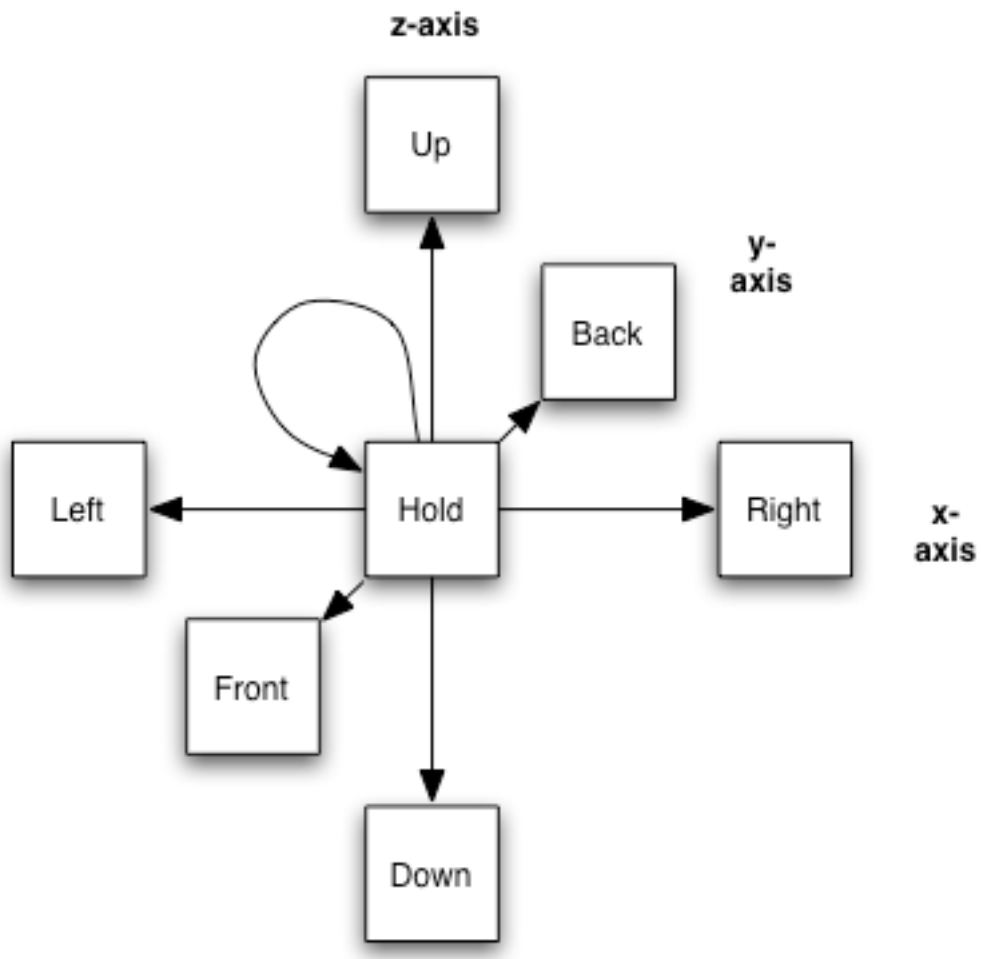
A 1-D Markov Chain

A 1-Dimensional Walk, or the walk of a drunkard:

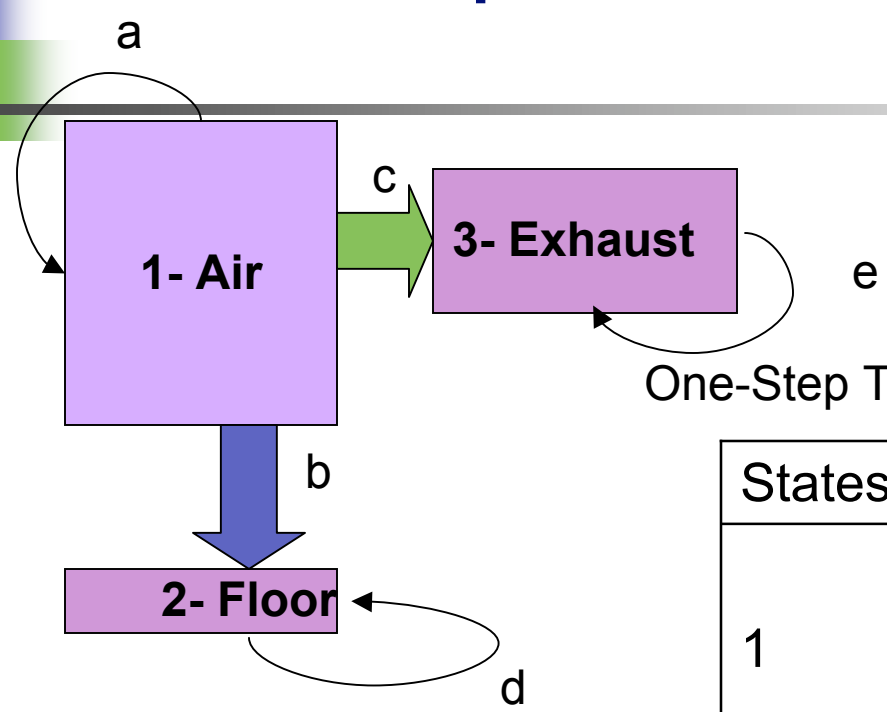


A 3-D Markov Model





An Example: One Step

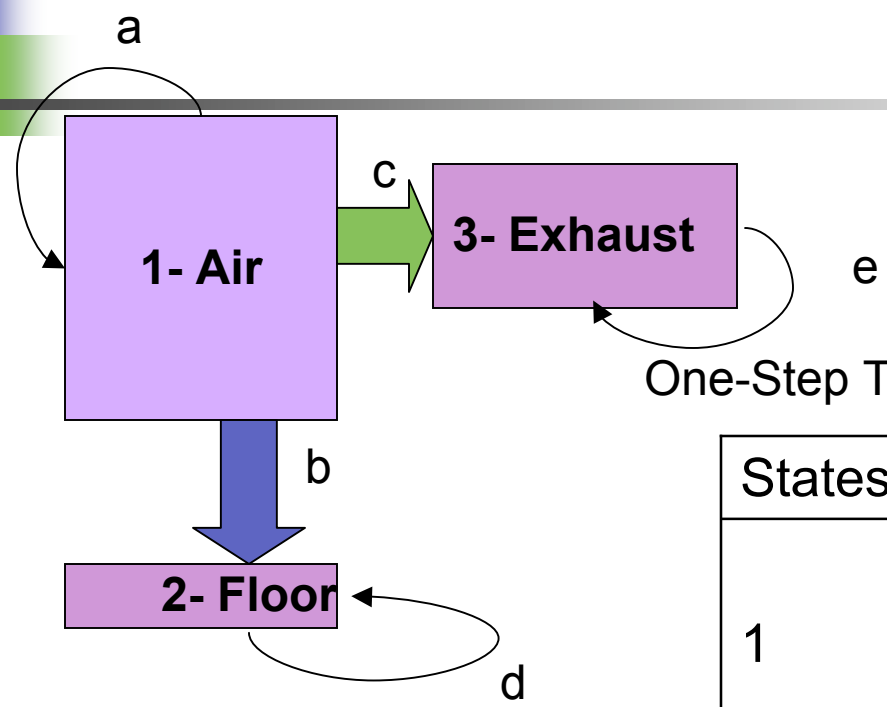


One-Step Transition Probability Matrix: **P**

States	1	2	3
1	a	b	c
2	0	d	0
3	0	0	e

Note: Each row in the matrix **P** must sum to one:
a+b+c=1; d=1; e=1
Also: Well-mixed box

An Example: One Step

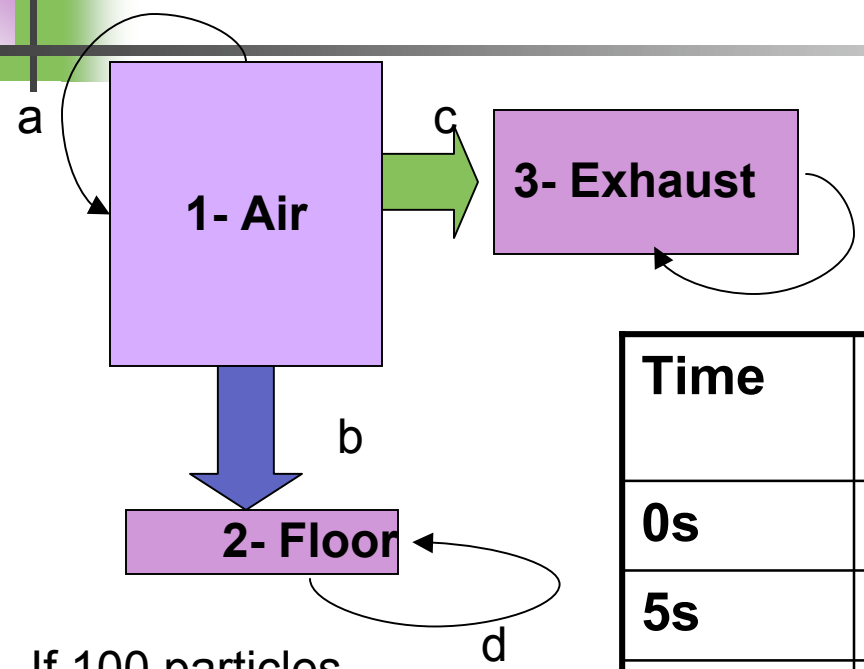


One-Step Transition Probability Matrix: **P**

States	1	2	3
1	0.982	0.003	0.016
2	0	1	0
3	0	0	1

Assume 5 second time steps: In one time step the transition probabilities are:

Over time… matrix multiplication



Probability that a particle initially present in state 1 (air) moves into 2 or 3

Time	1- Air	2- Floor	3- Exh
0s	1	0	0
5s	0.9816	0.003	0.0154
1min	0.80012	0.0326	0.01672
15min	0.0353	0.01573	0.8074
30min	0.0012	0.1628	0.8359

If 100 particles initially released, the expected number of particles in state 1 after 15 minutes is:
100 x 0.0353 = 3.5

Movement by 1st Order Rates

- Gravitational Settling, $\lambda_{TS} = V_{TS} / \Delta L$
- Turbulent Diffusion, $\lambda_{TD} = 6D_T/(\Delta L)^2$
- Advective Flow, $\lambda_{AF} = V_{AF}/\Delta L$
- Vertical Surface Deposition,
  - $P(dep) = V_D / \Delta L$
  - V_D from models

Transition Probabilities without Deposition

- $P(hold|TS)=\exp(-\lambda_{TS}\Delta t)$
- $P(hold|AF)=\exp(-\lambda_{AF}\Delta t)$
- $P(hold|TD)=\exp(-\lambda_{TD}\Delta t)$

- Assume independence:

$P(hold)= P(hold|TS)\, P(hold|AF)\, P(hold|TD)$

- $P(leave)=1-P(hold)$

Transition Probabilities without Deposition

The probability of leaving the a state in the Markov model in the **–z** or **+x** direction in one time step:

$$P(leave,-\mathbf{z}) = P(leave) \times \frac{\lambda_{TS} + \lambda_{AF,-\mathbf{z}} + \frac{1}{6}\lambda_{TD}}{\lambda_{TS} + \lambda_{AF} + \lambda_{TD}}$$

$$P(leave,+\mathbf{x}) = P(leave) \times \frac{\lambda_{AF,+\mathbf{x}} + \frac{1}{6}\lambda_{TD}}{\lambda_{TS} + \lambda_{AF} + \lambda_{TD}}$$

Advantages to the Markov Model

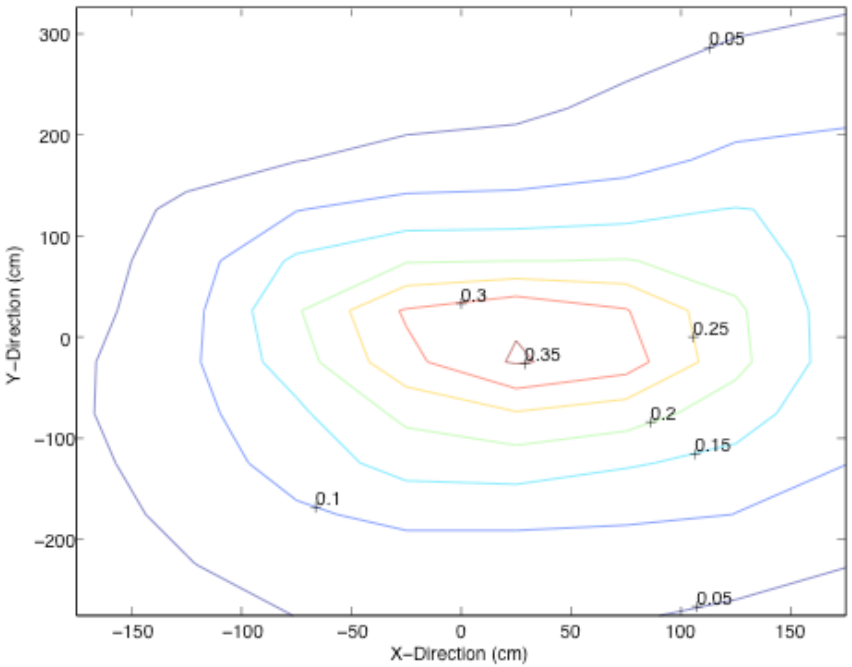
- Use **measured** or **simulated** data for velocity field, turbulent diffusion and deposition velocities
- **Long-duration** simulations
- Incorporate **gravitational settling**
- Improved **temporal and spatial resolution** over other zonal models
- **Quick** and **cheap** to simulate

Testing the Markov Model

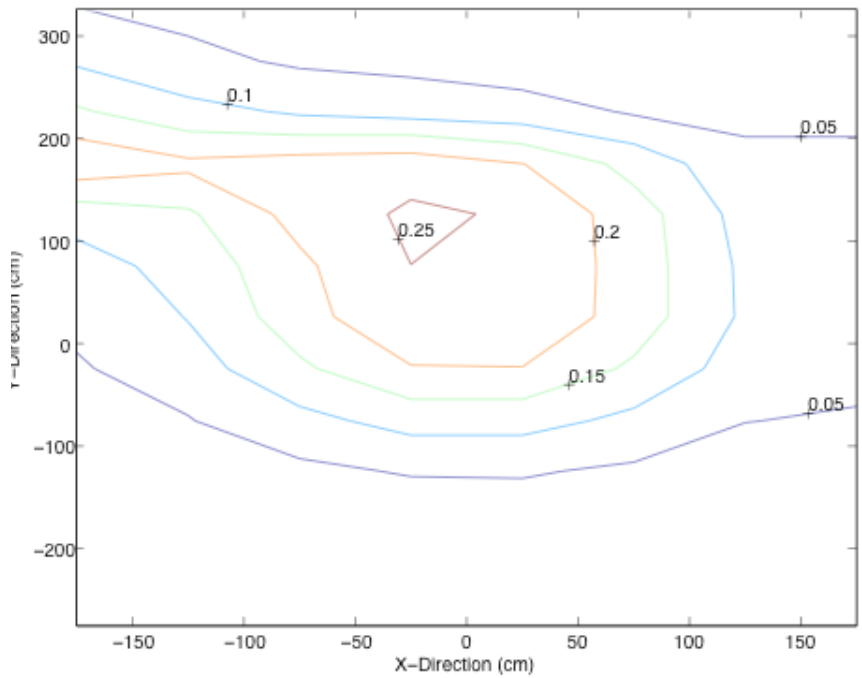
- Sajo et al (2002) Health Physics. 83: 871-883
- Repeated release of 10g of cobalt oxide in room 4 m by 6.6 m by 10.4 m
- Collected 36-55 samples on floor/walls per release
- Measured velocity field (quiescent, <0.25 cm/s)

Simulation I Results

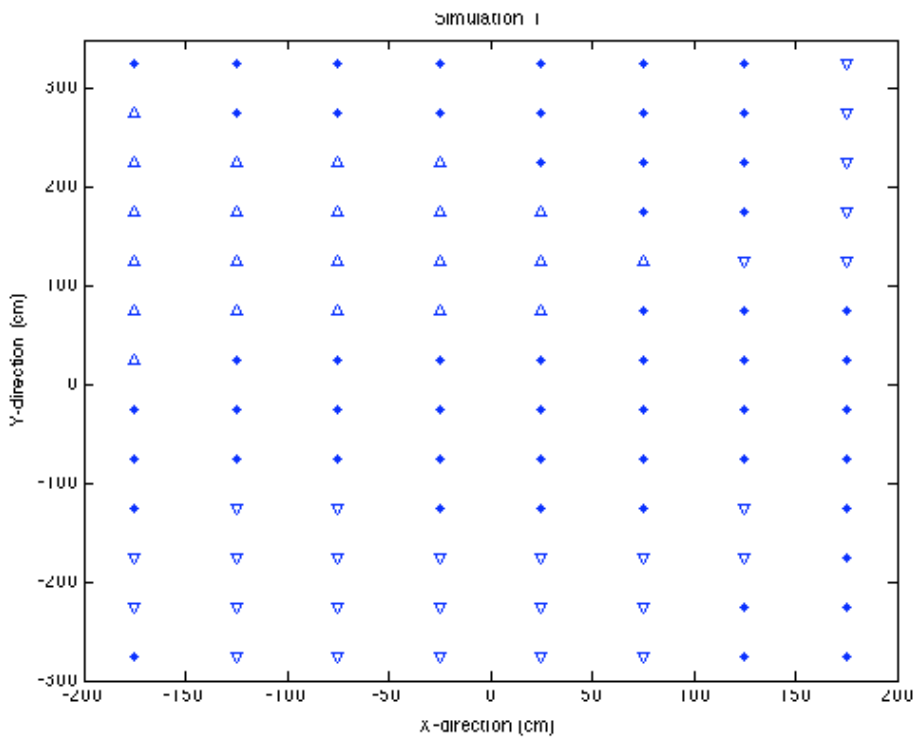
Experiment



Model



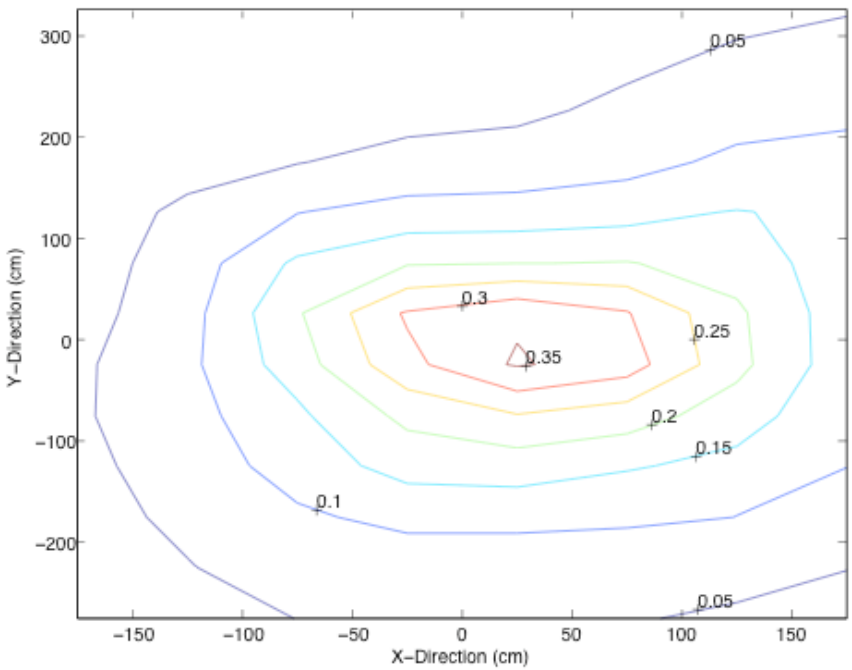
Simulation I Results



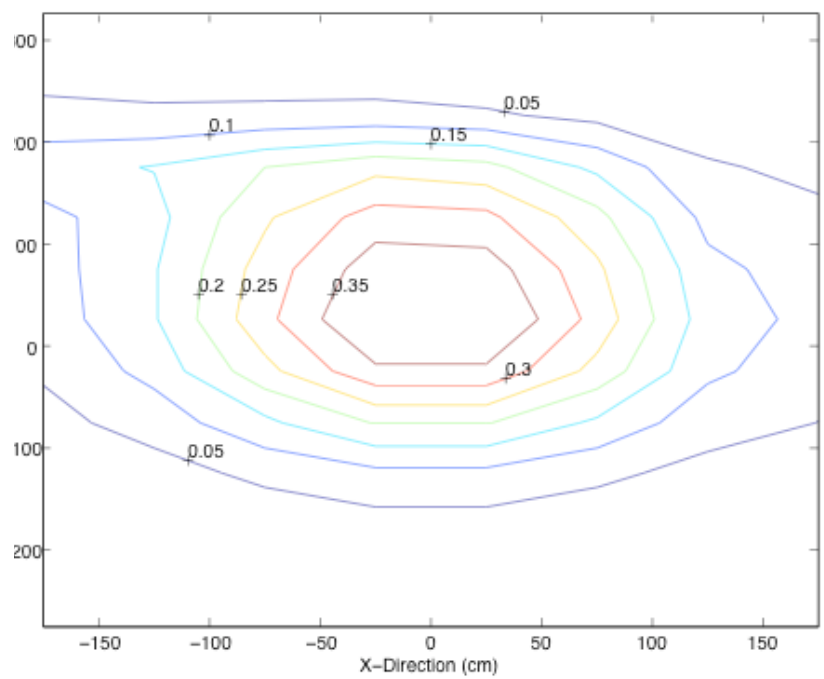
- 8.26 g predicted to deposit on floor
- 1.07g in air
- 55/104 (53%) floor areas within 95% CI of interpolated data;
- 22 (21%) >95%CI
- 27 (26%) <95%CI

Simulation II Results

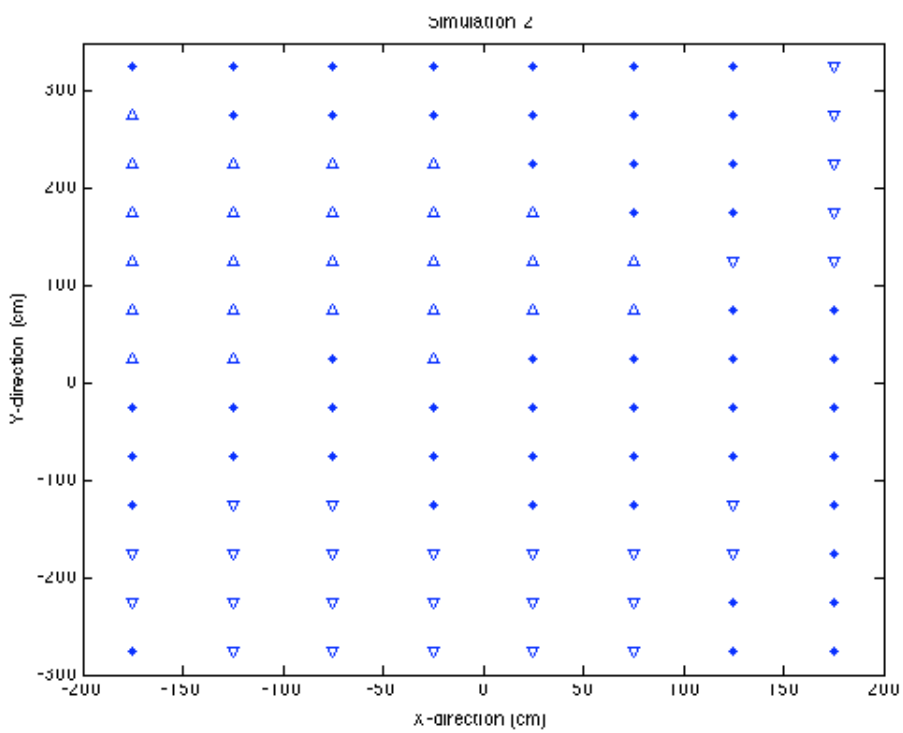
Experiment



Model

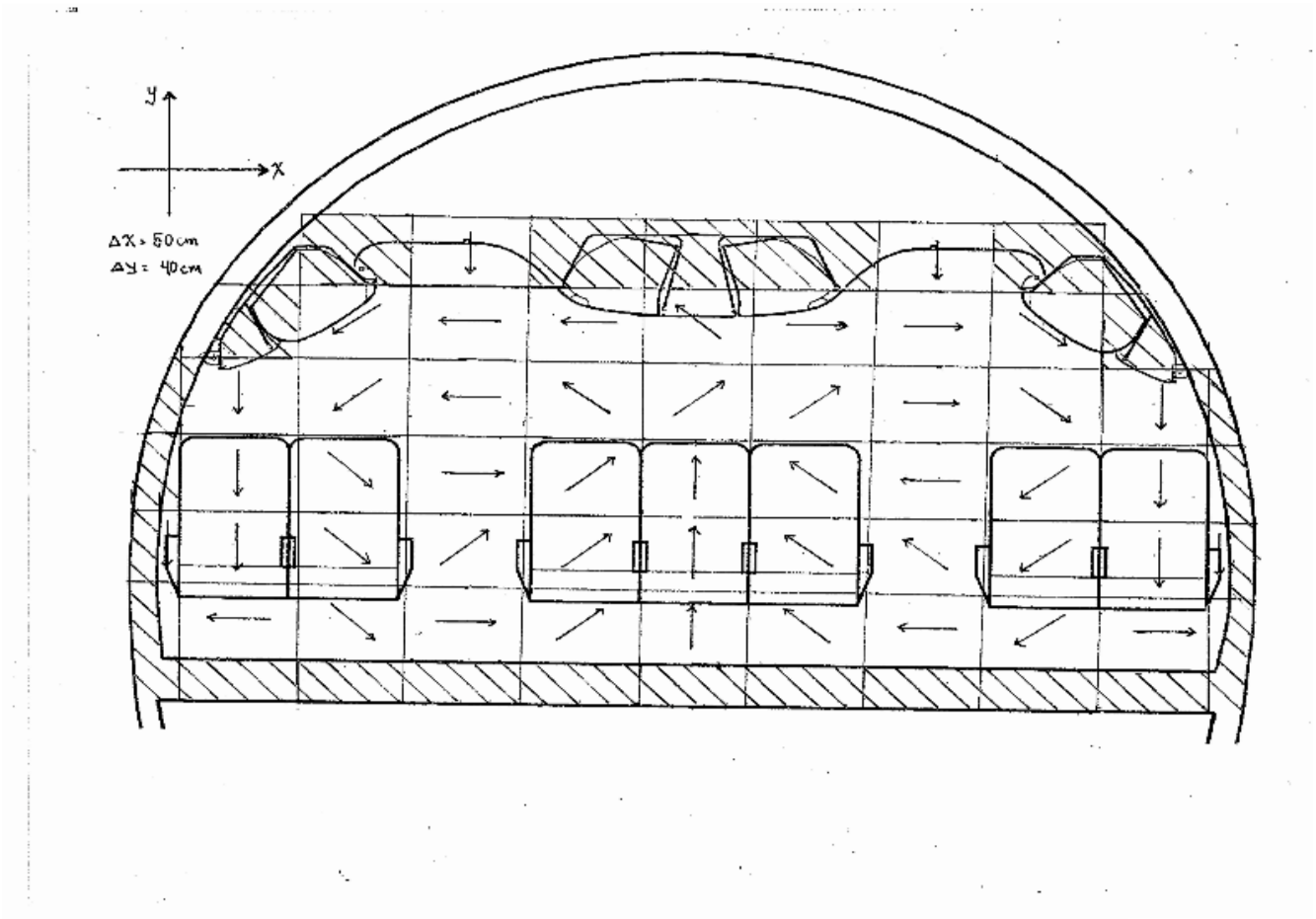


Simulation II Results



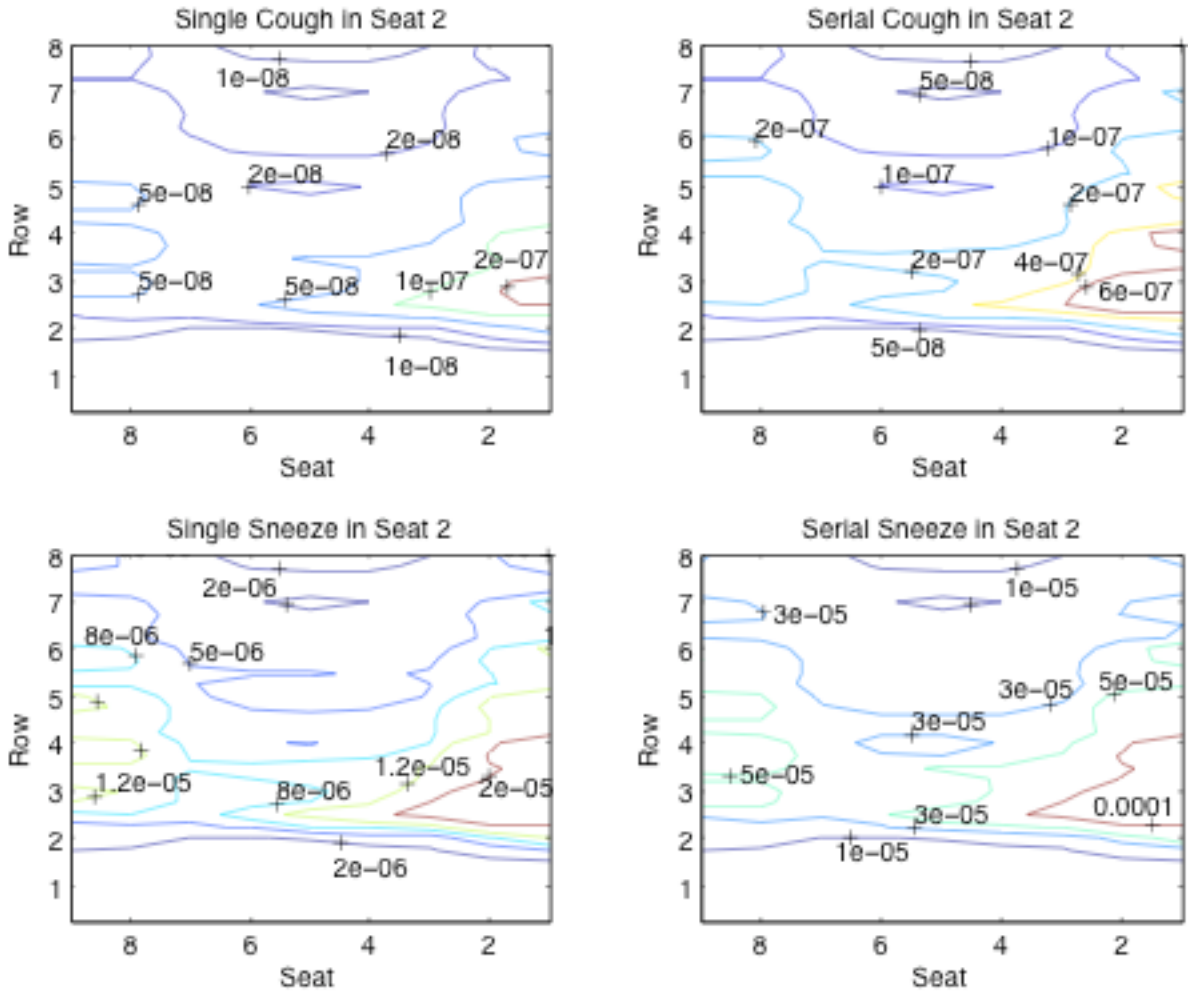
- 9.77 g predicted to deposit on floor
- 0.0007 g in air
- 53/104 (51%) within 95%CI of interpolated data
- 25% >95%CI
- 26% <95%CI

Boeing 767-300 Airplane



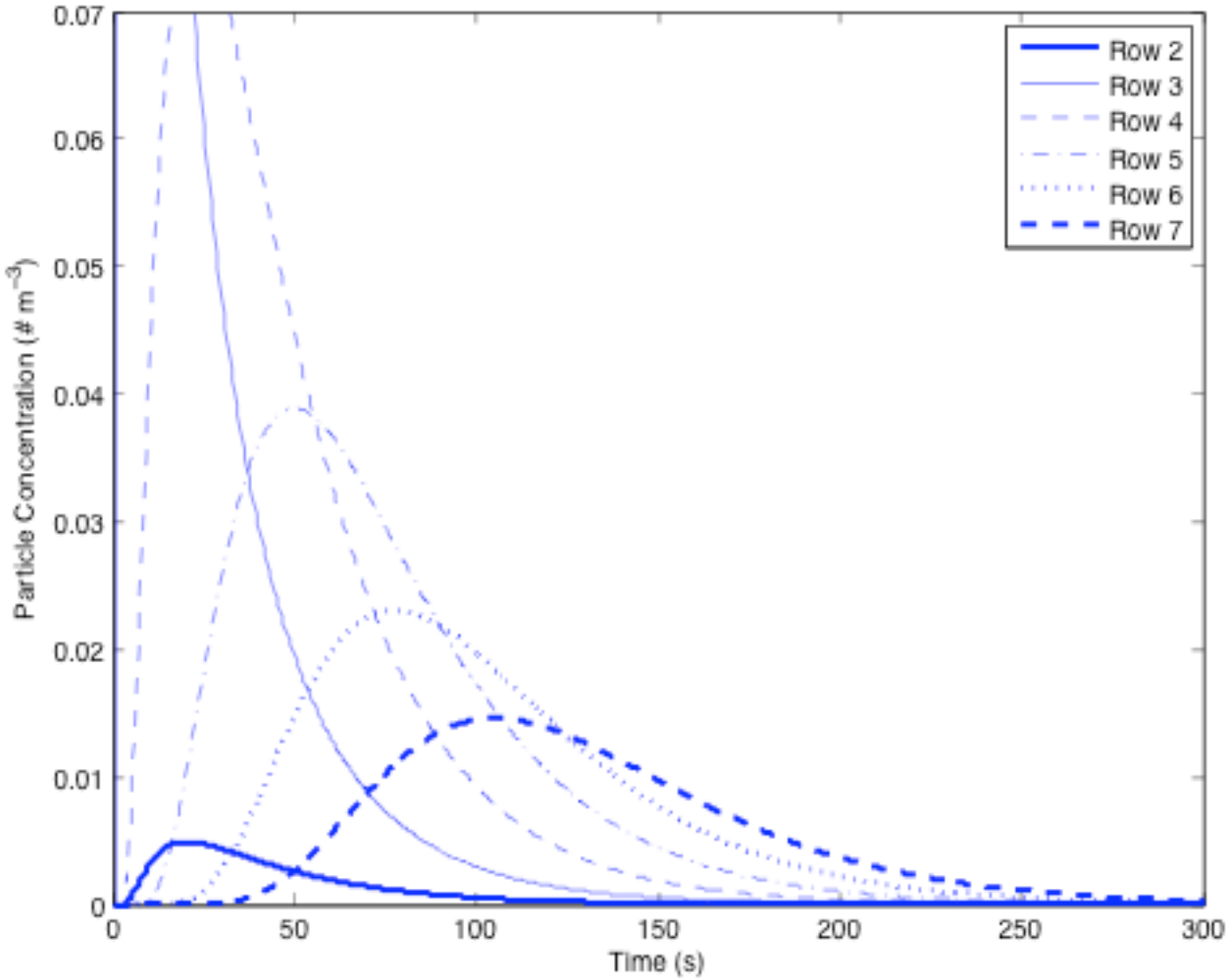
Boeing 767-300 Airplane

Contours of airborne concentrations, or expected inhaled dose (influenza A dose in 5 min)



Boeing 767-300 Airplane

Particle concentration as a function of time, in aisles at standing height



The Next Steps…

- Particle release experiments in controlled environments: the Water Village at the University of Arizona, Tucson
- Integration with CFD
- Application to microbial risk assessment

Acknowledgements

Mark Nicas, UCB
Bill Nazaroff, UCB
Mark Stacey, UCB
Chris Choi, UA
Pedro Romero, UA
Chuck Gerba, UA
Christine Sofge, NIOSH

CAMRA
Center for Advancing Microbial Risk Assessment

This research is funded by
U.S. EPA - Science To Achieve Results (STAR) Program and U.S. Department of Homeland Security University Programs
Grant # R83236201